Adding Charges to $N = 4$ Dyons

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Outline

Generalities

$N = 4$ models

- Spectrum of quarter BPS dyons in a class of four dimensional $N = 4$ supersymmetric string theories is reasonably well understood by now.
- These include toroidally compactified heterotic string theory, CHL models as well as asymmetric orbifolds of type II string theories.
- Method used for counting considers a specific description of quarter BPS dyons. (Dijkgraaf-Verlinde-Verlinde, Strominger-Shih, David-Jatkar-Sen, Dabholkar-Gaiotto-Nampuri)

Generalities contd..

Motivation

- This description does not give the most general charge vector corresponding to a quarter BPS dyon.
- While we expect the same counting formula to work for more general quarter BPS dyons, it is important to understand exactly how it works.
- Aim of our work is to consider a more general charge vector and see how the degeneracy formula takes these dyons into account.
- In what follows we will illustrate our results in terms of heterotic string compactified on T^6.

Two Descriptions

First Description

We consider type IIB string theory on $K3 \times S^1 \times \tilde{S}^1$. We call this to be the first description of the theory. A dyon is a specific brane configuration in this theory.

Second Description

We consider heterotic string theory on $T^4 \times S^1 \times \hat{S}^1$, we call this to be the second description of the theory. A dyon in this theory is denoted by a certain electric and magnetic charge vector.

These two descriptions are related by a chain of duality transformations.

Duality chain

$$\begin{pmatrix} IIB \\ K3 \times \tilde{S}^1 \end{pmatrix} \xrightarrow[S]{} \begin{pmatrix} IIB \\ K3 \times \tilde{S}^1 \end{pmatrix} \xrightarrow[T]{} \begin{pmatrix} IIA \\ K3 \times \hat{S}^1 \end{pmatrix} \xrightarrow[SS]{} \begin{pmatrix} Heterotic \\ T^4 \times \hat{S}^1 \end{pmatrix}$$

- Given any field configuration in the first description, we can follow the duality chain and find the corresponding field configuration in the second description.

Charge vector

- The compactified theory has 28 U(1) gauge fields.
- Any given state is characterised by 28 dimensional electric and magnetic charge vectors, $\vec{Q}$ and $\vec{P}$.

$$Q = \begin{pmatrix} \hat{Q} \\ k_1 \\ k_2 \\ k_3 \\ k_4 \\ k_5 \\ k_6 \end{pmatrix} \qquad P = \begin{pmatrix} \hat{P} \\ l_1 \\ l_2 \\ l_3 \\ l_4 \\ l_5 \\ l_6 \end{pmatrix}$$

T-duality

- T-duality symmetry is $SO(6, 22; Z)$, and the T-duality invariants are,

$$Q^2 = Q^T LQ \qquad P^2 = P^T LP \qquad Q.P = Q^T LP,$$

where L is a 28×28 matrix with $(6, 22)$ signature.

Special Charge vector

- The degeneracy formula was computed using the charge vectors (David,Sen)

$$Q = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ k_4 \\ 0 \\ k_6 \end{pmatrix} \qquad P = \begin{pmatrix} 0 \\ 0 \\ 0 \\ l_3 \\ l_4 \\ l_5 \\ 0 \end{pmatrix}$$

- They found the degeneracy of the states of the system as ,

$$d(Q,P) = f(\frac{Q^2}{2}, \frac{P^2}{2} + 1, Q.P)$$

Classification

- We shall use the second description of the theory to classify charges as electric or magnetic.
- An electrically charged state will correspond to an elementary string state.
- A magnetically charged state will correspond to a wrapped NS 5 brane or KK monopole.
- Electric/Magnetic charges appearing in the charge vector are related to the asymptotic values of the gauge field strength as,

$$F^i_{rt}|_\infty = \frac{k_i}{r^2} \qquad F^i_{\theta\phi}|_\infty = l_i sin\theta$$

Conventions Contd..

Normalisations

- Coordinates ψ, y, χ are along $\tilde{S}^1, S^1, \hat{S}^1$ directions respectively and are normalised to $2\pi\sqrt{\alpha'}$.
- K3 volume is normalised to $(2\pi\sqrt{\alpha'})^4$.
- We choose $\alpha' = 16$.

New Charges

Electric charge vector

- The elements of the 28-dimensional electric charge vector can be given the following interpretation in the second description.
  - $k_3, k_4 \quad \longrightarrow$ momenta along $\hat{S}^1$ and S^1 respectively,
  - $k_5, k_6 \quad \longrightarrow$ fundamental string winding along $\hat{S}^1$ and S^1 respectively,
  - $\hat{Q}, k_1, k_2 \longrightarrow$ momenta or winding charges along the internal directions.

