Thermodynamic Geometry and Extremal Black Holes in String Theory

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PLAN

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- Black hole thermodynamics as a theoretical laboratory for testing issues of quantum gravity arising from string theory. Major progress towards a microscopic statistical understanding for extremal and near extremal black holes.

- Extremal BPS black holes. D-brane bound states. Attractor mechanism. Special Geometry. Entropy function and extremization. Higher derivative contributions. Wald formula. Extremal non-BPS black holes. Gauge gravity correspondence.

- Central role of *attractor mechanism* to ensure validity of a microscopic statistical basis.

- Understand attractor mechanism and the attractor fixed point in relation to certain intrinsic geometrical structure of thermodynamics. Speculative and "experimental". Motivation from relation with scalar moduli spaces of sugra compactifications. "Thermodynamics" of extremal black holes.

Thermodynamic Geometry

- Space of equilibrium thermodynamic macrostates. Maxima of the entropy $S = S(x^i)$ of extensive variables x^i in the entropy representation or minima of $M = M({\prime}x^i)$ in the energy representation.

- A Riemannian metric in the state space is provided by the Hessian of internal energy $h_{ij} = \partial_i \partial_j M$ over the extensive variables. Weinhold Geometry.

- In the entropy representation a Riemannian structure given by the Hessian of the entropy as $g_{ij} = \partial_i \partial_j S$. Ruppeiner Geometry. Physical significance related to thermodynamic fluctuations connecting the equilibrium states.

- Probability of fluctuation connecting two states inversely proportional to line element ds^2 and given as $P(x^i) = Aexp(-1/2 g_{ij} dx^i dx^j)$ in the Gaussian approximation.

- For two variables the scalar curvature related to interactions in the system and proportional to correlation volume $R \sim \xi^d$ and d is the physical dimension of the system. At critical points of phase transition R diverges.

- Legendre transforms correspond to general coordinate transformations in the thermodynamic state space. Multiple variables define covariant thermodynamic fluctuation theory and $\mid R \mid \sim V$ signifies minimum real space volume beyond which CFT breaks down. Same as the correlation volume near critical points.

- Ruppeiner and Weinhold metrics are conformaly related $h_{ij}dx^{'i}dx^{'j} = Tg_{ij}dx^i dx^j$ with temperature T as the conformal factor. Inverse Weinhold metric given by the Hessian of the Gibbs potential and that of Ruppeiner by the Hessian of Legendre transform of the entropy S.

Thermodynamic Geometry of Black Holes

- Geometric approach first applied by *Ferrara, Gibbons and Kallosh* to the thermodynamics of extremal charged black holes in $N = 2$ $D = 4$ sugra coupled to vector multiplets. These involve gauge fields and complex scalar moduli ϕ.

- Solutions characterized by electric and magnetic charges q^I and p^J. BPS solitons interpolating between asymptoticaly flat and near horizon geometries $AdS^2 \times S^{D-2}$. ADM mass $M(p, q, \phi^\infty) =| Z_\infty |$ where Z is the SUSY central charge and ϕ^∞ are moduli values at infinity.

- Radial variation of the moduli show *attractor behaviour* with a fixed point at the horizon where the moduli are determined in terms of the charges by the stability (attractor) equations. The macroscopic entropy $S_{macro} = \pi \mid Z_{fix} \mid$.

- May be recast in terms of a *effective potential* $V(p, q, \phi^{\infty}) = M^2$ and the entropy $S = \pi V(p, q, \phi_{fix})$ at the attractor fixed point. Equivalence to *Sen* entropy function *Mahapatra, Astefanesi.*

- ϕ_{fix} are determined from the critical points of V such that $\frac{\partial V}{\partial \phi} = 0$. Each critical point is associated with a Bertotti-Robinson vaccuum with $AdS_2 \times S^{D-2}$ geometry.

- Extremal black holes are trajectories in the scalar moduli space$\mathcal{M}_\phi$ describing flow from ϕ_∞ to ϕfix at the horizon.

- The Hessian of the ADM mass or V w.r.t the rescaled scalar moduli $z, \bar{z}$ is proportional to the moduli space metric with the BPS mass as the constant of proportionality *Ferrara, Gibbons, Kallosh*
$\partial_{z_i}\partial_{z_j} M(p, q, z, \bar{z}) = 1/2 G_{i\bar{j}}(z_{cr}, \bar{z}_{cr}) M(p, q)$.

