AN INTRODUCTION TO CLOSED STRINGS

[STRING SYMPOSIUM OF THE NATIONAL ACADEMY OF SCIENCES, AT H.R.I. ALLAHABAD, OCT. 1-2]

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OUTLINE OF THE TALK

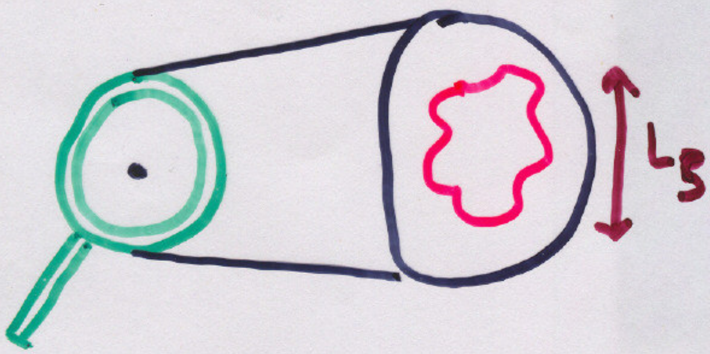
* WHY STRINGS ?
* CLASSICAL DYNAMICS OF POINT PARTICLES AND STRINGS
* QUANTUM STRINGS - THE SPECTRUM
* QUANTUM STRINGS - INTERACTIONS
* COMPACTIFICATIONS AND T-DUALITY
* SUMMARY

Why Strings?

→ To Cure Infinities In QFT

(specially Quantum Gravity) Caused By Point Like Interactions ($\Rightarrow$ short distance divergences)

Replace Notion Of Point Particle As Fundamental Object. By An Extended One Dimensional Object – A String.



* What Is The Extent l_s?

Characteristic Scale Of Quantum Effects In Gravity – Planck Scale.

$$M_P^2 = \frac{\hbar c}{G_N} \Rightarrow l_P^2 = \frac{\hbar G_N}{c^3}$$

$$\sim (10^{19}\ \mathrm{GeV}/c^2)^2 \qquad \sim (10^{-35}\ \mathrm{m})^2$$

$\therefore$ TO CURE INFINITIES IN QTM. GRAV.

EXPECT $L_s \sim L_P \Rightarrow$ AS OF NOW,

UNOBSERVABLY SMALL.

* ONE DIMENSIONAL STRING CHARACTERISED

BY MASS / UNIT LENGTH = TENSION.

BY UNCERTAINTY PRINCIPLE, ENERGY $\sim \frac{1}{L_s}$

$\Rightarrow$ TENSION $= \frac{1}{L_s^2} \equiv \frac{1}{2\pi\alpha'}$ ($\hbar = c = 1$ UNITS)

* WHY NOT OTHER EXTENDED OBJECTS?

AS OF NOW, WE DO NOT KNOW HOW TO QUANTIZE MEMBRANES OR OTHER HIGHER DIM. OBJECTS, TREATING THEM AS THE FUNDAMENTAL ENTITIES.

HOWEVER, EXTENDED OBJECTS OF VARIOUS DIMENSIONS ARISE COLLECTIVELY (BOUND STATES) FROM FUNDAMENTAL STRINGS.

Classical Dynamics Of Particles And Strings

* Describe Dynamics By Specifying Action

e.g. Free Particle (Spinless)

ⓐ $S = \frac{1}{2} m \int (\dot{x}^i)^2 dt \xrightarrow[\text{Of Motion}]{\text{Eqn.}} m\ddot{x}^i = 0$ (Non-Relativistic)

$(i = 1 \ldots D-1)$

ⓑ $S = -mc^2 \int \sqrt{1 - \frac{1}{c^2}\left(\frac{dx^i}{dt}\right)^2}\, dt.$ (Relativistic)