Magnetic charge vector

- The elements of the magnetic charge vector can be given the following interpretation in the second description,
  - $l_3, l_4 \quad \longrightarrow$ number of NS 5 branes wrapped along $\hat{S}^1 \times T^4$ and $S^1 \times T^4$ respectively.
  - $l_5, l_6 \quad \longrightarrow$ KK-monopole charges associated with $\hat{S}^1$ and S^1 respectively,
  - $\hat{P}, l_1, l_2 \longrightarrow$ monopole charges associated with the internal directions.

Interpretation in the first description

Electric charge vector

- Following the duality chain we can interpret the electric charges in the first description as follows,
  - k_3 -> D-string winding along $\tilde{S}^1$,
  - k_4 -> momentum along S^1,
  - k_5 -> D5 brane charge along $\tilde{S}^1 \times K3$,
  - k_6 -> number of KK-monopole associated with $\tilde{S}^1$,
  - k_1 -> fundamental IIB string winding charge along $\tilde{S}^1$,
  - k_2 -> number of NS 5 brane wrapped along $\tilde{S}^1 \times K3$,
  - $\hat{Q}$ -> D3 brane charges wrapped along 22 two-cycles of K3 and $\tilde{S}^1$.

Interpretation in the first description

Magnetic charge vector

- Following the duality chain we can interpret the magnetic charges in the first description as follows,
  - l_3 -> D-string winding along S^1,
  - l_4 -> momentum along $\tilde{S}^1$,
  - l_5 -> D5 brane charge along $S^1 \times K3$,
  - l_6 -> number of KK-monopole associated with S^1,
  - l_1 -> fundamental IIB string winding charge along S^1,
  - l_2 -> number of NS 5 brane wrapped along $S^1 \times K3$,
  - $\hat{P}$ -> D3 brane charges wrapped along 22 two-cycles of K3 and S^1.

S-duality

- The electric - magnetic duality in the second description corresponds to a geometric transformation $\tilde{S}^1 \longleftrightarrow S^1$ in the first description.

Original configuration

- The original configuration studied by David and Sen has charge vectors,

$$Q = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -n \\ 0 \\ -1 \end{pmatrix} \qquad P = \begin{pmatrix} 0 \\ 0 \\ 0 \\ Q_1 - 1 \\ -J \\ 1 \\ 0 \end{pmatrix}$$

- The T-duality invariants are,

$$Q^2 = 2n \qquad P^2 = 2(Q_1 - 1) \qquad Q.P = J$$

- The field configuration in the first description is,
  - $-n$ units of momentum along S^1 and J units of momentum along $\tilde{S}^1$,
  - A single KK-monopole associated with $\tilde{S}^1$, a single D5 brane wrapped on $K3 \times S^1$ and Q_1 number of D1 branes wrapped along S^1 direction.
- A D5 brane wrapped on $K3 \times S^1$ also carries -1 unit of D1 brane charge along S^1.

How can we generalise the charge vectors?

- We shall do this by adding charges to the existing system by exciting appropriate collective modes of the system.
- It is easiest to study this in the first description of the theory.
- There are 3 sources of the collective modes, excitations of KK monopole, and two different types of flux configurations on D5 brane.

Source 1: Excitations of KK monopole

- We have a KK-monopole associated with $\tilde{S}^1$ in IIB. The solution is,

$$\begin{aligned} ds^2 &= \left(1+\frac{K\sqrt{\alpha'}}{2r}\right)\left(dr^2+r^2(d\theta^2+\sin^2\theta d\phi^2)\right) \\ &+ K^2\left(1+\frac{K\sqrt{\alpha'}}{2r}\right)^{-1}\left(d\psi+\frac{\sqrt{\alpha'}}{2}\cos\theta d\phi\right)^2 \end{aligned}$$

- This geometry is known as Taub-NUT space.This space is transverse to the KK-monopole world volume $K3\times S^1\times t$.

- TN geometry admits a self-dual normalizable harmonic 2 form, ω,

$$\begin{aligned}\omega &\propto \frac{2}{\sqrt{\alpha'}}\frac{r}{r+\frac{1}{2}K\sqrt{\alpha'}}d\sigma_3+\frac{K}{(r+\frac{1}{2}K\sqrt{\alpha'})^2}dr\wedge\sigma_3\\ \sigma_3 &\equiv \left(d\psi+\frac{\sqrt{\alpha'}}{2}\cos\theta d\phi\right).\end{aligned}$$

- Type IIB in ten dimensions has following form fields,
  - 2 form fields: B and $C^{(2)}$,
  - 4 form field $C^{(4)}$, which after reducing along 2 cycles of K3 gives additional 2 form fields.
- Given such a 2 form field C_{MN}, we introduce a scalar field ϕ,

$$C = \phi(y, t)\omega,$$

where y is along S^1.

