FREE FERMIONS + AdS/CFT - I (w/ SUVANKAR DUTTA) - HRI (Oct. '07)

- INTRODUCTION + MOTIVATION
- REVIEW OF THERMAL GAUGE THEORY ON $S^1 \times S^3$.
- OVERVIEW OF RESULTS

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① INTRODUCTION + MOTIVATION:

- Gauge theory known to be a holographic description in many cases. But How does it encode a local diff. invt. theory in one higher dim? Is there a natural way in the gauge theory to restore the redundancies which characterise geometrical description of the bulk?

Partial hint to an answer: Work of LLM: Geometry of a class of ½BPS solns (asymp. $AdS_5 \times S^5$) determined by a fn. $u(x_1, x_2)$ s.t. $u = 1$, $(x_1, x_2) \in R \subset \mathbb{R}^2$

(x_1, x_2) identified w/ phase space of free fermions. Geometry encoded in Shape R. ~~In fact~~, configuration space of SUGRA ~~becomes~~ identified w/ phase space of bdy. theory d.o.f.

Note that the phase space descr. is also redundant - area preserving diffeomorphism - dynamics really on boundary (Holography!).

Of course, ½ BPS special. Not clear how this generalises to other situations.

Here, however, for a non-SUSY context of finite temp. gauge theory will see a free fermionic phase space picture emerge. Can therefore hope that this may be the right description to holographically reconstruct the bulk ~~theory~~ geometry. (Precedent also of $C=1$ string theory).

~~Concretely~~ Technically, this comes about in the following route.

- Finite temp. gauge theory on S^3 can be pert. descr. by a unitary matrix model.
- Exact solution for these matrix models at finite N.
- Take large N limit in a novel way. (like 2D Yang-Mills)

Review of Thermal AdS/CFT

Consider (Euclidean) gauge theory on $S^3 \times S^1$ (radius β) in the limit as $\lambda \to 0$.
Because of the Kaluza-Klein redn. on S^3 - most modes are massive except for $\alpha = \frac{1}{V_{S^3}} \int_{S^3} A_0$ — zero mode.

Can integrate out all other (massive) fields to obtain

$$Z_{YM} = e^{-\beta F_{YM}} = \int [DU] e^{-S_0[U]} \qquad (U = e^{i\beta\alpha}) \hookrightarrow \text{Polyakov loop.}$$

$$S_0[U] = \sum_{m=1}^{\infty} \frac{1}{m} a_m^{(T)} (\mathrm{Tr}\, U^m)(\mathrm{Tr}(U^+)^m) \qquad \text{(Sundborg, Aharony et. al. Alvarez-Gaume et. al)}$$

where the coeff. $a_n(T)$ are determined by the field content of the theory.

This was as $\lambda \to 0$.

For non-zero λ, can continue to integrate out the massive modes to get an eff. action $S_{eff}[U]$.

$$S_{eff}[U] = \sum_{\{n_i\}} \underbrace{a_{\{n_i\}}(\lambda, T, N)}_{\text{can be computed in pert. theory.}} \prod_{i=1}^{k} \left(\frac{1}{N} \mathrm{Tr}\, U^{n_i}\right) \qquad (\text{w/ } \textstyle\sum n_i = 0).$$

Further simplification: The order parameter for the finite temperature phases is $\mathrm{Tr}\, U$ - the mode which condenses. So can imagine integrating out $\mathrm{Tr}\, U^n$ $(n \neq \pm 1)$ and getting an action in terms of $(\mathrm{Tr}\, U \, \mathrm{Tr}\, U^+)$.

So can consider

$$\int [DU] e^{N^2 S_{eff}[U]}$$

$$\text{w/ } S_{eff}(U) = a_1(\lambda, T)\, \mathrm{Tr}\, U \, \mathrm{Tr}\, U^+ + \frac{b_1(\lambda, T)}{N^2} (\mathrm{Tr}\, U \, \mathrm{Tr}\, U^+)^2 + \cdots$$

[$S(x)$ convex and $S'(x)$ concave].

These are very "effective" effective actions - in particular, the (a, b) model w/ only $(a_1, b_1) \neq 0$ studied as a toy model.

Can take the large N limit of these models and analyse using eigenvalue density. $\sigma(\theta) = \frac{1}{N}\sum_{i=1}^{N}\delta(\theta-\theta_i)$. where $U = \mathrm{diag}(e^{i\theta_i})$.

