PERTURBATIVE STUDY OF THE LEIGH-STRASSLER DEFORMED $\mathcal{N}=4$ SYM

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- K. Madhu and S. Govindarajan, JHEP **0705** (2007) 038 [arXiv:hep-th/0703020].
- K. Madhu and S. Govindarajan, *To appear.*

Introduction

- The AdS/CFT correspondence as a gauge theory-string theory duality, was conjectured for $\mathcal{N} = 4$ supersymmetric Yang-Mills theory
- This Leigh-Strassler theory is a marginal deformation of $\mathcal{N} = 4$ SYM is interesting to extend the correspondence to cases with lesser supersymmetry, i.e, $\mathcal{N} = 1$ supersymmetry.
- The gravity dual of this is not yet known in generality, however special cases like the β-deformation is better studied. (Lunin, Maldacena)

Introduction

- Chiral primary states in superconformal gauge theory are particularly useful in studying the AdS/CFT correspondence.
- We construct chiral primary states of the LS theory upto dimension 6.
- These states are found to exist in representations of a discrete symmetry group of the theory, the trihedral group $\Delta(27)$.
- We study the perturbative properties of the LS theory like conformal invariance, holomorphicity and symmetries to higher order.

Leigh-Strassler deformation of $\mathcal{N}=4$ SYM

- Consider the LS deformation of $SU(N)$ $\mathcal{N}=4$ SYM

$$
\begin{aligned}
\mathcal{L} &= \int d^2\theta d^2\bar{\theta}\,\mathrm{Tr}\Big(e^{-gV}\bar{\Phi}e^{gV}\Phi\Big) + \Big[\frac{1}{2g^2}\int d^2\theta\,\mathrm{Tr}\Big(W^\alpha W_\alpha\Big) \\
&+ ih\int d^2\theta\,\mathrm{Tr}\Big(q\Phi_1\Phi_2\Phi_3 - \bar{q}\Phi_1\Phi_3\Phi_2\Big) \\
&+ \frac{ih'}{3}\int d^2\theta\,\mathrm{tr}\Big(\Phi_1^3+\Phi_2^3+\Phi_3^3\Big) + c.c.\Big]
\end{aligned}
$$

- The theory is conformal invariant provided the following condition is obeyed

$$
|h|^2\Big(1 + \frac{1}{N^2}(q-\bar{q})^2\Big) + |h'|^2\frac{N^2-4}{2N^2} = g^2 .
$$

The trihedral group: $\Delta(27)$

- The theory has a symmetry of the trihedral group $\Delta(27) \sim (\mathbb{Z}_3)_R \times \mathbb{Z}_3 \rtimes \mathcal{C}_3$ acts on the fields (Aharony et al.)

$$\begin{aligned} \mathbb{Z}_3 \quad &: \quad \Phi_1 \longrightarrow \Phi_1\,, \Phi_2 \longrightarrow \omega\Phi_2\,, \Phi_3 \longrightarrow \omega^2\Phi_3 \\ \mathcal{C}_3 \quad &: \quad \Phi_1 \longrightarrow \Phi_2 \longrightarrow \Phi_3 \longrightarrow \Phi_1 \end{aligned}$$

  with ω, a non-trivial cube-root of unity and $(\mathbb{Z}_3)_R$ is a sub-group of $U(1)_R$.

- $\Delta(27)$ has nine one-dimensional representations $\mathcal{L}_{Q,j}$ and two three-dimensional representations $\mathcal{V}_1$ and $\mathcal{V}_2$.
- In particular, there are polynomials which exist in the above mentioned representations, which we make use of in our work.

Representations of $\Delta(27)$

- $\Delta(27)$ has nine one-dimensional representations $\mathcal{L}_{Q,j}$ and two three-dimensional representations $\mathcal{V}_1$ and $\mathcal{V}_2$.
- The generators h and τ act on the one-dimensional representation v as follows:

$$h \cdot v = \omega^Q \, v \, , \quad \tau \cdot v = \omega^j \, v \tag{1}$$

where $v \in \mathcal{L}_{Q,j}$. The 'charges' $Q = 0, 1, 2$ and $j = 0, 1, 2$ both are clearly valued modulo three. The singlet corresponds to $\mathcal{L}_{0,0}$ in this notation.