- We consider configuration carrying momentum conjugate to this scalar field OR winding number along y of this scalar field .
  $\Longrightarrow$ represented by solutions where ϕ is linear in t or y. So, we need,

$$dC \propto dt \wedge \omega \qquad dC \propto dy \wedge \omega$$

  - First one has a component proportional to $dt \wedge dr \wedge d\psi$, so it represents strings electrically charged under C, wrapped along $\tilde{S}^1$,
  - Second one has a component proportional to $dy \wedge d\theta \wedge d\phi$, so it represents strings magnetically charged under C wrapped along $\tilde{S}^1$.

Deformations

C	State
RR	D1 brane $\tilde{S}^1$ (k_3)
RR	D5 brane wrapped on $K3 \times \tilde{S}^1$ (k_5)
NSNS	fundamental string of IIB (k_1)
NSNS	NS5 brane wrapped on $K3 \times \tilde{S}^1$ (k_2)
$C^{(4)}$	D3 brane along 2 cycles of $K3 \times \tilde{S}^1$ $(\hat{Q})$

- In this way, we can produce the entire electric charge vector from these collective modes.
- k_2 represents NS5 brane charge wrapped along $k3 \times \tilde{S}^1$.
- For a weakly coupled IIB theory, this can produce a large backreaction to the geometry, therefore we set $k_2 = 0$.

Source 2: Gauge Field Fluxes on D5 Brane

- The original configuration also contains one D5 brane wrapped along $K3 \times S^1$.
- We can turn on flux $\mathcal{F}$ of world volume gauge fields on D5 brane through various 2-cycles of K3.
- The coupling of RR fields to D5 brane is,

$$\int \left[C^{(6)} + C^{(4)} \wedge \mathcal{F} + \frac{1}{2} C^{(2)} \wedge \mathcal{F} \wedge \mathcal{F} + \cdots \right],$$

  the integral is over the D5 brane world volume which spans (t,y,K3).

- The coupling $\int C^{(4)} \wedge \mathcal{F}$ gives charges of the D3 branes wrapped along the 2-cycles of K3 and S^1.
- These are the 22 dimensional $\hat{P}$ sitting in magnetic charge vector $\vec{P}$ and the gauge field strength is proportional to,

$$\mathcal{F} \propto \sum \hat{P}_\alpha \Omega_\alpha,$$

Ω_α is the 2-cycle of K3.

Source 3: Electric Flux along S^1

- D5 brane can also carry electric flux along S^1 direction.
- This induces charges of a fundamental type IIB strings wrapped along S^1 direction.
  This gives charge l_1 to the charge vector.
- We do not produce any deformation of our original configuration which produces l_2 or l_6, so these two charges are zero.
- So, we get the magnetic charge vector.

Deformed charge vector

- So, we finally get the deformed charge vector as,

$$Q = \begin{pmatrix} \hat{Q} \\ k_1 \\ 0 \\ k_3 \\ k_4 \\ k_5 \\ k_6 \end{pmatrix} \qquad P = \begin{pmatrix} \hat{P} \\ l_1 \\ 0 \\ l_3 \\ l_4 \\ l_5 \\ 0 \end{pmatrix}$$

- It is clear that the value of Q^2, P^2 and $Q.P$ changes.

Additional shift in charge vector

- We need to consider the effect of the interaction of these deformations produced by the collective modes with:
  - the background fields already present in the system, and
  - the background fields produced by them, i.e. among themselves.
- This produce further shifts in the charge vector. While first effect is linear in deformations, second one has a quadratic dependence on new charges.

Electric charge vector

- Let $C^{(2)}$ be any 2 form field in the 6D theory (IIB on K3) and $F = dC$ is the field strength.
- Switching on various components of electric charge vector $\vec{Q}$ requires F to be proportional to $dt \wedge \omega$ or $dy \wedge \omega$. This gives a coupling,

$$-\int \sqrt{-\det g}\, g^{yt} F_{ymn} F_t{}^{mn},$$

  where m,n are TN space indices.
- This produces a source for g^{yt}, momentum along S^1 (k_4).

Shift

- Thus, we get a shift in k_4 quantum number given as,

$$\Delta k_4 = k_3 k_5 + \hat{Q}^2/2, \qquad k_2 = 0$$

  - $k_3 k_5$ comes from RR 2-form field.
  - $\hat{Q}^2/2$ comes from RR 4-form field reduced along 2-cycles of K3.

- There is no other shifts in the electric vector.

Magnetic charge vector

1.

