DYNAMICS OF SUPERTUBES

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PLAN OF THE TALK

1. DBI description of Supertubes

2. Gravity description of 2-charge microstates

3. Perturbation of Supertubes

4. Thin ring limit

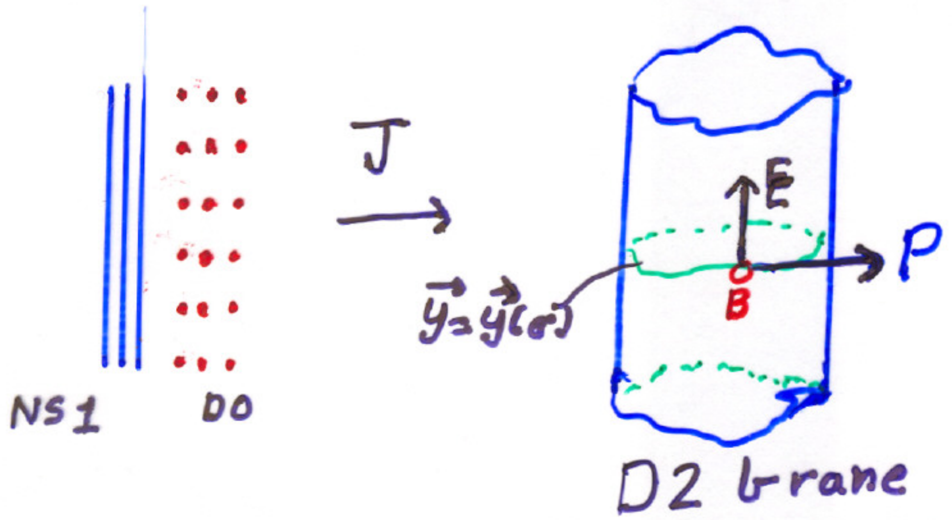
5. Excitations at large coupling

6. Conjecture & Summary

2-charge Supertubes : World-vol. description

Brane Expansion in flat space, preserving susy

Supertubes describe D0-F1 bound states [Townsend + Mateos]



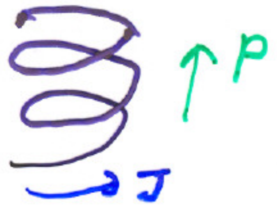
$$\mathcal{L} = -\sqrt{\det(\eta + F)}$$

$$F = E\, dt \wedge dz + B(\sigma)\, dz \wedge d\sigma$$

* Preserves $\frac{1}{4}$ susy for arbitrary cross-section if $E^2 = 1$ & $\mathrm{sgn}(B) = \text{Constant}$
* Crossed electric and magnetic fields → Poynting Angular momentum $|J| \leq \frac{q_1 q_0}{N_{D2}}$
* No tension due to D2 brane, no susy constraint either
$$H = |q_1| + |q_0|$$
* No net D2 brane charge ; D2 acts as dipole charge $\quad q_{D2} \sim N_{D2}$

* Supertubes are dual to helical string carrying left moving wave

$$\begin{pmatrix} NS1-D0 \\ D2 \end{pmatrix} \xrightarrow{T,S} \begin{pmatrix} NS1-P \\ NS1 \end{pmatrix}$$



$$\downarrow \begin{pmatrix} D1-D5 \\ KKM \end{pmatrix}$$

* For circular supertubes, (Marolf - Palmer) linearized DBI action & did quantum counting of states

Becomes same as 1+1 dim. gas

$$S = 2\pi\sqrt{2(q_1 q_0 - J)}$$

* Classically, a continuous family of $\frac{1}{4}$ BPS configurations.

Q. Take supertube in any configuration & add some energy. What's the low energy behavior?

Is there a 'drift' among allowed configurations?

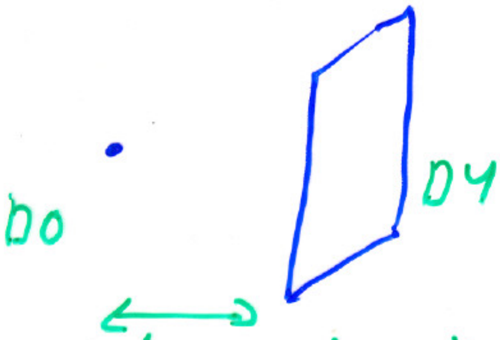
Or some oscillatory behavior?

