Mapping G-structures and Supersymmetric vacua of 5 dimensional $\mathcal{N}$=4 supergravity

by

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Motivation

Supergravity solutions are important

- describe low mass d.o.f. of super-string/M theory
- Gauge-gravity correspondence
- Classical solutions such black holes, black rings, p-branes and pp waves

Two popular methods

- Make ansatz for the metric based on isometries.
- Analyzing Killing spinor equations (e.g. finding G-structure).

For large number of supersymmetries

- G-structure method more involved
- New solutions

Simple case: N=4, D=5 supergravity

{Awada and Townsend NPB255(1985)617}

Method applied to N=2, D=5 case by

{Gauntlett, Martelli, Sparks, Waldram: Class. Quan. Grav. **20**(2003)4587}

Generalize method to N=4, D=5 case with Lagrangian (Bosonic part)

$$e^{-1}\mathcal{L} = R - \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{8} e^{\frac{2\phi}{\sqrt{6}}} (F^{ij}_{\mu\nu})^2 - \frac{1}{4} e^{\frac{-4\phi}{\sqrt{6}}} (G_{\mu\nu})^2 + \frac{1}{16} \varepsilon^{\mu\nu\rho\sigma\lambda} F^{ij}_{\mu\nu} F_{\rho\sigma\ ij} B_\lambda$$

Content

R symmetry : USp(4)

Content:	e_μ^m	$(A_\mu^{ij}$	$B_\mu)$	ϕ	Ψ_μ^i	χ^i
USp(4) rep.	$\mathbf{1}$	$\mathbf{5}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{4}$	$\mathbf{4}$

$$f^{[ij]} = i\bar{\epsilon}^i\epsilon^j \qquad f, f^a \qquad \mathbf{1}+\mathbf{5}$$

$$V_\mu^{[ij]} = \bar{\epsilon}^i\gamma_\mu\epsilon^j \qquad K_\mu, V_\mu^a \qquad \mathbf{1}+\mathbf{5}$$

$$\Phi^{(ij)} = i\bar{\epsilon}^i\gamma_{\mu\nu}\epsilon^j \qquad \Phi^{ab} \qquad \mathbf{10}$$

Related by $\delta\Psi_\mu^i = 0$, $\delta\chi^i = 0$ and Fierz relations.

Properties

Killing vector $$\boxed{K_\mu = \frac{1}{4}\Omega_{ij} V_\mu^{ij}}$$

$$K_\mu K^\mu = -f^{a2} \quad \begin{array}{ll} = 0 & \text{null case} \\ < 0 & \text{time-like case} \end{array}$$

Identification of a Killing vector naturally separates the metric into

a Killing direction and a 3(4)-dimensional base in (null) time-like case

$$\mathcal{L}_K G = 0 \qquad \mathcal{L}_K F^a = 0$$
$$\mathcal{L}_K (e^{-\frac{4}{\sqrt{6}}\phi} * G) = 0 \qquad \mathcal{L}_K (e^{\frac{4}{\sqrt{6}}\phi} * F^a) = 0$$
$$i_K d\phi = 0 \qquad \mathcal{L}_K (e^{\frac{1}{\sqrt{6}}\phi} V^a) = 0$$

Isometry generated by K_μ extends to entire solution.

Null case : R^3 structure

$$\boxed{ds^2 = H^{-1}du(2dv + \mathcal{F}du) + H^2 h_{mn}(dx^m + a^m du)(dx^n + a^n du)}$$

$$\boxed{\begin{aligned} G &= G_{+m} e^+ \wedge e^m - H^{-2} *_3 d\mathcal{H}_1 \\ F^a &= F^a_{+m} e^+ \wedge e^m + \frac{1}{\sqrt{2}} H^{-2} *_3 [u^a d\mathcal{H}_2 - \mathcal{H}_2 du^a] \end{aligned}}$$

$$\boxed{V^a_\mu = u^a K_\mu} \qquad \boxed{e^{\sqrt{\frac{3}{2}}\phi} = \frac{\mathcal{H}_1}{\mathcal{H}_2}}$$

$\mathcal{H}_1$ and $\mathcal{H}_2$ are harmonic.

u^a points out $SO(5) \supset SO(4) \simeq SU(2)_L \times SU(2)_R$.

Time-like case : SU(2) structure

$$f^2 = (f^a)^2$$

$$ds^2 = -(\mathcal{H}_1\mathcal{H}_2^2)^{-2/3}(dt+\omega)^2 + (\mathcal{H}_1\mathcal{H}_2^2)^{1/3}h_{mn}dx^m dx^n,$$

$$G = -d[\mathcal{H}_1^{-1}(dt+\omega)] - G_1^+, \qquad F^a = d[u^a\mathcal{H}_2^{-1}(dt+\omega)] + u^a G_2^+,$$

$$e^{\frac{3}{\sqrt{6}}\phi} = \mathcal{H}_2/\mathcal{H}_1,$$

where $dG_1^+ = 0$, $d(u^a G_2^+) = 0$,
$\Box_4\mathcal{H}_1 = \frac{1}{2}(G_2^+)^2$ and $(\Box_4 - \hat{R})\mathcal{H}_2 = \frac{1}{2}G_1^+G_2^+$.

Time-like case : $SU(2)$ cont'd

$$V^a = u^a K \qquad\qquad \Phi = \Phi^{(6)ab}$$

$$i_K \Phi^{(6)ab} = 0$$
$$\Phi^{(6)ab} = \frac{1}{2}\epsilon^{abcde} u^c \Phi^{(6)de}$$
$$f\Phi^{(6)ab} + *(K \wedge \Phi^{(6)ab}) = 0$$

$\Phi^{(6)ab}$ resides in

Tangent space group $SU(2)_- \subset SO(4) \subset SO(4,1)$,

and internal symmetry group $SU(2)_+ \subset SO(4) \subset SO(5)$.

In $\mathcal{N} = 2$ case, we have SU(2) holonomy, here only SU(2) structure.

Time-like case : Id structure

$$(f^a)^2 > f^2$$

$$u^a = f^a/|f^a| \qquad\qquad V^a_\mu = u^a V^{(1)}_\mu + V^{(4)a}_\mu$$

Metric: $ds^2 = -(f^a)^2(dt + \omega^2) + ((f^b)^2 - f^2)^{-1} V^{(4)a}_\mu V^{(4)a}_\nu$

$$u^a V^{(4)a}_\mu = 0.$$

Local frame is completely determined by Killing spinor.

The gauge fields can be written explicitly in terms of various forms.

Summary

- We analyzed $\mathcal{N}$=4, d=5 ungauged supergravity starting from fermionic e.o.m.s
- We identified 3 broad classes into which the solutions can be catagorized based on G-structure.
- Previously known solutions for N=2 belong to a particular case of the solutions with rigid u^a.
- We also found solutions for non-rigid u^a case, though they are singular.