- D5-brane wrapped on $K3 \times S^1$ OR the magnetic flux on this brane along any of the 2-cycles of K3 can produce a magnetic 2 form field with field strength,

$$F \equiv dC \propto \sin\theta \, d\psi \wedge d\theta \wedge d\phi,$$

  C is any 2-form field of 6D.
- In TN background F satisfies equation of motion, $dF = 0$ and Bianchi identity $*dF = 0$.
- For various 2-form fields, the coefficient of this term is either proportional to 1, as we have taken only one D5-brane OR $\hat{P}$.

- This field strength together with the field strength coming from KK-monopole excitations, $F \propto dt \wedge \omega$ generate a source for the component $g^{\psi t}$ via the coupling,

$$-\int \sqrt{-\det g} g^{\psi t} F_{\psi mn} F_t{}^{mn}.$$

Shift

- This shifts momentum along $\tilde{S}^1$, i.e. l_4 quantum number as,

$$\Delta l_4 = k_3 + \hat{Q}.\hat{P}.$$

  - k_3 comes from F of RR 2-form field of IIB,
  - $\hat{Q}.\hat{P}$ comes from F of RR 4-form field of IIB along various 2-cycles of K3.

2.

- In D5-brane world volume theory, we also had a coupling proportional to,

$$\int C^{(2)} \wedge \mathcal{F} \wedge \mathcal{F}.$$

- It acts as a source for D1-brane charge wrapped along S^1.

Shift

- So, it shifts the corresponding quantum number l_3 by an amount quadratic in $\mathcal{F}$ or $\hat{P}$,

$$\Delta l_3 = -\hat{P}^2/2.$$

3.

- shifted l_3 quantum number is,

$$l_3 = Q_1 - 1 - \hat{P}^2/2.$$

- Hence, the charge of the D1-brane is shifted and the corresponding 2-form field $C^{(2)}$ is :

$$dC^{(2)} \propto (Q_1 - 1 - \hat{P}^2/2)\, r^{-2}\, dr \wedge dt \wedge dy\,,$$

  the right hand side is both closed and co-closed.
- We also have a component from the excitation of the collective coordinate of KK-monopole,
  $dC^{(2)} \propto k_5 \wedge dy \wedge \omega.$

- Combining these two field strengths, we get a source term for $g^{\psi t}$ via the coupling,

$$-\int \sqrt{-\det g}\, g^{\psi t} F_{\psi r y} F_{t}{}^{ry}.$$

Shift

- This produce an additional shift in l_4 as,

$$\Delta l_4 = k_5(Q_1 - 1 - \hat{P}^2/2).$$

These are all possible shifts in the magnetic charge vector.

Shifted charge vector

- Combining all these, the final charge vectors are,

$$\begin{pmatrix} \hat{Q} \\ k_1 \\ 0 \\ k_3 \\ -n + k_3 k_5 + \frac{\hat{Q}^2}{2} \\ k_5 \\ -1 \end{pmatrix}, \begin{pmatrix} \hat{P} \\ l_1 \\ 0 \\ Q_1 - 1 - \hat{P}^2/2 \\ -J + k_3 + \hat{Q}\cdot\hat{P} + k_5(Q_1 - 1 - \frac{\hat{P}^2}{2}) \\ 1 \\ 0 \end{pmatrix}.$$

- This has, $Q^2 = 2n, \quad P^2 = 2(Q_1 - 1)$ and $Q.P = J$.
- $d(Q, P) = f(\frac{1}{2}Q^2, \ \frac{1}{2}P^2 + 1, \ Q.P)$ also does not change.

Dyon spectrum

Question

- Can we still justify d(Q,P) as the Dyon degeneracy function of the system ?

- Dyon spectra was computed from three mutually non-interacting parts,
  - dynamics of KK-monopole,
  - overall motion of D1-D5 system in the background of KK-monopole,
  - motion of D1-branes relative to D5-branes.
- Q. Can the additional charges from fluxes on D5-brane and winding and momenta of collective excitation of KK-monopole affect this analysis ?

a.

- The KK-monopole dynamics is not affected.

b.

- In the weakly coupled IIB theory, the overall motion of the D1-D5 system in KK background is also not affected. In weak coupling limit, the additional background fields due to additional charges are small compared to the one due to KK-monopole.

  For this it is important to keep $l_2 = l_6 = 0$.

c.

- The precise dynamics of D1-branes relative to D5-brane is affected by the presence of the gauge field flux on the D5-brane.
- However, in the degeneracy formula what enters is the elliptic genus of the corresponding conformal field theory. This does not change due to the fluxes.

Conclusion

- Using collective excitations we can turn on more charges on a dyon.
- Supergravity is clever enough to understand the effects of these new charges.
- New excitations adjust original dyon charges, which ensures consistency of the degeneracy formula for general charge vectors.