* Expand DBI action around some general configuration & solve linearized equations, keeping charges fixed.

Q. What is the low energy dynamics of 2-charge systems?

Examples of possible behaviors

(i) 'Drift' on moduli space



3 charge black hole

Any separation allowed

Slow motion described by motion of a moduli space

$v \sim \epsilon$, $\Delta t \sim \frac{1}{\epsilon}$, $\Delta x \sim 1$ $(\epsilon \to 0)$

(ii) Oscillations

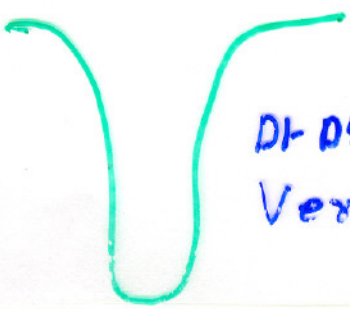
Vibration modes of a single brane.

$v \sim \epsilon$, $\Delta t \sim 1$, $\Delta x \sim \epsilon$ $(\epsilon \to 0)$

(iii) Radiation

Energy given to system flows to infinity due to coupling with SUGRA modes

(iv) Trapped SUGRA excitations



D1-D5 system: long throat

Very slow leaking of radiation to infinity

2-charge fuzzball geometries

$$ds^2 = \sqrt{\frac{H}{1+K}}\left\{-(dt - A_i dx^i)^2 + (dy + B_i dx^i)^2\right\}$$
$$+ \sqrt{\frac{1+K}{H}}\, dx\cdot dx + \sqrt{H(1+K)}\, dz\cdot dz$$

Type IIB theory: D1 wrapped on S^1_y

D5 " " $S^1_y \times T^4$

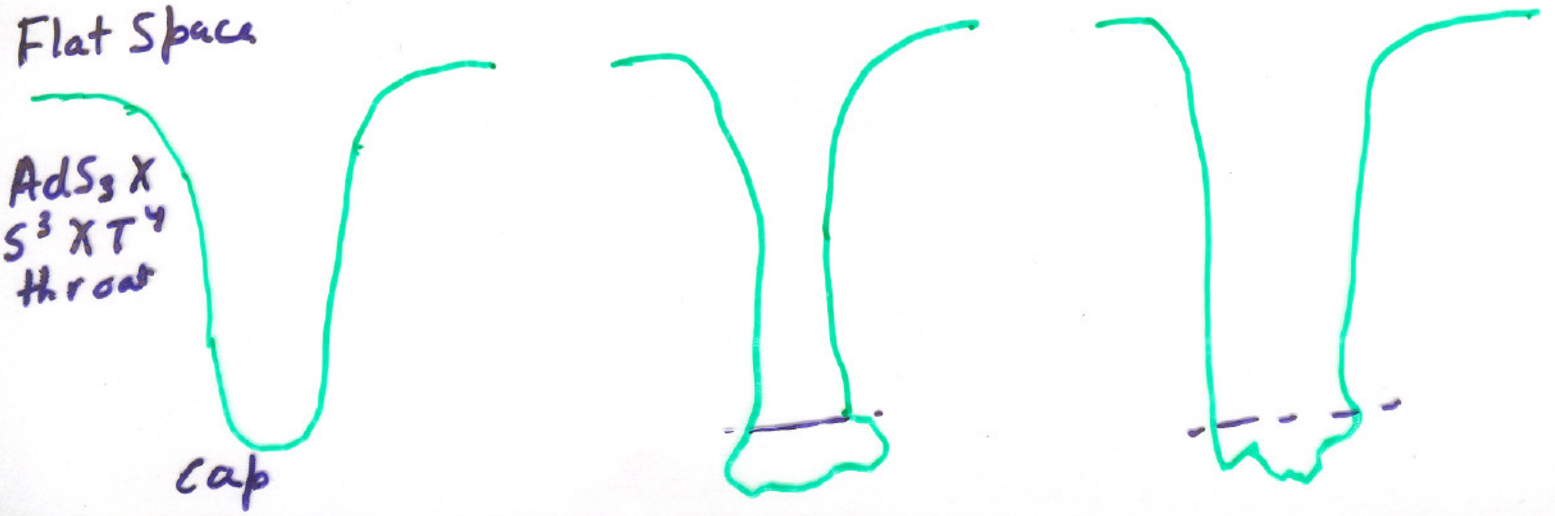
$$H^{-1} = 1 + \frac{Q}{L}\int_0^L \frac{dv}{|\vec{x}-\vec{F}(v)|^2}, \quad K = \frac{Q}{L}\int_0^L \frac{dv\,|\dot{F}(v)|^2}{|\vec{x}-\vec{F}(v)|^2}$$