- This is a reduced Weinhold geometry and captures the moduli space geometry at the attractor fixed point. Away from the fixed point ?

- Augment thermodynamic configuration space with the scalar moduli also as extensive variables and corresponding chemical potentials as $-\Sigma$, the scalar charges *Ferrara, Gibbons, Kallosh and Kol.*
$dM = Tds + \psi^a dq_a + \chi_b dp^b - \Sigma_c d\phi^c_\infty$.

- Consider thermodynamic geometries of this extended state space in terms of the covariant Hessians w.r.t the scalar moduli of appropriate thermodynamic potentials. Well defined as a geometrical object even if conventional thermodynamics breaks down at extremality.

- Scalar curvature should provide information about the moduli space and its divergences may describe phase transitions amongst distinct vaccua in moduli space or trace singularities of the moduli space.

- State space of extremal black holes as reduced space comprising of states respecting extremality (BPS) condition. The moduli variables lose all thermodynamic significance at this point and defines a geometry fixed in terms of the charges.

- First step to examine thermodynamic geometries at the attractor fixed point for extremal black holes with the macrostates as maxima of $S = S(p, q)$.

- The Hessian of the entropy $g_{ij} = \partial_i \partial_j S(p.q)$(or any other suitable potentials) w.r.t to the conserved charges (or other extensive variables) should give provide a non degenerate thermodynamic metric.

- Microscopicaly extremal black holes describe degenerate ground state of a quantum system. Gauge gravity correspondence seemingly indicates a limiting (zero temperature) version of conventional thermodynamics *Pedro Silva, De Boer, Hanany, Minwalla*

- Thermodynamic geometry of extremal black holes as a geometrical description of such a possible (zero temperature) limiting thermodynamic chracterization.

- Divergences in scalar curvature ? (Zero temperature) phase transitions ? Geometric issues may also be relevant to boundary gauge theories especialy the quiver gauge theories which have limited countable set of states.

Electricaly charged Black Holes.

- Black holes in D=5 N=2 sugra with 3 electric charges q_1, q_2, q_3. Moduli space with special geometry and $S_{macro} =\mid z \mid^{3/2}= 2\pi\sqrt{q_1 q_2 q_3}$. State space is 3-dimensional.

- The Hessian $\partial_i \partial_j S(p,q)$ is negative definite ensuring stability of the canonical ensemble and a positive definite non singular Ruppeiner metric. Scalar Curvature $R = \frac{3}{4\pi\sqrt{q_1 q_2 q_3}}$ is regular everywhere and non zero which signify and underlying interacting system.

- 4 charged extremal (BPS) black holes in D=4 with 4 electric charges and a four dimensional thermodynamic state space. Entropy S again proportional to the square root of the product of the charges.

- Thermodynamic metric is unchanged and the scalar curvature R of the state space retains the same form modulo a factor of half and is regular everywhere.

D1-D5-P System.

- IIB Sugra in D=10. N_1 D-1 branes, N_5 D-5 branes and p units of of KK momentum along a compact direction. The charges Q_1, Q_5, Q_p related to N_1, N_5, N_p. D=10 extremal black hole but with a near horizon geometry of $M_3 \times S^3 \times T^4$ where M_3 is a boosted AdS_3.

- Entropy at the two derivative level computed through the entropy function is $S = 2\pi\sqrt{N_1 N_5 N_p}$. The Hessian of S provides the positive definite Ruppeiner metric. Stae space scalar curvature is $R = \frac{3}{4\pi\sqrt{N_1 N_5 N_p}}$ and regular everywhere.

- Higher derivative R^4 corections may be incorporated through the entropy function and the corrected entropy is $S = 2\pi\sqrt{N_1 N_5 N_p}[1 + \frac{\gamma}{N_1 N_5}^{3/2}]$.

- The Ruppeiner metric is modified and leads to a complicated scalar curvature R involving a cubic equation in the denominator and diverges at the roots. Significance of divergences in R ?

- The D=10 sugra solution may be compactified to D=5 on $S^1 \times T^4$ to the standard $D-1, D-5, P$ extremal black hole. At the two derivative level the entropy S is identical to the D=10 case and the thermodynamic geometry is unchanged.