Representations of $\Delta(27)$

- $\Delta(27)$ has nine one-dimensional representations $\mathcal{L}_{Q,j}$ and two three-dimensional representations $\mathcal{V}_1$ and $\mathcal{V}_2$.
- On three-dimensional representation $\mathcal{V}_a$ with $a = 1, 2$

$$h \cdot \begin{pmatrix} v_0 \\ v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^a & 0 \\ 0 & 0 & \omega^{2a} \end{pmatrix} \begin{pmatrix} v_0 \\ v_1 \\ v_2 \end{pmatrix}$$

$$\tau \cdot \begin{pmatrix} v_0 \\ v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} v_0 \\ v_1 \\ v_2 \end{pmatrix}$$

Polynomials as irreps of $\Delta(27)$

- Consider three complex variables (z_1, z_2, z_3) that transform in the three-dimensional representation $\mathcal{V}_1$ of $\Delta(27)$.
- For $\Delta_0 = 0$ mod 3, polynomials are in the one-dimensional representations $\mathcal{L}_{Q,j}$. For example,

$$(z_1^3 + \omega^j z_2^3 + \omega^{2j} z_3^3) \in \mathcal{L}_{0,j} \ ,$$

  with $j = 0$ giving rise to a singlet. Similarly, $z_1 z_2 z_3$ is also a singlet.

Polynomials as irreps of $\Delta(27)$

- Consider three complex variables (z_1, z_2, z_3) that transform in the three-dimensional representation $\mathcal{V}_1$ of $\Delta(27)$.
- For $\Delta_0 \neq 0$ mod 3, polynomials are in the three dimensional representations $\mathcal{V}_a$, where $a = \Delta_0$ mod 3. For example,

$$\begin{pmatrix} z_2 z_3 \\ z_3 z_1 \\ z_1 z_2 \end{pmatrix} \in \mathcal{V}_2 \ .$$

Chiral Primary States

- Chiral primary states have zero anomalous dimensions.
- We will consider states made up of scalar fields.
- The anomalous dimension for such operators can be written as a modulus square.
- We find anomalous dimensions of operators $\mathcal{O}$ diagramatically by computing $\langle \mathcal{O}(x)\bar{\mathcal{O}}(0)\rangle$ at planar ($N \longrightarrow \infty$), one-loop order. This calculation is done in component fields.

Interaction potential LS theory

- In terms of component fields V_F is the F-term potential

$$
\begin{aligned}
V_F \;=\;& \mathrm{Tr}\Big(|h'|^2 \bar{Z}_1{}^2 Z_1^2 + \; h\bar{h}'[Z_2, Z_3]_q \bar{Z}_1{}^2 - \bar{h}h'[\bar{Z}_2, \bar{Z}_3]_q Z_1^2 \\
& -|h|^2 [Z_2, Z_3]_q [\bar{Z}_2, \bar{Z}_3]_q \Big) - \frac{1}{N}\Big[|h'|^2\, \mathrm{Tr}(\bar{Z}_1{}^2)\, \mathrm{Tr}(Z_1^2) \\
& + h\bar{h}'\, \mathrm{Tr}([Z_2, Z_3]_q)\, \mathrm{Tr}(\bar{Z}_1{}^2) - \bar{h}h'\, \mathrm{Tr}(Z_1^2)\, \mathrm{Tr}([\bar{Z}_2, \bar{Z}_3]_q) \\
& + |h|^2\, \mathrm{Tr}([Z_2, Z_3]_q) \left(\mathrm{Tr}[\bar{Z}_2, \bar{Z}_3]_q \right) \Big] + \text{cyclic permutations}
\end{aligned}
$$

- V_F has a double trace interaction which is suppressed by a factor $\frac{1}{N}$. We will see that this is important even in planar ($N \longrightarrow \infty$) limit.

Anomalous dimensions at planar one-loop

- For the simplest candidate operator $\mathcal{O} = \text{Tr}(Z_1^k Z_2^l Z_3^m)$, we compute $\langle \mathcal{O}(x)\bar{\mathcal{O}}(0)\rangle$ to one-loop at large N.
- We observe that the contributions to anomalous dimensions from all the interactions other than V_F vanishes when we impose conformal invariance condition. This is a demonstration of the holomorphicity of the theory. (d'Hoker et al.)
- Dimension $\Delta_0 = 2$ operators get a contribution from the $\frac{1}{N}$ suppressed term in V_F. This cancels exactly with the contributions from the rest of the terms. Hence dimension $\Delta_0 = 2$ operators are always protected.