$$A_i = -\frac{Q}{L}\int_0^L \frac{dv\,\dot{F}_i(v)}{|\vec{x}-\vec{F}(v)|^2}, \quad dB = -*_4 dA$$

[First such metric: De Boer, Ross, Balasubramanian et al, Townsend, Mateos, Emparan

General metrics: Lunin - Mathur]

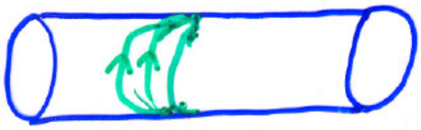
* Smooth, horizonless geometries
* Correspond to chiral primaries of D1-D5 CFT
* These geometries have same mass, charge & are $\frac{1}{4}$ BPS



* Geometries obtained by dualizing from

$NS1_y - P_y \xrightarrow{T,S} D1_y - D5_{y\ T^4}$

Multiply wound string carrying



$\vec{F}(v)$ Traveling waves

* Oscillations cause strands to separate from eachother & hence bound state has transverse size.

* In D1-D5 frame, size is due to KK monopole supertube.

Q. What is the low energy dynamics if some energy is added to such a system?

If Charges are separated.. then 'drift' on moduli space?

Since we only have threshold bound, does the bound state break into few unbound states?

Perturbation at zero coupling

$$R = \bar{R}(\sigma) + r(\sigma,t) \qquad Z_a = Z_a(\sigma,t)$$

$$E = \bar{E} + \partial_t a_y \qquad B = \bar{B} - \partial_\sigma a_y$$

EOM & gauss law imply that $a_\sigma = 0$

Expanding the DBI lagrangian up to second order, we get following EOM

① $$\omega^2 \ddot{r} + 2\dot{r}' + (\omega^2 \ddot{a}_y + 2\dot{a}_y')\frac{\bar{R}'}{\bar{B}} - \frac{2\bar{R}\,\dot{a}_y}{\bar{B}} + 2\partial_\sigma\frac{(\partial_\sigma \bar{R})}{\bar{B}}\dot{a}_y = 0$$

② $$(\omega^2 \ddot{a}_y + 2\dot{a}_y')\left(\frac{\bar{R}^2 + \bar{R}'^2}{\bar{B}^2}\right) + (\omega^2\ddot{r} + 2\dot{r}')\frac{\bar{R}'}{\bar{B}} + 2\frac{\bar{R}}{\bar{B}}\dot{r} + \partial_\sigma\left(\frac{\bar{R}^2 + \bar{R}'^2}{\bar{B}^2}\right)\dot{a}_y = 0$$

③ $$\omega^2 \ddot{z}_a + 2\dot{z}_a' = 0$$

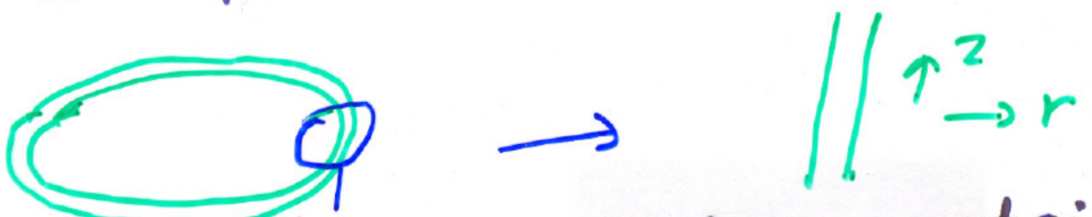
where $$\omega^2 = \frac{\bar{R}^2 + \bar{R}'^2 + \bar{B}^2}{\bar{B}}$$

