# GENERAL THEORY OF RELATIVITY

1) NEWTONIAN GRAVITY

2) SPECIAL RELATIVITY

3) GEODESIC EQUATION

4) TENSORS & NON-TENSORS
   - RIEMANN, RICCI & EINSTEIN TENSORS

5) EINSTEIN EQUATION

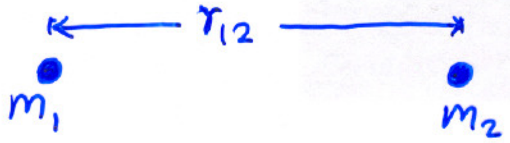
6) BLACK HOLE SOLUTION

7) GRAVITY & QUANTUM THEORY

8) QUANTUM THEORY & DIVERGENCES

# THEORY OF GRAVITY

## NEWTONIAN MECHANICS




GRAVITATIONAL FORCE

$$\vec{F}_g = -\frac{G_N m_1 m_2}{r_{12}^2}\,\hat{r}_{12}$$

$G_N$ : NEWTON'S CONSTANT

THIS FORCE ACTS INSTANTANEOUSLY

- GRAVITY IS UNIVERSAL

  i.e. A UNIVERSAL INTERACTION BETWEEN ALL MASS

- GRAVITY IS ALWAYS ATTRACTIVE FORCE

  – NO NEGATIVE GRAVITATIONAL CHARGE

- GRAVITY IS THE WEAKEST FORCE

$$\frac{F_g}{F_e} = \frac{G_N m_p^2 / r^2}{e^2/4\pi\epsilon_0 r^2} = \frac{G_N m_p^2}{e^2/4\pi\epsilon_0} \sim 10^{-36}$$

$m_p$ & $e$ MASS & CHARGE OF THE PROTON

# SPECIAL RELATIVITY

- SPEED OF LIGHT IS CONSTANT
- ALL INERTIAL FRAMES ARE EQUIVALENT

INSTANTANEOUS FORCE IS NOT CONSISTENT WITH POSTULATES OF SPECIAL RELATIVITY

NEWTON'S LAW OF GRAVITY SHOULD BE MODIFIED IN THE RELATIVISTIC SITUATIONS

NOTATION:

EUCLIDEAN SPACE: LINE ELEMENT

$$dS^2 = (dx)^2 + (dy)^2 + (dz)^2$$

MINKOWSKI SPACE (SPECIAL RELATIVITY):
LINE ELEMENT

$$(ds)^2 = -(c\,dt)^2 + (dx)^2 + (dy)^2 + (dz)^2$$

C: SPEED OF LIGHT

$$= \eta_{\mu\nu}\, dx^\mu dx^\nu$$

$\mu, \nu = (0, 1, 2, 3)$

$x^0 = ct\,,\ x^1 = x,\ x^2 = y,\ x^3 = z$

$\eta_{\mu\nu} = \mathrm{diag}(-1, 1, 1, 1) \rightarrow$ METRIC

REPEATED INDICES ARE SUMMED.

$$a_\mu b^\mu = -a_0 b_0 + a_1 b_1 + a_2 b_2 + a_3 b_3 = \eta_{\mu\nu} a^\mu b^\nu$$

# LORENTZ TRANSFORMATIONS

$$x^{0\prime} = \gamma(x^0 - \beta x^1) \qquad x^{1\prime} = \gamma(x^1 - \beta x^0)$$
$$x^{2\prime} = x^2 \quad , \quad x^{3\prime} = x^3$$

$$x'^{\mu} = \Lambda^{\mu}{}_{\nu}\, x^{\nu} \qquad \Lambda = \begin{pmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & & 1 \end{pmatrix} = \frac{\partial x'}{\partial x}$$

$$\beta = \frac{v}{c} \ , \ \gamma = \frac{1}{\sqrt{1-\beta^2}}$$

- PRESERVE THE MINKOWSKI METRIC $\eta_{\mu\nu}$.

$$\rightarrow \quad \eta'_{\mu\nu} = \eta_{\mu\nu} = \Lambda_{\mu}{}^{\rho}\, \eta'_{\rho\sigma}\, \Lambda^{\sigma}{}_{\nu}$$