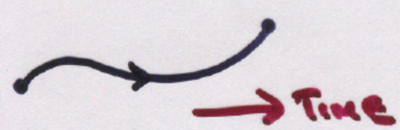
$$\left[\simeq -mc^2 \int dt + \frac{1}{2} m \int (\dot{x}^i)^2 dt + \ldots \right]$$

$$= -mc \int \sqrt{\left(\frac{dx^\mu}{d\tau}\right)^2}\, d\tau. \xrightarrow[\text{Of Motion}]{\text{Eqn.}} m\frac{d^2 x^\mu}{d\tau^2} = 0.$$

$(\mu = 0 \ldots D-1)$

$$= -mc \underbrace{\int d\tau}$$

Proper Length Of Worldline Of Particle



Thus Relativistic Action Has Natural Geometric Interpretation. Equation Of Motion That Of A Geodesic. (even in a general background metric $g_{\mu\nu}(x)$)

Actions For Strings:

(a) Free String - Nonrelativistic

Obeys wave eqn. $\dfrac{\partial^2 X^\perp}{\partial \tau^2} - \dfrac{1}{v^2}\dfrac{\partial^2 X^\perp}{\partial \sigma^2} = 0$

Transverse Dirns.

Spatial dirn. along string

Follows from an action

$$S = \frac{1}{2}\underbrace{T}_{\text{Tension of the string}}\int\left[\left(\frac{\partial X^\perp}{\partial \tau}\right)^2 - \frac{1}{v^2}\left(\frac{\partial X^\perp}{\partial \sigma}\right)^2\right] d\tau\, d\sigma$$

Speed of String oscillations $\ll c$

Note that the Energy of the string is

$$E = \frac{1}{2}T\int\Big[\underbrace{\left(\frac{\partial X^\perp}{\partial \tau}\right)^2}_{\text{Kinetic energy}} + \frac{1}{v^2}\underbrace{\left(\frac{\partial X^\perp}{\partial \sigma}\right)^2}_{\text{Potential energy.}}\Big] d\tau\, d\sigma$$

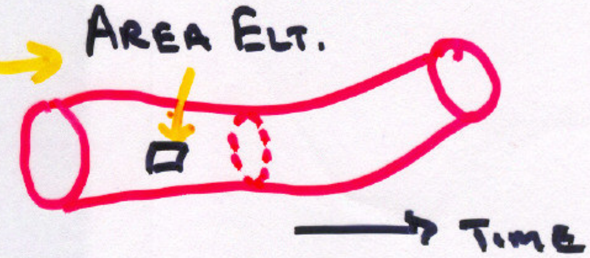
In D spacetime dimensions, $X^\perp$ has <u>D-2</u> components, e.g. 2 transverse components in 4 spacetime dimensions.

(b) FREE STRING – RELATIVISTIC

HOW DO WE DESCRIBE DYNAMICS OF A RELATIVISTIC STRING?

NATURAL ACTION GIVEN BY GEOMETRIC PRINCIPLE GENERALISING POINT PARTICLE.

$$S = -\underbrace{T}_{\text{TENSION}} \int dA$$



i.e. ACTION PROPORTIONAL TO PROPER AREA OF "WORLDSHEET" OF STRING. [NAMBU-GOTO ACTION]

GEOMETRICALLY, FOR A STRING IN D SPACETIME DIM., PROPER AREA GIVEN BY

$$\int dA = \int \sqrt{(\partial_\tau X^\mu \partial_\sigma X_\mu)^2 - (\partial_\tau X^\mu \partial_\tau X_\mu)(\partial_\sigma X^\nu \partial_\sigma X_\nu)} \times d\tau\, d\sigma$$

$$= \int d\tau\, d\sigma \sqrt{-\det(\gamma_{\alpha\beta})}$$

where $\gamma_{\alpha\beta}$ is the induced metric on the world sheet

$$\gamma_{\alpha\beta} = \eta_{\mu\nu} \frac{\partial X^\mu}{\partial \xi^\alpha} \frac{\partial X^\nu}{\partial \xi^\beta} \qquad \left[\begin{array}{l} \xi^1 = \tau \\ \xi^2 = \sigma \end{array}\right]$$

QUANTIZATION

TWO APPROACHES TO QUANTISING A CLASSICAL SYSTEM – ILLUSTRATE IN PT. PARTICLE CASE.