- Only time derivative of fields occurs
- Solving, we get following period

$$\Delta t = \frac{1}{2}\left(\frac{n_0 T_0 + n_1 T \bar{L}_y}{T_2 L_y}\right)$$

where $L_y = 2\pi R_y$

Small but non-zero coupling g. Supertube in a 'thin ring'. Move from worldvolume description to spacetime description.



In the NS1-P picture, metric around the 'thin tube' is

$$ds^2 = H^{-1}[-2\,dt\,dv + K\,dv^2 + 2A\,dv\,dz] + dz^2 + dx\cdot dx + dz_a\,dz_a$$

$$B = (H^{-1}-1)\,dt\wedge dv + H^{-1}A\,dv\wedge dz$$

$$e^{2\Phi} = H^{-1}$$

$$H = 1 + \frac{Q_1}{r} \qquad K = 1 + \frac{Q_p}{r} \qquad A = \frac{\sqrt{Q_1 Q_p}}{r}$$

Perturb string along one of the torus directions

$$ds^2_{1+r} \to ds^2_{1+r} + 2A^{(1)}dz_a$$

$$B \to B + A^{(2)}\wedge dz_a$$

Writing $A^{\pm} = A^{(1)} \pm A^{(2)}$ & $F^{\pm} = dA^{\pm}$ in the dimensionally reduced 6-dim. action, we get following equations for perturbation

$$\nabla^{\mu}(e^{-2\Phi}F^{\pm}_{\mu\lambda}) \pm \frac{1}{2}e^{-2\Phi}F^{\pm}_{\mu\nu}H^{\mu\nu}{}_{\lambda} = 0$$

Here $H = d\tilde{B} - \frac{1}{2}(A^{(1)} \wedge F^{(2)} + A^{(2)} \wedge F^{(1)})$

$\tilde{B} = B + \frac{1}{2}(A^{(1)}_\mu A^{(2)}_\nu - A^{(2)}_\mu A^{(1)}_\nu)$

Solution to these equations is

$$A^+_v = (\alpha - \beta) H^{-1} \frac{Q_1}{r} e^{ik(z+ir)} + c.c$$

$$A^-_v = (\alpha + \beta) H^{-1} (Q_1 - Q_p) e^{ikz - i\omega t} \frac{e^{-|\tilde{k}|r}}{r}$$

$$A^-_t = -2(\alpha + \beta) H^{-1} Q_1 e^{i(kz - \omega t)} \frac{e^{-|\tilde{k}|r}}{r}$$

$$A^-_z = -2(\alpha + \beta) H^{-1} \sqrt{Q_1 Q_p}\, e^{i(kz - \omega t)} \frac{e^{-|\tilde{k}|r}}{r}$$

$$\omega = \frac{2k\sqrt{Q_1 Q_p}}{Q_1 + Q_p}$$

$$\tilde{k}^2 = k^2 \left(\frac{Q_1 - Q_p}{Q_1 + Q_p}\right)^2$$

Time period $\Delta t = \int_0^{L_z} \frac{dz}{v} = \int_0^{L_z} dz \frac{k}{\omega}$

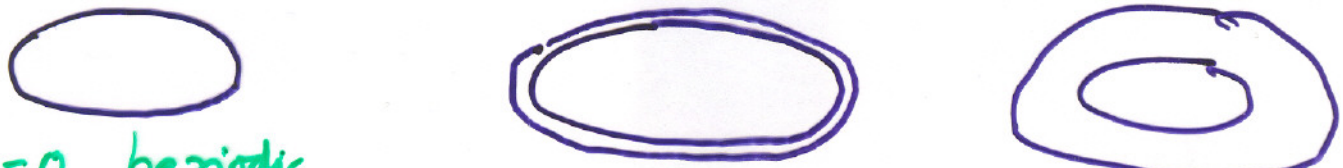
$$= \frac{1}{2T}(M_{NS1} + M_p)$$

Same as the one found in flat space limit $g = 0$ supertube

* For thin ring, energy leaks very slowly to infinity since amplitude in radiation zone is small.

