AdS_3 Black holes and higher derivative corrections

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Reference:

J. High Energy Phys. JHEP07(2007)058 (with Justin David and Ashoke Sen)

Introduction

- BTZ solution describes a black hole in 3 dimesnional theory of gravity with negative cosmological constant and often appears as a near horizon factor in higher dimensional black holes in string theory.
- The computation of entropy of BTZ black hole has been an important problem both in 3 dimensional theory of gravity as well as string theory.
- One can use either Walds formalism or Euclidean action formalism to compute the entropy of BTZ black hole in the presence of arbitrary higher derivative terms as well as Chern simons terms.
- Although these techniques tells us how to compute entropy given the higher derivative terms but it does'nt say what these terms are.

- Krauss and Larsen (hep-th/0506176,0508218) argued using ADS/CFT correspondence that if the three dimensional theory has at least (0,4) Supersymmetry then the entropy is completely determined in terms of the coefficients if Gauge as well as gravitational Cherns simons terms and does'nt recieve any higher derivative corrections.
- But it is surprising and leave one to wonder how the requirement of (0,4) supersymmetry make the dependence on these higher derivative terms vanish.
- We propose a simple explanation for this fact that (0,4) supersymmetry prevents the addition of higher derivative terms except for those which can be removed by field redefinition and hence the entropy dependence on these higher derivative terms vanishes.

- In ADS/CFT correspondence the boundary operators dual to the fields in the supergravity multiplet are just the superconformal currents associated with the (0,4) Susy algebra.
- The correlation function of these operators are completely determined by c_L and c_R: The central charges of left moving Virasoro algebra and the right moving supervirasoro algebra.
- c_R is determined in terms of the coefficient of the Chern-Simons term of the associated SU(2) gauge fields and $c_L - c_R$ is determined in terms of the gravitational Chern-Simons term.

- Thus the Knowledge of the Chern-Simons terms in the bulk theory determines all the correlation functions of (0,4) superconformal currents in the boundary theory and hence completely determine the boundary S-Matrix of this theory.
- Since the boundary S-matrix are the only perturbative observables of the bulk theory, we expect that two theories with the same boundary S-matrix must be equivalent.
- In other words two different bulk theories with the same gauge and gravitational chern simons should be equivalent i.e they must be related by field redefinition.

- So, Once we have constructed a bulk theory with (0,4) Supersymmetry, there cannot be any further higher derivative correction to the action apart from those which can be removed by field redefinition
- The Non-renormalizabilty of the Black hole entropy trivially follows from this observation
- We shall make our observation concrete by wrting down the unique (0,4) Supergravity action and compute the entropy of BTZ black hole using this action and then discuss how the relationship between the Non-renormalization of boundary S-Matrix and Non-Renormalization of classical action works for the Gauge sector of the theory.

$(0,4)$ SUGRA action

- This action was constructed by M. Nishimura and Y. Tanii (hep-th/9904010) and J.R. David (hep-th/9904068) by regarding the SUGRA as a gauge theory based on $SU(1,1) \times SU(1,1|2)$ algebra
- The action is a Chern-Simons action and takes the form:

$$\begin{aligned}
\mathcal{S} &= -a_L \int d^3x \left[Tr(\Gamma_L \wedge d\Gamma_L + \frac{2}{3}\Gamma_L \wedge \Gamma_L \wedge \Gamma_L \right] \\
&\quad + a_R \int d^3x \left[Str(\Gamma_R \wedge d\Gamma_R + \frac{2}{3}\Gamma_R \wedge \Gamma_R \wedge \Gamma_R \right] \quad (1)
\end{aligned}$$

  - Where Γ_L and Γ_R denote the (super)connections in the $SU(1,1)$ and $SU(1,1|2)$ algebras respectively

- After expressing the action in component notation and eliminating the auxilliary fields, it takes the form:

$$\begin{aligned}\mathcal{S} &= \int d^3x[\sqrt{-\det G}[R+2m^2] + K\,\Omega_3(\widehat{\Gamma}) \\ &\quad -\frac{k_R}{4\pi}\,\epsilon^{MNP} Tr(\mathbf{A}_M \partial_N \mathbf{A}_P + \frac{2}{3}\mathbf{A}_M\mathbf{A}_N\mathbf{A}_P)]\,,\end{aligned} \quad (2)$$

where

$$\frac{1}{m} = \frac{1}{2}(a_R + a_L), \qquad K = \frac{1}{2}(a_L - a_R)\,, \quad (3)$$

$$k_R = 4\pi a_R = 4\pi\left(\frac{1}{m} - K\right)\,, \quad (4)$$

- We Can now use the techniques mentioned before to compute the entropy of a BTZ black hole with mass M and angular momentum J in this theory and we land up withe result as:

$$\mathbf{S}_{BH} = 2\pi\sqrt{\frac{c_L q_L}{6}} + 2\pi\sqrt{\frac{c_R q_R}{6}}\,, \tag{5}$$