(a) HAMILTONIAN APPROACH (Heisenberg / Schrodinger)

* Start w/ worldline action, construct canonical momenta and impose commutation relations.
* Needs care since action has an invariance under $\tau \to \tau' = f(\tau)$ ["Reparametrisation inv."]
* Result: Physical states are of the form

$|\vec{p}\rangle$ (where $\vec{p}$ satisfies $(p^0)^2 = \vec{p}^{\,2} + m^2$)

These are all the expected Single particle states of a spinless particle of mass m.

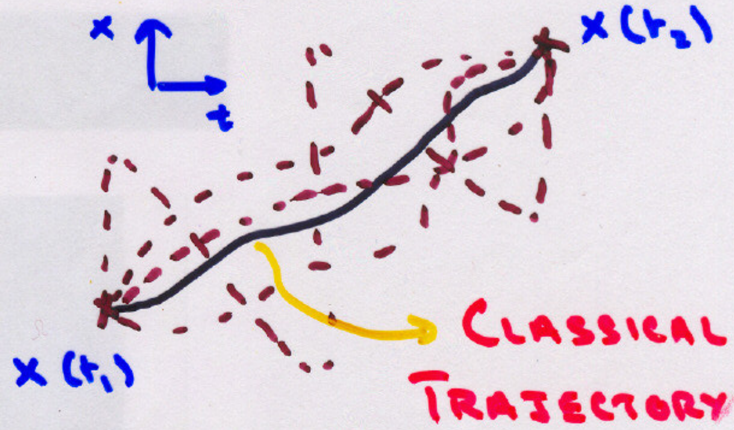
* For a particle with spin, one has to add fermionic degrees of freedom to the worldline action. $(\psi^\mu(\tau))$.

Quantisation gives states $|\vec{p}, s\rangle$

spin qtm.#

(b) Lagrangian Approach (Feynman)

$$\underbrace{\langle \vec{x}(t_2) | \vec{x}(t_1) \rangle}_{\text{Qtm. Amplitude}} = \sum_{\text{paths}} e^{i \frac{S}{\hbar} \underbrace{[x(t_2), x(t_1)]}_{\text{action on a given path}}}$$



THUS QTM. AMPLITUDE GETS CONTRIBUTIONS FROM ALL TRAJECTORIES JOINING $\vec{x}(t_1)$ TO $\vec{x}(t_2)$. EACH TRAJECTORY WEIGHTED BY A PHASE. $\hbar \to 0 \Rightarrow$ CLASSICAL TRAJECTORY DOMINATES.

GIVES AN INTUITIVE, GEOMETRIC PICTURE e.g. FOR FREE PARTICLE, DIFFERENT TRAJECTORIES WEIGHTED BY $e^{i \frac{[\text{Proper length}]}{\hbar}}$.

LAGRANGIAN APPROACH MORE SUITABLE FOR STUDYING INTERACTIONS (PARTICLES, STRINGS ..)

THUS PROCEDURE WE WILL FOLLOW FOR STRINGS:

* USE HAMILTONIAN APPROACH TO FIND PHYSICAL STATES AND SPECTRUM
* USE LAGRANGIAN APPROACH TO STUDY INTERACTIONS OF STRINGS.

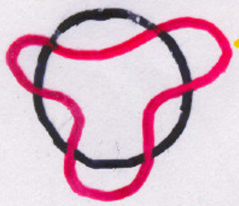
HAMILTONIAN APPROACH

* START w/ NAMBU-GOTO WORLDSHEET ACTION. CONSTRUCT MOMENTA CONJUGATE TO $X^\mu(\tau,\sigma)$ AND IMPOSE CANONICAL COMMUTATION RELATIONS.
* AS IN PARTICLE CASE, NEEDS TO BE DONE CAREFULLY BECAUSE OF REPARAMET-RISATION INVARIANCE OF ACTION UNDER

$$\tau \to \tau' = f(\tau,\sigma)$$
$$\sigma \to \sigma' = g(\tau,\sigma)$$

i.e. freedom to choose intrinsic coordinates (τ,σ) on worldsheet.