* At larger coupling, oscillation mode mixes with radiation modes & energy loss increases

* At still larger values of g. there is no oscillation mode as energy radiates away on the same timescale as period of oscillation.

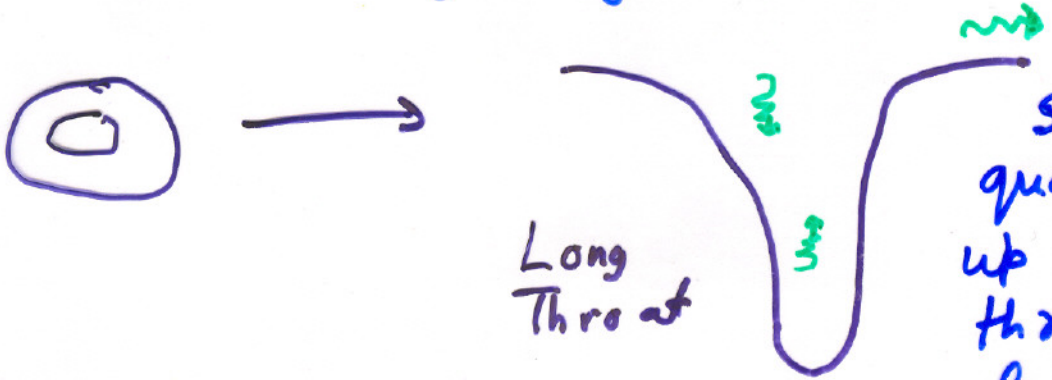


$g=0$ periodic oscillations

small g, long lived oscillations

larger g, no oscillations

At still larger g.

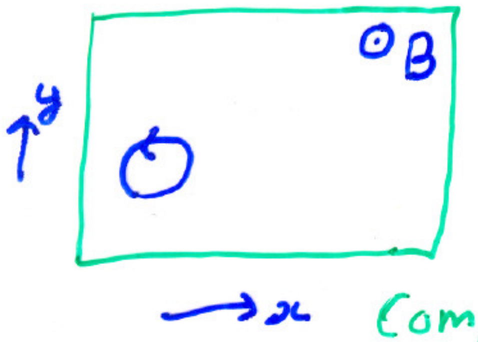


SUGRA quanta bounce up & down the throat. Slow leakage to infinity

These 'long' lived excitations are different from previous

* We find that even though we had a family of degenerate configurations, the system didn't 'drift' along this family. But, we don't get usual oscillations

Quasi-oscillations : Like particle in a magnetic field



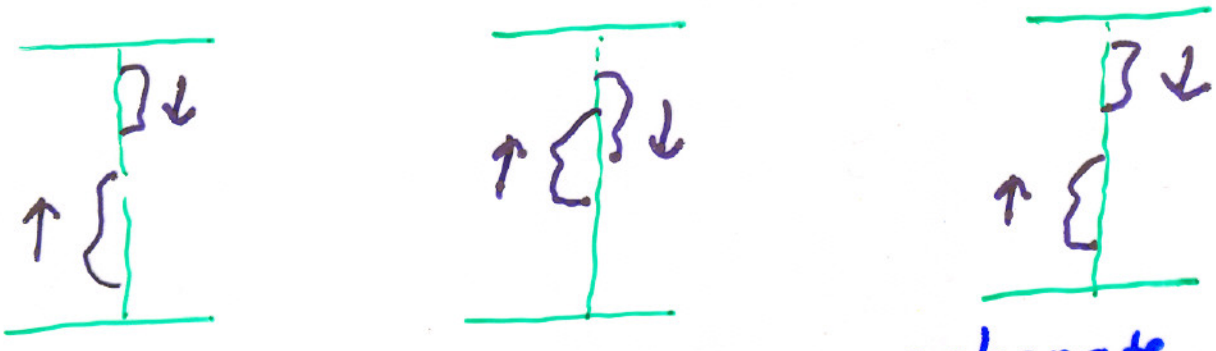
$\ddot{x} = \frac{e}{m}\dot{y}$, $\ddot{y} = -\frac{e}{m}\dot{x}$