- MAP ONE INERTIAL FRAME ONTO OTHER INERTIAL FRAME

- PRESERVES FORM OF RELATIVISTIC EQUATIONS OF MOTION

→ e.g. FREE PARTICLE

$$m\frac{d^2 x'^{\mu}}{d\tau^2} = 0 \longrightarrow m\frac{d^2 x^{\mu}}{d\tau^2} = 0$$

THIS EQUATION CAN BE DERIVED FROM THE LAGRANGIAN

$$L = \left(-\eta_{\mu\nu}\frac{dx^{\mu}}{ds}\frac{dx^{\nu}}{ds}\right)^{1/2}$$

$$= \left[\left(\frac{dx^0}{ds}\right)^2 - \left(\frac{dx^1}{ds}\right)^2 - \left(\frac{dx^2}{ds}\right)^2 - \left(\frac{dx^3}{ds}\right)^2\right]^{1/2}$$

LET, $x^\mu \to x'^\mu = f^\mu(x)$ ARBITRARY COORDINATE TRANSFORMATION

$$\frac{d^2 x'^\mu}{d\tau^2} = 0 \longrightarrow \frac{d^2 x^\mu}{d\tau^2} + \frac{\partial x^\mu}{\partial f^\nu}\frac{\partial^2 f^\nu}{\partial x^\rho \partial x^\sigma}\frac{dx^\rho}{d\tau}\frac{dx^\sigma}{d\tau} = 0$$

DEFINE $\Gamma^\mu_{\rho\sigma} = \frac{\partial x^\mu}{\partial f^\nu}\frac{\partial^2 f^\nu}{\partial x^\rho \partial x^\sigma}$ CHRISTOFFEL CONNECTION

$$\to \frac{d^2 x^\mu}{d\tau^2} + \Gamma^\mu_{\rho\sigma}\frac{dx^\rho}{d\tau}\frac{dx^\sigma}{d\tau} = 0$$ GEODESIC EQUATION

$f^\mu(x)$ A LINEAR FUNCTION OF $x \Rightarrow \Gamma^\mu_{\rho\sigma} = 0 \quad \forall \mu, \rho, \sigma$

$\Gamma \neq 0$ FOR GENERAL $f^\mu(x)$.

$\Rightarrow$ FREE PARTICLE IN $x'$-FRAME IS NOT A FREE PARTICLE IN $x$-FRAME ($\eta_{\mu\nu}$ IS NOT PRESERVED)

$\Gamma \neq 0 \Rightarrow$ NON-ZERO ACCELERATION

$\Rightarrow f^\mu$ IS A MAP FROM AN ACCELERATED FRAME

FREE PARTICLE TRAVELS ON A STRAIGHT LINE (A SHORTEST PATH IN MINKOWSKI SPACE): $x'^\mu = a^\mu \tau + b^\mu$

SOLUTION TO GEODESIC EQUATION $\Rightarrow$ SHORTEST PATH IN AN ACCELERATED FRAME

EXAMPLES:

1) TWO DIMENSIONAL PLANE IN POLAR COORDINATES

$(r, \theta)$ COORDINATES

METRIC

$$ds^2 = g_{\mu\nu}\, dx^\mu dx^\nu = dr^2 + r^2 d\theta^2$$

$\Rightarrow \quad g_{rr} = 1 , \quad g_{\theta\theta} = r^2 , \quad g_{r\theta} = 0 \qquad g = \begin{pmatrix} 1 & 0 \\ 0 & r^2 \end{pmatrix}$

INVERSE METRIC: $\quad g^{rr} = 1 , \quad g^{\theta\theta} = \frac{1}{r^2} , \quad g^{r\theta} = 0$

ACTION FOR A PARTICLE

$$I = \int d\tau \left( \left(\frac{dr}{d\tau}\right)^2 + r^2 \left(\frac{d\theta}{d\tau}\right)^2 \right)^{1/2}$$

GEODESIC EQUATIONS

$$\frac{d^2 r}{d\tau^2} - r \left(\frac{d\theta}{d\tau}\right)^2 = 0 \; ; \qquad \frac{d^2\theta}{d\tau^2} + \frac{2}{r}\frac{dr}{d\tau}\frac{d\theta}{d\tau} = 0$$