Chiral primaries of LS theory

- There are no further chiral primaries of LS theory of the above simple form.
- We need to consider linear combinations of permutations of monomials constructed out of operators.
- These operators are in the polynomials representations of trihedral $\Delta(27)$.
- For example, dimension $\Delta_0 = 4$ operator looks like

$$\begin{aligned}\mathcal{O}_4^1 &= \mathrm{tr}\Big(Z_2^4 + b\, Z_1^3 Z_2 + c\, Z_1^2 Z_3^2 + c_1 Z_1 Z_3 Z_1 Z_3 \\ &\quad + d\, Z_1 Z_2^2 Z_3 + \ d_1 Z_1 Z_2 Z_3 Z_2 + d_2 Z_3 Z_2^2 Z_1 \\ &\quad + f\, Z_3^3 Z_2 \Big)\end{aligned}$$

Chiral Primaries of Leigh-Strassler theory

- This operator is in the three dimensional representation $\mathcal{V}_1$ of $\Delta(27)$.
- Calculating the anomalous dimensions and writing it as a modulus square, we find that there exists a unique solution for generic values of couplings to the condition for vanishing of anomalous dimensions. Thus we obtain the chiral primary at dimension $\Delta_0 = 4$.

Chiral Primaries at planar one-loop

- At dimension $\Delta_0 = 6$ we find that there are operators $\mathcal{O}_6^{(0,0)}$ in the $\mathcal{L}_{0,0}$ representation of $\Delta_0 = 4$ that are protected. There are precisely two independent solutions, for generic values of couplings.
- For operators in the $\mathcal{L}_{i,j}$, $i \neq 0$ and/or $j \neq 0$ there are no generic solutions. But solutions exist on specific sub-loci of the coupling space.
- In general, if an operator has dimension $\Delta_0 = a$ mod 3, for generic values of couplings, when $a = 0$ mod 3, there are two independent chiral primaries in the one-dimensional $\mathcal{L}_{0,0}$ representation of $\Delta(27)$. When $a \neq 0$ mod 3, the chiral primaries are in the three-dimensional representation $\mathcal{V}_a$.

Chiral Primaries at planar one-loop

- We can organize the chiral primaries of $\mathcal{N}=4$ SYM, the β-deformed theory as well, since the $\Delta(27)$ symmetry of the LS theory was obtained by breaking down the bigger invariance groups of these theories. For $\Delta_0 > 2$, we have the following conjecture :

Scaling dim.	$\Delta_0 = 3r$	$\Delta_0 = a$ mod 3
$\mathcal{N}=4$ theory	$\mathcal{L}_{0,0} \oplus \frac{r(r+1)}{2}\left[\oplus_{i,j} \mathcal{L}_{i,j}\right]$	$\frac{(\Delta_0+1)(\Delta_0+2)}{6}\ \mathcal{V}_a$
β-def. theory	$\mathcal{L}_{0,0} \oplus_j \mathcal{L}_{0,j}$	$\mathcal{V}_a$
LS theory	$2\mathcal{L}_{0,0}$	$\mathcal{V}_a$

Anomalous dimension of chiral superfi eld

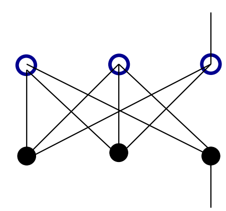
- Using $SU(N)$ gauge invariance and the $\Delta(27)$ symmetry we can argue that γ^I_J must be proportional to δ^I_J.
- The only gauge invariant $SU(N)$ tensor is δ^a_b. Hence gauge invariance of the matrix γ^I_J implies $\gamma^I_J \equiv \gamma^i_j \delta^a_b$.
- $\Delta(27)$ invariance implies $c_{112} = \ldots = 0$ and $c_{111} = c_{222} = c_{333}$
- For the quantum theory to preserve $\Delta(27)$ symmetry, $\beta(c_{112}) = 0$. and $\beta(c_{111}) = \beta(c_{222})$.
- Using the JJN β-function

$$\beta(Y_{IJK}) \sim Y_{LJK}\, \gamma^L_I + Y_{ILK}\, \gamma^L_J + Y_{IJL}\, \gamma^L_K$$

we easily see that $\gamma^i_j \equiv \gamma \delta^i_j$.