So H_0 Is An Infinite Set Of Simple Harmonic Oscillators (w/ each of the (D-2) oscillators $x_n^\perp$ and $\tilde{x}_n^\perp$ having identical frequency $\omega_n = n$)



An Oscillation with $n = 3$.

The Quantum String States Can Have Any Of These Internal Oscillators Excited. (to arbitrary level).

In Terms Of Creation / Annihilation Ops.

$$(a_n^\perp)^\dagger \equiv \frac{1}{\sqrt{2}}(p_n^\perp + i n x_n^\perp) \quad , \quad (\tilde{a}_n^\perp)^\dagger \equiv \frac{1}{\sqrt{2}}(\tilde{p}_n^\perp + i n \tilde{x}_n^\perp)$$

Can Construct States

$$(a_n^\perp)^\dagger \ldots\ldots (\tilde{a}_m^\perp)^\dagger \ldots |0\rangle$$

Labelled By Occupation Number Of Osc.

Eigenvalue Spectrum Of $H_0 = N - 2$

Total occupation number

zero pt. energy.

* **RESULT:** PHYSICAL STATES OF THE FORM

$$|p_0 - p_{\parallel}, \underbrace{\vec{p}_{\perp}}_{\text{transverse momentum to string}}, \{m\}\rangle \equiv |p_0 - \underbrace{p_{\parallel}}_{\text{longitudinal mom.}}, \vec{p}_{\perp}\rangle \otimes |\{m\}\rangle$$

The $\vec{p}$ label the centre of mass momenta.

While $\{m\}$ labels the <u>internal</u> oscillation modes of the string — arises as eigenvalues of a Hamiltonian H_0 describing the oscillations.

Classically H_0 is obtained from the Energy density $E = \frac{1}{2}\int_0^{2\pi} d\sigma \left[(\partial_\tau X^\perp)^2 + (\partial_\sigma X^\perp)^2\right]$

$$= \frac{1}{2}\sum_{n>0}\left[\dot{x}_n^{\perp 2} + n^2 x_n^{\perp 2}\right] + \frac{1}{2}\sum_{n>0}\left[(\dot{\tilde{x}}_n^\perp)^2 + n^2(\tilde{x}_n^\perp)^2\right]$$

w/ $X^\perp(\sigma) = \sum_{n>0} \underbrace{x_n}_{\text{Oscillation modes}} e^{in\sigma} + \sum_{n>0} \underbrace{\tilde{x}_n}_{\text{Oscillation modes}} e^{-in\sigma}$

i.e.

$$\boxed{H_0 = \frac{1}{2}\sum_{n>0}\left[(p_n^\perp)^2 + n^2(x_n^\perp)^2\right] + \frac{1}{2}\sum_{n>0}\left[(\tilde{p}_n^\perp)^2 + n^2(\tilde{x}_n^\perp)^2\right]}$$

FINALLY, THE STATES $|p_0 - p_{\parallel}, \vec{p}_\perp, \{m\}\rangle$

HAVE ENERGIES SATISFYING.

$$p_0^2 - p_{\parallel}^2 = \vec{p}_\perp^{\,2} + \underbrace{\frac{2(N-2)}{\alpha'}}_{M_N^2}$$

Occupation number of the states $\{m\}$

$$\boxed{\text{THUS DIFFERENT EXCITATIONS OF THE QUANTUM STRING CORRESPOND TO DIFFERENT SINGLE PARTICLE STATES w/ MASSES } M_N^2 = \frac{2(N-2)}{\alpha'}}$$

THERE ARE AN INFINITE NUMBER OF STATES w/ ARBITRARILY HIGH MASSES.