CHRISTOFFEL CONNECTIONS

$\Gamma^r_{\theta\theta} = -r , \quad \Gamma^\theta_{r\theta} = \frac{1}{r} \qquad$ REST ALL VANISH

RIEMANN TENSOR: $R_{\mu\nu\rho\sigma} = 0 \quad \forall\, \mu, \nu, \rho, \sigma = r, \theta$

$\Rightarrow$ RICCI TENSOR: $R_{\mu\nu} = 0$ & RICCI SCALAR $R = 0$

# TWO DIMENSIONAL SPHERE

EMBEDDED IN 3 DIMENSIONS

$$(x^1)^2 + (x^2)^2 + (x^3)^2 = 1$$

METRIC ON FLAT 3 DIMENSIONAL SPACE

$$ds^2 = (dx^1)^2 + (dx^2)^2 + (dx^3)^2$$

ANY POINT ON A SPHERE IS PARAMETRIZED BY TWO ANGLES $\theta, \phi$.

THE INDUCED METRIC ON THE SPHERE IS

$$ds^2 = \sum_{i=1}^{3} \left(\frac{\partial x^i}{\partial \theta}\right)^2 d\theta^2 + \sum_{i=1}^{3} \left(\frac{\partial x^i}{\partial \phi}\right)^2 d\phi^2$$

PARAMETRIZE

$$x^1 = \sin\theta \cos\phi$$

$$x^2 = \sin\theta \sin\phi$$

$$x^3 = \cos\theta$$

THEN THE INDUCED METRIC BECOMES

$$ds^2 = d\theta^2 + \sin^2\theta \, d\phi^2$$

## 2) TWO DIMENSIONAL SPHERE (CONTINUED)

COORDINATES – $(\theta, \phi)$

METRIC:

$$ds^2 = g_{\mu\nu}\, dx^\mu dx^\nu = d\theta^2 + \sin^2\theta\, d\phi^2$$

$$g_{\theta\theta} = 1, \quad g_{\phi\phi} = \sin^2\theta, \quad g_{\theta\phi} = 0 \qquad g = \begin{pmatrix} 1 & 0 \\ 0 & \sin^2\theta \end{pmatrix}$$

INVERSE METRIC: $g^{\theta\theta} = 1, \quad g^{\phi\phi} = \dfrac{1}{\sin^2\theta}, \quad g^{\theta\phi} = 0$

ACTION FOR A PARTICLE

$$I = \int d\tau \left( \left(\frac{d\theta}{d\tau}\right)^2 + \sin^2\theta \left(\frac{d\phi}{d\tau}\right)^2 \right)^{1/2}$$

GEODESIC EQUATIONS

$$\frac{d^2\theta}{d\tau^2} - \sin\theta\cos\theta \left(\frac{d\phi}{d\tau}\right)^2 = 0, \qquad \frac{d^2\phi}{d\tau^2} + 2\frac{\cos\theta}{\sin\theta}\frac{d\theta}{d\tau}\frac{d\phi}{d\tau} = 0$$

CHRISTOFFEL CONNECTIONS

$$\Gamma^\theta_{\phi\phi} = -\sin\theta\cos\theta, \quad \Gamma^\phi_{\theta\phi} = \frac{\cos\theta}{\sin\theta}, \quad \text{REST ALL VANISH}$$

RIEMANN TENSOR: $R_{\theta\phi\theta\phi} = -\sin^2\theta$

RICCI TENSOR: $R_{\theta\theta} = 1, \quad R_{\phi\phi} = \sin^2\theta, \quad R_{\theta\phi} = 0$

RICCI SCALAR: $R = 2$

$$\frac{d^2x^\mu}{d\tau^2} = -\Gamma^\mu_{\rho\sigma}\frac{dx^\rho}{d\tau}\frac{dx^\sigma}{d\tau}$$ GEODESIC EQUATION

$$\frac{d^2x^\mu}{d\tau^2} = -\frac{e}{m}F^\mu{}_\nu\frac{dx^\nu}{d\tau}$$ LORENTZ FORCE EQ$^{\underline{N}}$