Conformal invariance of LS theory

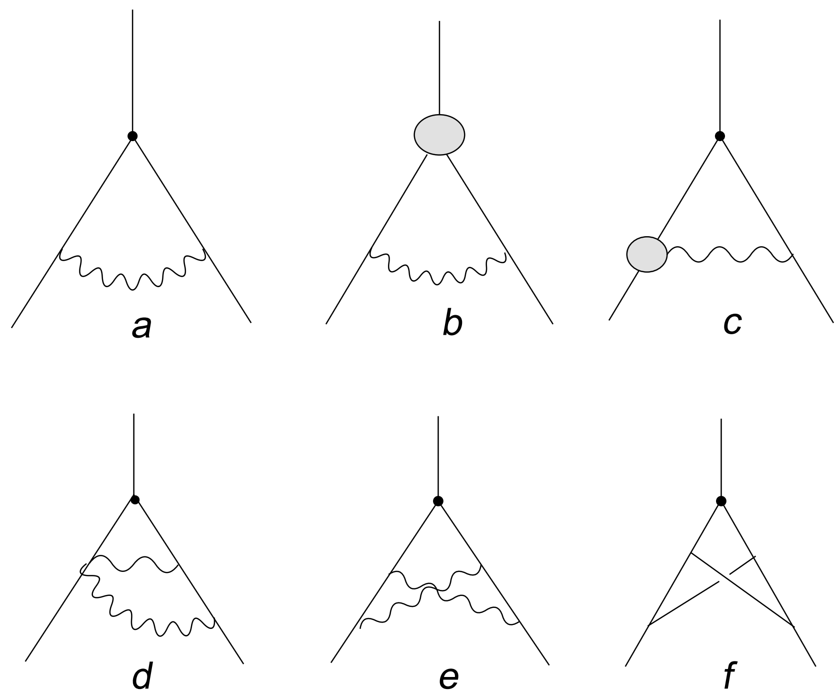
- Hence in LS theory, there is a single condition in the coupling space which ensures marginality of the couplings. This holds true at two-loop too.
- At three-loop the anomalous dimension has a contribution (Jack, Jones, North)



- However, there exists redefinitions of coupling constants that ensure the vanishing of anomalous dimension at three-loop also.

Effective Superpotential

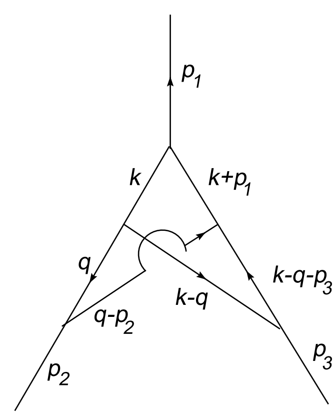
- The contributions from the D-terms to the effective superpotential at two-loop are from the following Feynman diagrams



- The contribution from this is proportional to W once we impose the conformal invariance condition.

Effective Superpotential

- There are no corrections possible at one-loop from chiral interaction vertices.
- Holomorphy restricts the possible diagrams that can contribute to the effective action. The only diagram that gives a non-trivial contribution (that is not proportaional to the classical superpotential)



This is a finite contribution

Effective Superpotential

- This contribution is related to the three-loop anomalous dimension $\gamma^{(3)}$ calculated. It is expected that the coupling constant redefinitions that make $\gamma^{(3)}$ vanish, remove this contribution. Thus the two-loop effective action is expected to be identical to the tree level action
- Hence holomorphy and conformal invariance can be preserved.

Summary of results

- The symmetry group trihedral $\Delta(27)$ of the LS theory helps to classify the chiral primary states with dimension $\Delta_0 > 2$ in its representations, for generic values of the couplings. The multi trace interactions in V_F ensure that dimension 2 operators are always protected
- $\Delta(27)$ is essential in preserving the conformal invariance of the theory. The effective Kähler potential as well as the superpotential shows that $\Delta(27)$ is preserved in quantum theory as well.

Summary of results

- We observe that there are coupling constant redefinitions which ensure vanishing of anomalous dimensions to three-loop, preserving conformal invariance.
- The same coupling constant redefinitions can eliminate two-loop corrections to the superpotential hence also preserving the holomorphicity of the theory.