Small Black Holes

- As an example we may consider 2-charge extremal BPS black holes in IIA compactified on $K_3 \times T^2$ with a single electric charge q_0 and a magnetic charge p^1.

- General expression for the entropy is $S = 2\pi\sqrt{\mid q_0 \mid \frac{c_L}{6}}$ where $c_L = C_ABCp^Ap^Bp^C + c_2Ap^A$ and the second term involves the higher derivative contribution.

- For small black hole $C_{ABC} = 0$ and $c_2A = 24p^1$ for K_3. So the entropy is zero at the two derivative level and arises purely from the higher derivative contributions as $S = 4\pi\sqrt{| q_0 p^1 |}$.

- The thermodynamic metric given by the Hessian of $S = S(p, q)$ w.r.t the charges. The scalar curvature of the state space is $R = -\frac{11}{8}(\frac{\pi}{(qp)^{3/2}})$ regular everywhere. Note that the curvature is small in the limit of large charges.

- A microscopic description of the small black hole involves Heterotic compactification on T^6 and described by electricaly charged heterotic state with charge $q_0 = n$ and $p^1 = w$ where n, w are KK momentum and winding respectively.

- The entropy from microscopic state counting in the large charge limit is $S = 4\pi\sqrt{\mid nw \mid} - \frac{27}{4} ln \mid nw \mid + O(\frac{1}{\sqrt{nw}})$ involving a correction to the macroscopic entropy S_{macro}.

• The correction term requires introduction of *non holomorphic terms* in the prepotential and leads to the macroscopic entropy $S = 4\pi\sqrt{\mid qp \mid} - 12 ln \mid nw \mid + O(\frac{1}{\sqrt{qp}})$. This shows a mismatch. Grand canonical ensemble *Sen* in the heterotic description.

• The thermodynamic metric for the state space computed both for the microscopic and macroscopic entropy. For the former the scalar curvature is $R = -\frac{\pi}{(qp)^{3/2}}(\frac{4\pi\sqrt{qp}-27}{8\pi\sqrt{qp}-27})$ and diverges at $\sqrt{qp} = \frac{27}{8\pi}$. Significance ?

- For the state space based on the macroscopic entropy corrected by *non holomorphic* contribution we have the scalar curvature $R = -\frac{\pi}{2(qp)^{3/2}}(\frac{\pi\sqrt{qp}-6}{\pi\sqrt{qp}-3})$ which diverges at $\sqrt{qp} = \frac{3}{\pi}$.

- It is possible to include other $\alpha^{'}$ corrections to the microscopic entropy involving the state counting from topological string and the OSV formula. The corrected entropy reads $S = 4\pi\sqrt{\mid nw \mid} - \frac{27}{4}ln \mid nw \mid + \frac{15}{2}ln2 - \frac{675}{32\pi\sqrt{|nw|}} + ...$

- The scalar curvature R based on the state space given by the corrected entropy has a complicated form and again involves a cubic equation in $\sqrt{nw}$ in the denominator and hence diverges at the roots.

- Further corrections to the entropy of 2 charge small black holes arise from string loop corrections *Sinha, Suryanarayana* and the modified entropy has the form $S = \sqrt{anw + bn}$ with $n >> w >> 1$ and a is an arbitrary constant and b depends on the loop corrections.

- Curiously *Kallosh and Linde* also conjectures a simmilar form for the entropy of two charged small black holes based on quantum information theory as $S = \sqrt{anw + b(n+w)}$.

- The thermodynamic scalar curvature on the state space based on this conjectured entropy from QIF is regular everywhere.

Summary

- Geometric description of thermodynamic systems through Riemannian geometry of the state space. Application to extremal black holes in string theory.

- For extremal black holes in N=2 D=4 SUGRA, reduced thermodynamic geometries capture moduli space geometry at the attractor fixed points. Phase transitions in moduli space ?

- Significance of scalar moduli as extensive thermodynamic variables. Thermodynamic significance away from the attractor fixed points.

- (Zero temperature) thermodynamic description of extremal black holes. Thermodynamic geometry of the state space of extremal black holes as a possible example of such limiting geometries for limiting thermodynamics.

- Examples of 3 charged extremal black holes, D1-D5,P systems and Small Black Holes. Non degenerate state space geometries. Divergences of scalar curvature. Significance ?

- Relation to extremal black holes described by Gauge-gravity correspondence. Thermodynamic geometry of boundary quiver gauge theories ?