$e$ & $m$ : CHARGE & MASS OF THE PARTICLE

## LAGRANGIAN FOR GEODESIC EQUATION

$$L = \left(-g_{\mu\nu}(x)\frac{dx^\mu}{ds}\frac{dx^\nu}{ds}\right)^{1/2}$$

$\boxed{g^{\mu\nu} \rightarrow \text{INVERSE METRIC}}$

$$\Rightarrow \Gamma^\mu_{\rho\sigma} = \frac{1}{2}g^{\mu\nu}\left(\frac{\partial g_{\nu\sigma}}{\partial x^\rho} + \frac{\partial g_{\rho\nu}}{\partial x^\sigma} - \frac{\partial g_{\rho\sigma}}{\partial x^\nu}\right)$$

NON-RELATIVISTIC LIMIT : $\Gamma^\mu_{0i}$, $\Gamma^\mu_{ij}$ SMALL

$$\Gamma^\mu_{00} \longrightarrow \Gamma^i_{00} = -\frac{1}{2}\nabla^i g_{00} \qquad \frac{dx^\rho}{d\tau} \rightarrow \frac{dx^0}{d\tau} = 1$$

$g_{\mu\nu}$ AS A SMALL DEVIATION FROM $\eta_{\mu\nu}$

$$\Rightarrow \quad g_{00}(x) = -1 - 2\phi(x)$$

$$\Gamma^i_{00} = \nabla^i\phi(x)$$

GEODESIC EQ$^{\underline{N}}$ $$\frac{d^2x^i}{dt^2} = -\nabla^i\phi(x)$$

$\phi(x)$ : NON-RELATIVISTIC GRAVITATIONAL POTENTIAL.

$\Rightarrow$ ACCELERATING FRAMES IMITATE EFFECTS OF GRAVITATIONAL FIELD. IN PARTICULAR, UNIFORM ACCELERATION IS INDISTINGUISHABLE FROM UNIFORM GRAVITATIONAL FIELD.

WE NEED A CRITERION TO DISTINGUISH DIFFERENT METRICS $g_{\mu\nu}(x)$.

SINCE COORDINATE CHANGE DO NOT CHANGE PROPERTIES OF SPACE-TIME, WE CLASSIFY METRICS UP TO COORDINATE CHANGE.

CATEGORISE FUNCTIONS OF $g_{\mu\nu}$ AND ITS DERIVATIVES INTO TWO SETS UNDER COORDINATE TRANSFORMATIONS

SET A: FUNCTIONS TRANSFORMING HOMOGENEOUSLY (TENSORS)

$$f'_{\mu_1 \cdots \mu_n}(x') = \left(\frac{\partial x^{\nu_1}}{\partial x'^{\mu_1}}\right) \cdots \left(\frac{\partial x^{\nu_n}}{\partial x'^{\mu_n}}\right) f_{\nu_1 \cdots \nu_n}(x)$$

SET B: FUNCTIONS TRANSFORMING INHOMOGENEOUSLY (NON-TENSORS)

$$\tilde{f}'_{\mu_1 \cdots \mu_n}(x') = \left(\frac{\partial x^{\nu_1}}{\partial x'^{\mu_1}}\right) \cdots \left(\frac{\partial x^{\nu_n}}{\partial x'^{\mu_n}}\right) \tilde{f}_{\nu_1 \cdots \nu_n}(x) + \sum_{\substack{k \geq 2 \\ i \neq p}}^{n-1} \frac{\partial^k x^{\nu_i}}{\partial x'^{\mu_{j_1}} \cdots \partial x'^{\mu_{j_k}}} \frac{\partial x^{\nu_p}}{\partial x'^{\mu_\ell}} \cdots$$

FUNCTIONS IN SET-B CAN BE SET TO ZERO BY COORDINATE TRANSFORMATIONS.

PROPERTIES OF <u>FNS</u> IN SET-A DO NOT CHANGE.

CONCENTRATE ON SET-A: IF A FUNCTION IS NON-VANISHING IN ONE FRAME, IT REMAINS SO UNDER ALL NON-SINGULAR COORDINATE CHANGES.