$v \sim \epsilon$ $\Delta t \sim 1$ $\Delta x \sim \epsilon$

Compare with drift

$v \sim \epsilon$ $\Delta t \sim \frac{1}{\epsilon}$ $\Delta x \sim 1$

* In NS 1-P frame, it's easy to see why there is no drift



Left & right movers separate. Waves travel around the string & come back.

In all duality frames

Time period $\Delta t = \dfrac{\text{Total Energy}}{2 \times \text{mass of dipole/length}}$

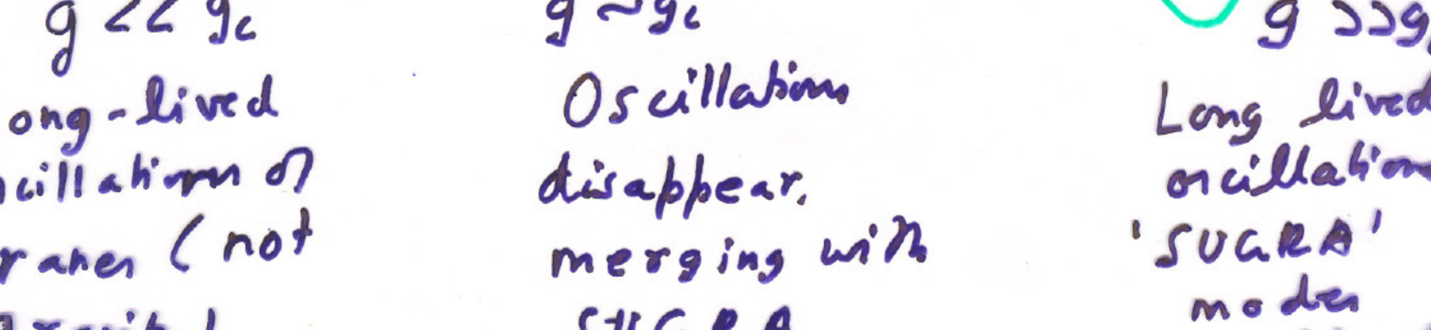
* By considering wave equation in deep throat geometry, we get

$$\alpha = \frac{\Delta t_{escape}}{\Delta t_{oscillation}} = \frac{1}{(2\pi)^2}\left(\frac{Q_1 Q_p}{a^4}\right)$$

$$\beta = \frac{(Q_1 Q_p)^{1/4}}{a}$$ gives the no. of AdS radii that we can go before reaching the 'neck'

So when the throat becomes deep, the quanta trapped become long lived excitations.

Transition pt. $g \sim g_c = \sqrt{\frac{M V R_y}{\alpha'^2}}$



$g \ll g_c$
Long-lived oscillations of branes (not gravity)

$g \sim g_c$
Oscillations disappear, merging with SUGRA mode

$g \gg g_c$
Long lived oscillations 'SUGRA' modes trapped

Summary

1. At $g=0$, perturbations of supertubes correspond to 'quasi-oscillations' without drift even though there is a family of degenerate configurations.

2. For 'thin ring' limit, gravity reaches some distance away from ring ($\sim Q$) & we have $Q << a$ (Charge radius; a: Ring radius)

   Behavior is still oscillitory & time period matches with $g=0$ case.

3. Thick ring $Q \sim a$. Energy leaks to infinity & hence no oscillation

4. Deep throat: $Q >> a$

   New long-lived excitations emerge

5. Distinction between bound & unbound states appears to be the absence or presence of 'drift' modes. Possibly this is a general feature of bound states (absence of drift modes)