(Beyond Planck scale)

NOTE THAT THERE IS A STATE

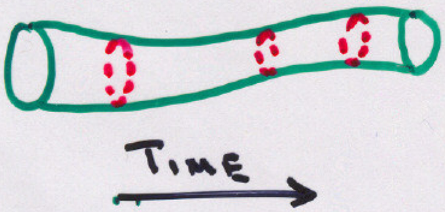
$$|ij\rangle \equiv (a_1^i)^+ (\tilde{a}_1^j)^+ |0\rangle \quad (i,j = 1 \ldots . D-2)$$

w/ $N = 1+1 = 2 \Rightarrow M^2 = 0$

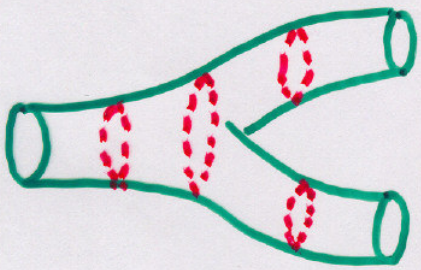
$(|ij\rangle + |ji\rangle)$ is symmetric in (i,j) – HAS THE RIGHT QTM. #'s TO BE A GRAVITON! (g_{ij})

INTERACTIONS OF QUANTUM STRINGS

CAN INCORPORATE INTERACTIONS EASILY IN LAGRANGIAN APPROACH.



FREE STRING PROPAGATION.



SINGLE STRING SPLITS INTO TWO STRINGS

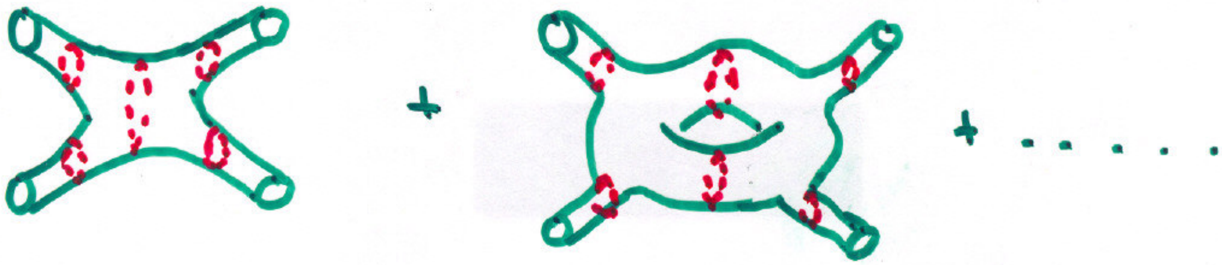
ALL AMPLITUDES CAN BE GIVEN BY THE FEYNMAN PRESCRIPTION $e^{iS/\hbar} = e^{i(\text{Area})/\hbar}$

CAN WEIGHT PROCESSES IN WHICH SINGLE STRING $\longrightarrow$ TWO STRINGS BY FACTOR $\underbrace{g_s}_{\text{STRING COUPLING}}$

CAN THEN HAVE PROCESSES WHERE STRING SPLITS AND REJOINS - LOOP DIAGRAMS

COMES WITH WEIGHT g_s^2

Can Now Study Quantum Scattering Amplitudes For Candidate Gravitons.



[These Are The String Feynman Diagrams Which Are Analogues Of

+ Other Such Pt. Particle Diagrams]

* Tree Level (i.e. without loops) Scattering Of Gravitons Shows Coupling As In General Relativity ($E \ll M_p$)

* Loop Diagrams Of Strings Are Finite – No Short Distance Divergence As In Field Theory. Because Of Lack Of Point Like Interaction For Strings.

* HOWEVER, POTENTIAL PROBLEM WITH PRESENCE OF UNPHYSICAL STATES

(HAVING $\| \; \|^2 < 0$)

CONSISTENCY REQUIREMENT GIVES $D = 26$!!