where

$$q_L = \frac{1}{2}(M - J), \qquad q_R = \frac{1}{2}(M + J)\,, \tag{6}$$

$$c_L = 24\,\pi\,a_L, \qquad c_R = 24\,\pi\,a_R\,, \tag{7}$$

- The above calculated entropy does'nt change by our previous arguement in the presence of higher derivative terms as long as they obey (0,4) Supersymmetry.
- Let us now focus on the relationship between the non-renormalization of the boundary S-matrix and of the classical action by seeing how this works for the gauge sector of the theory.
- In this case the Chern-Simons theory has equation of motion $\mathbf{F}_{MN} = 0$ where

$$\mathbf{F}_{MN} \equiv \partial_{[M}\mathbf{A}_{N]} + [\mathbf{A}_M, \mathbf{A}_N] \tag{8}$$

is the gauge field strength.

- Any additional gauge invariant term in the action will involve the gauge field strength and hence will vanish when $\mathbf{F}_{MN} = 0$ and hence such terms can be removed by field redefinition
- Lets see how the vanishing of the additional terms in the action for $\mathbf{F} = 0$ is related to the non-renormalization of the boundary S-matrix. We shall compute the boundary S-matrix from pure Chern-Simons theory for this purpose.
- Euclidean AdS_3 metric in the poincare patch:

$$ds^2 = \frac{l^2}{(x^0)^2}\left((dx^0)^2 + (dx^1)^2 + (dx^2)^2\right) , \tag{9}$$

- The gauge field action takes the form:

$$\mathbf{S}_{gauge} = -i\frac{k_R}{4\pi}\int d^3x\epsilon^{MNP}Tr\left(\mathbf{A}_M\partial_N\mathbf{A}_P + \frac{2}{3}\mathbf{A}_M\mathbf{A}_N\mathbf{A}_P\right) + \frac{k_R}{2\pi}\int d^2zTr(\mathbf{A}_z\mathbf{A}_{\bar{z}})|_{x^0=0}\,. \tag{10}$$

- The last term is a boundary term required for the consistency of the theory and its effect is that under a variation of the gauge fields, the result only depends on $\delta\mathbf{A}_z$ at the boundary and not on $\delta\mathbf{A}_{\bar{z}}$. Thus we just need boundary condition on $\mathbf{A}_z$.

- Let $I[\mathbf{A}_z^{(0)}]$ denote the value of the action for an on-shell field configuration subjected to boundary condition

$$\mathbf{A}_z(x^0 = 0, \vec{z}) = \mathbf{A}_z^{(0)}(\vec{z})\,. \tag{11}$$

- Then according to AdS/CFT conjecture:

$$\begin{aligned} &\left\langle \bar{J}^{a_1}(\vec{z}_1) \cdots \bar{J}^{a_n}(\vec{z}_n) \right\rangle \\ = \quad &(i\pi)^n \; \frac{\delta}{\delta A_z^{(0)a_1}(\vec{z}_1) \cdots \delta A_z^{(0)a_n}(\vec{z}_n)} e^{-I[\mathbf{A}_z^{(0)}]} \Bigg|_{\mathbf{A}_z^{(0)}(\vec{z})=0} \end{aligned} \tag{12}$$

where $\bar{J}^a(\vec{z})$ are the $SU(2)$ currents of the CFT at the boundary

- So we need to compute $I[\mathbf{A}_z^{(0)}]$ for which we need to evaluate gauge field configuration that satisfies the equation of motion $\mathbf{F}_{MN} = 0$ and the boundary condition $\mathbf{A}_z(x^0 = 0, \vec{z}) = \mathbf{A}_z^{(0)}(\vec{z})$
- This is given by:

$$\begin{aligned} \mathbf{A}_M(\vec{z}, x^0)\, dx^M &= e^{-\Phi}\, d\, e^{\Phi} \\ &= d\Phi + \frac{1}{2}(d\Phi\Phi - \Phi d\Phi) + \cdots \end{aligned} \tag{13}$$

where

$$\Phi(\vec{z}, x^0) = \int d^2w K(\vec{z}, x^0; \vec{w})\, \mathbf{B}_z^{(0)}(\vec{w})\,, \tag{14}$$

- $$K(\vec{z}, x^0; \vec{w}) = \frac{1}{\pi} \left[\frac{z - w}{(x^0)^2 + |z - w|^2} \right] , \qquad (15)$$

  and $\mathbf{B}_z^{(0)}$ is chosen such that $\mathbf{A}_M$ satisfies the boundary condition

- $K(\vec{z}, x^0; \vec{w})$ satisfies the following conditions:

$$\lim_{x^0 \to 0} \partial_z K(\vec{z}, x^0; \vec{w}) = \delta^{(2)}(\vec{z} - \vec{w}) , \qquad (16)$$

$$\lim_{x^0 \to 0} \partial_{\bar{z}} K(\vec{z}, x^0; \vec{w}) = -\frac{1}{\pi} \frac{1}{(\bar{z} - \bar{w})^2} . \qquad (17)$$