We have a functional $S[\sigma(\theta)] = \int d\theta_1 d\theta_2\, \sigma(\theta_1)\sigma(\theta_2) V(\theta_1-\theta_2)$ (for $\lambda = 0$ case)

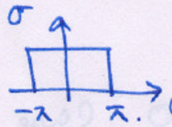
w/ $V(\theta) = C) + \sum_{n=1}^{\infty} \frac{1}{n}(1-a_n(T))\cos n\theta$

Find essentially 3-saddle points to $\frac{\delta S}{\delta\sigma} = 0$.

In fact, can consider models e.g. (a,b) model and it has a very detailed phase diagram.

(a) $T < T_0$.

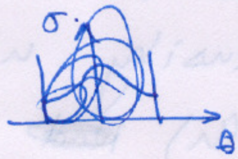
$\sigma(\theta) = \frac{1}{2\pi}$



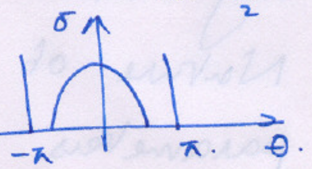
– Only saddle pt. (Thermal AdS)

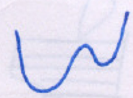
(b) $T_0 < T < T_H$ – 2 new saddle pts. (1 stable and one unstable)

~~$\sigma(\theta) = \frac{1}{2\pi}(1+2\xi\cos\theta)$~~



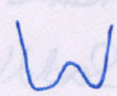
$\sigma(\theta) = \frac{1}{\pi\sin^2\frac{\theta_0}{2}}\sqrt{\sin^2\frac{\theta_0}{2} - \sin^2\frac{\theta}{2}}\,\cos\frac{\theta}{2}$





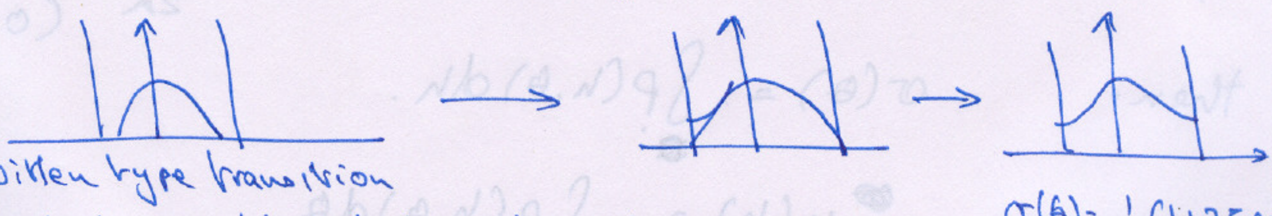
w/ two different values of θ_0 – SBH + BBH.

(c) For some $T_1 > T_0$, the stable saddle above (BBH) has lower free energy than Thermal AdS.



– Hawking-Page transition

(d) Also for some $T_c > T_0$, the unstable saddle above goes from gapped to ungapped saddle pt.



Gross-Witten type transition

Interpreted as black hole–string transition.

$\sigma(\theta) = \frac{1}{2\pi}(1+2\xi\cos\theta)$

$\xi \leq 1/2$.

② For $T > T_H$, the unstable saddle merges w/ Thermal AdS saddle and becomes tachyonic. — Hagedorn temp.

~~PB~~ Quite remarkable how it generically reproduces details of Thermal history in AdS space. Generalises non-trivially to charged case. (Basu + Wadia). — there is no Thermal AdS saddle point.

Overview Of Our Results

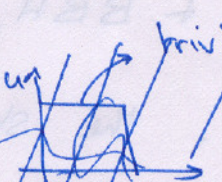
- Give an exact solution at any N of the matrix model (as a sum over representations.

$$\sum_{k}\sum_{R} \frac{a_1^{k}}{k!}\, d_R^2(S_k) \qquad - \quad a_1 \neq 0 \text{ case.}$$

- In the $N\to\infty$ limit, get a dominant repn. R_0. — characterised by a Young tableau density $u(h)$ — profile of Young tableau

. Nature of $u(h)$ ~~had~~ changes qualitatively as parameters varied.

① $n\{$  $n<N$. or ② N 



Reproduce the same thermal history — not surprising

~~In terms of u(h)~~

— Saddle pt. values of $u(h)$ and $\sigma(\theta)$ have a remarkable relation — they are ~ functional inverses of each other.

— In fact can define $\rho(h,\theta) = \frac{1}{2\pi}$ in region R ~~or 0~~ (0 outside)

then $\sigma(\theta) = \int \rho(h,\theta)\,dh$.

$u(h) = \int \rho(h,\theta)\,d\theta$.

Thus we have a phase space picture in terms of free fermions. — specifying a region R ~~which is~~ for each of the saddle points — Thermal AdS, SBH, BBH etc.