* ANOTHER PROBLEM – ALREADY PRESENT IN THE SPECTRUM. $M_N^2 = \frac{2(N-2)}{\alpha'}$

FOR $N = 0$ (Ground state - no excitations)

$M_0^2 < 0$. MIGHT SEEM UNPHYSICAL BUT ACTUALLY A SIGN OF INSTABILITY.

A TERM IN THE POTENTIAL: $-|M_0|^2 \phi^2$

* SPECTRUM OF THIS STRING ACTION ONLY BOSONIC – "BOSONIC STRING".

BUT REAL WORLD FULL OF FERMIONS.

THUS BOSONIC STRINGS WHILE CONTAINING GRAVITY HAVE A PROBLEM IN REALISING OTHER REALISTIC PARTICLES.

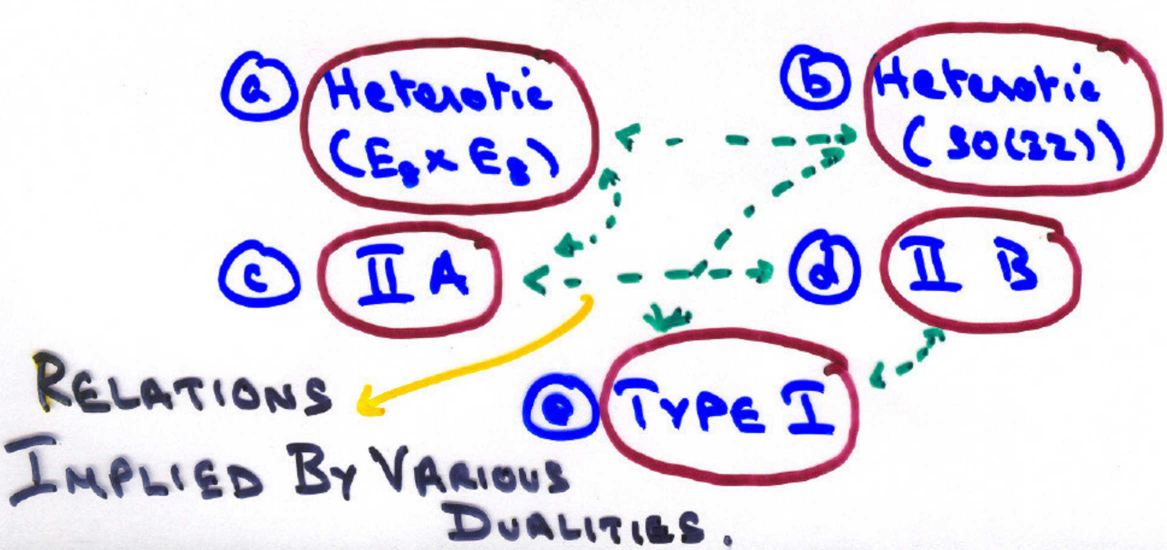
CAN CURE SOME OF THESE PROBLEMS BY INTRODUCING FERMIONIC d.o.f. ON WORLDSHEET – like on worldline.

QUANTISATION PROCEEDS SIMILARLY

– OSCILLATOR SPECTRUM w/ $m_N^2 = \frac{2N}{\alpha'}$

* NO "TACHYONIC" MODE. MASSLESS MODE CONTAINS GRAVITY.
* FERMIONS IN SPECTRUM AS WELL.
* CONSISTENCY CONDITION FOR NO UNPHYSICAL STATES $\Rightarrow$ D=10 !

ACTUALLY <u>FIVE</u> DIFFERENT CONSISTENT FERMIONIC ("SUPERSTRING") THEORIES.



COMPACTIFICATION + T-DUALITY

HOW CAN A THEORY IN 10 DIMENSIONS BE RELEVANT IN DESCRIBING 4D PHYSICS?

ANSWER: IF SIX SPATIAL DIMENSIONS ARE VERY SMALL - "COMPACT".