- Using the above relations we see that to leading order $\mathbf{B}_z^{(0)} = \mathbf{A}_z^{(0)}$ boundary condition for $\mathbf{A}$ is satisfied. The higher order contributions can be obtained iteratively and we get:

$$\mathbf{B}_z^{(0)}(\vec{z}) \;\; = \;\; \mathbf{A}_z^{(0)}(\vec{z})\frac{1}{2\pi}\int \frac{d^2w}{(\bar{z}-\bar{w})}\mathbf{G}(\vec{z},\vec{w}) + \cdots \quad (18)$$

where

$$\mathbf{G}(\vec{z},\vec{w}) = \Big(\mathbf{A}_z^{(0)}(\vec{w})\mathbf{A}_z^{(0)}(\vec{z}) - \mathbf{A}_z^{(0)}(\vec{z})\mathbf{A}_z^{(0)}(\vec{w})\Big) \quad (19)$$

and $\cdots$ denote higher order terms

- We can now substitute everything to evaluate the on-shell action $I[\mathbf{A}_z^0]$ and the final result is:

$$
\begin{aligned}
I[\mathbf{A}_z^{(0)}] &= \frac{k_R}{4\pi^2}\int d^2z d^2w\,(\bar{z}-\bar{w})^{-2}\,A_z^{(0)a}(\vec{z})\,A_z^{(0)a}(\vec{w}) \\
&\quad -\frac{k_R}{12\pi^3}\int d^2z d^2w d^2v\,\epsilon^{abc}H_z^{abc} \\
&\quad +\cdots\,. \qquad (20)
\end{aligned}
$$

where

$$
H_z^{abc} = (\bar{z}-\bar{w})^{-1}(\bar{w}-\bar{v})^{-1}(\bar{v}-\bar{z})^{-1}\,A_z^{(0)a}(\vec{z})A_z^{(0)b}(\vec{w})A_z^{(0)c}(\vec{v}) \qquad (21)
$$

- We can now thus evaluate arbitrary correlation function of the SU(2) currents. Particularly for the two and three point functions we get:

$$\left\langle \bar{J}^{a_1}(\bar{z}_1)\bar{J}^{a_2}(\bar{z}_2)\right\rangle = \frac{k_R}{2}\,\delta_{a_1 a_2}\,(\bar{z}_1-\bar{z}_2)^{-2}\,, \tag{22}$$

$$\begin{aligned}\left\langle \bar{J}^{a_1}(\bar{z}_1)\bar{J}^{a_2}(\bar{z}_2)\bar{J}^{a_3}(\bar{z}_3)\right\rangle &= -\frac{ik_R}{2}\,\epsilon^{a_1a_2a_3}(\bar{z}_1-\bar{z}_2)^{-1}(\bar{z}_2-\bar{z}_3)^{-1}\\ &\qquad (\bar{z}_3-\bar{z}_1)^{-1}\,,\end{aligned} \tag{23}$$

which are the expected conformal field theory correlation functions.

- If we add additional gauge invariant term in the action the solution satisfying $\mathbf{F}_{MN} = 0$ continues to be the solution of equation of motion.
- The on-shell action is also not modified since the additional terms vanish on-shell.
- As a result the correlation functions remain unchanged.
- Thus we see that correlation function remaining unchanged is intimately related to the vanishing of additional terms on-shell which in turn implies that such terms can be removed by field redefinition

- By supersymmetry the correlators of the currents and stress tensor are related in the CFT
- Hence the boundary S-Matrix is also Not-renormalized in the gravity sector
- This implies that action in the gravity sector is also not renormalized
- Supersymmetry is crucial for this arguement.
- In the absence of supersymmetry relating the current correlators to stress tensor correlators, there is no reason for the later to be renormalized.
- Hence the action can receive corrections which do not vanish on-shell and hence cannot be removed by field redefinition.

Summary

- We looked at the origin of the Non-renormalizability of the Entropy of a BTZ black hole which was earler argued by Karuss and Larsen (hep-th/0506176,0508218) using AdS/CFT correspondence and Euclidean action formalism.
- We argued that the Non-renormalizability of the entropy comes as a result of the action being not renormalized except for the terms which can be removed by field redefinition.
- We saw that the S-matrix of the boundary CFT are the only perturbative observables and the non-renormalizabilty of the action is intimately related to the non-renormalizabilty of the boundary S-matrix.

- We worked out the boundary S-matrix of the gauge sector of the theory and saw that any further corrections just depending on the field strength wont renormalize the boundary S-matrix and hence these terms can be removed by field redefinition in the effective action.
- We argued that because of Supersymmetry the boundary S-matrix of the currents are related to that of the stress tensor and hence stress tensor correlators are also not renormalized which in turn implies that even in the gravity sector one cannot add further corrections except for those which can be removed by field redefinition.

- Supersymmetry was crucial for our arguement because in its absence the renormalization of boundary stress tensor wont follow which in turn would imply that we could add corrections to the action which could not be removed by field redefinition