A SMALL CYLINDER LOOKS LIKE A ONE DIMENSIONAL LINE IF RESOLUTION IS LESS THAN SIZE OF CIRCULAR DIMENSION.

$\therefore$ IF THERE ARE SIX ADDITIONAL SMALL DIMENSIONS (e.g. of size $L_s \sim l_p \sim 10^{-35}$m) THEN WE WOULD NOT BE SENSITIVE TO THEIR PRESENCE AT LOW ENERGY ($E \ll \frac{1}{L_s}$)

THIS IDEA IS VERY OLD – KALUZA + KLEIN IN 1930'S – PROPOSED TO UNIFY GRAVITY w/ ELECTROMAGNETISM ($A_\mu = g_{\mu 5}$)

STRING THEORY ADMITS SUCH "COMPACTIFICATIONS" TO $D=4$. MANY INEQUIVALENT WAYS TO COMPACTIFY. SIMPLEST IS ON SIX CIRCLES. ALSO OTHER SIX DIMENSIONAL GEOMETRIES – PARTICULARLY USEFUL CLASS CALLED CALABI-YAU SPACES.

CAN GIVE RISE TO REALISTIC STANDARD MODEL LIKE THEORIES w/ LEFT HANDED FERMIONS, THREE GENERATIONS etc.

T-DUALITY: A NOVEL SYMMETRY EXHIBITED BY STRINGS ON A COMPACT SPACE. e.g. A CIRCLE OF RADIUS R.

* STRING THEORY ON CIRCLE OF RADIUS R IS EQUIVALENT TO STRING THEORY ON CIRCLE OF RADIUS α'/R.

$\Rightarrow$ STRINGS DO NOT "SEE" LENGTHS IN SPACE TIME $< \sqrt{\alpha'} \sim l_s$ (QUANTUM SPACETIME)

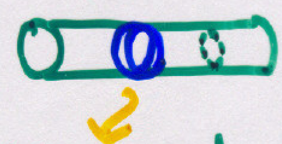
HOW DOES THIS HAPPEN?

* STATE w/ MOMENTUM P ALONG CIRCLE OF RADIUS R. $\Rightarrow$ $P = \frac{n}{R} \times 2\pi$ (i.e. quantised)

IF STRING OSCILLATORS NOT EXCITED AND ALL OTHER MOMENTA ZERO THEN

$$P_0^2 = P^2 = \frac{n^2}{R^2} \times (2\pi)^2$$

* STATE w/ NO MOMENTUM BUT WRAPPING AROUND THE CIRCLE n TIMES.



AGAIN, ASSUME NO STRING OSCILLATORS EXCITED BUT

$X(\sigma) = m\sigma R$ (So that $X(2\pi) = X(0) + 2\pi m R$)

FROM THE $\int (\partial_\sigma X)^2 d\sigma$ term one gets THE ENERGY OF THIS WINDING # STATE.

$$P_0^2 = n^2 (2\pi)^2 \frac{R^2}{\alpha'^2}$$

THUS EXCHANGING WINDING m STATES WITH MOMENTUM m STATES TOGETHER w/ $R \to \frac{\alpha'}{R}$ KEEPS SPECTRUM UNCHANGED.

SUMMARY

* Replaced Point Particle Notion By 1D Extended Object - String - To Cure Infinities In Quantum Theory.
* Dynamics Of String Governed By Action $\propto$ (Area) Of String Worldsheet.
* Quantisation $\Rightarrow$ Oscillation Of String = Bunch Of Quantum Harmonic Oscillators
* Energy Levels Give Infinite Tower Of Equally Spaced Particle Masses $\propto M_p$
* Naturally Includes Massless Graviton (has the right interactions). Gravity As Output With Only Relativistic String As Input!
* Consistency (of fermionic string) $\Rightarrow$ $D=10$!
* Realistic String Theories Obtained By Compactification. (shows novel symmetries and minimum length of spacetime)