$\Gamma$ BELONGS TO SET-B. LOOK FOR OTHER DERIVATIVES OF $g_{\mu\nu}$.

RIEMANN CURVATURE TENSOR

$$R^{\mu}{}_{\nu\rho\sigma} = \partial_\sigma \Gamma^{\mu}_{\nu\rho} - \partial_\rho \Gamma^{\mu}_{\nu\sigma} + \Gamma^{\alpha}_{\nu\rho}\Gamma^{\mu}_{\alpha\sigma} - \Gamma^{\alpha}_{\nu\sigma}\Gamma^{\mu}_{\alpha\rho}$$

RICCI CURVATURE TENSOR RICCI SCALAR

$$R_{\mu\nu} = R^{\lambda}{}_{\mu\lambda\nu} \qquad\qquad R = R^{\mu}{}_{\mu}$$

$R^{\mu}{}_{\nu\rho\sigma}$, $R_{\mu\nu}$, $R$ BELONG TO SET-A.

- $R_{\mu\nu\rho\sigma} = R_{\rho\sigma\mu\nu} = -R_{\nu\mu\rho\sigma} = -R_{\mu\nu\sigma\rho}$
- $R_{\mu\nu\rho\sigma} + R_{\mu\rho\sigma\nu} + R_{\mu\sigma\nu\rho} = 0$
- BIANCHI IDENTITY
$$D_\alpha R_{\mu\nu\rho\sigma} + D_\sigma R_{\mu\nu\alpha\rho} + D_\rho R_{\mu\nu\sigma\alpha} = 0$$

WHERE $D_\alpha V^{\mu} = \dfrac{\partial V^{\mu}}{\partial x^{\alpha}} + \Gamma^{\mu}_{\alpha\nu} V^{\nu}$ COVARIANT DERIVATIVE

$$g^{\alpha\nu} g^{\mu\rho}(\text{BIANCHI}) \Rightarrow D^{\mu}(\underbrace{R_{\mu\nu} - \tfrac{1}{2} g_{\mu\nu} R}) = 0$$

EINSTEIN CURVATURE TENSOR $\longleftarrow G_{\mu\nu}$

NEWTONIAN GRAVITY IN FREE SPACE

$$\nabla^2 \phi(x) = 0 \qquad \phi(x) \rightarrow \text{GRAVITATIONAL POTENTIAL}$$

IN PRESENCE OF MASS

$$\nabla^2 \phi(x) = -4\pi G_N \rho \qquad \begin{array}{l} G_N \rightarrow \text{NEWTON'S CONSTANT} \\ \rho \rightarrow \text{MASS DENSITY} \end{array}$$

WE NEED TO WRITE THE COVARIANT VERSION OF THIS EQUATION.

ELECTROMAGNETIC ANALOGY

COULOMB'S LAW

$$\nabla^2 \phi_e(x) = -(4\pi\epsilon_0)^{-1} \rho_e(x)$$

$\phi_e(x) \rightarrow$ ELECTROSTATIC POTENTIAL

$\rho_e(x) \rightarrow$ ELECTRIC CHARGE DENSITY

RELATIVISTIC VERSION:

$$\nabla_\mu F^{\mu\nu} = -(4\pi\epsilon_0)^{-1} J^\nu$$

$F_{\mu\nu}$ : ELECTROMAGNETIC FIELD STRENGTH
$(\partial_\mu A_\nu - \partial_\nu A_\mu)$ , $A_\mu = (\phi_e, \vec{A})$

$J^\mu$ : ELECTROMAGNETIC CURRENT $(\rho_e, \vec{J})$

BY ANALOGY, MASS DENSITY $\rho(x) \rightarrow T_{\mu\nu}(x)$
THE ENERGY MOMENTUM TENSOR

$\nabla^2\phi(x) \rightarrow G_{\mu\nu}(x)$ THE EINSTEIN TENSOR

EINSTEIN EQUATION : $\boxed{G_{\mu\nu}(x) = -4\pi G_N T_{\mu\nu}(x)}$

THE EINSTEIN EQUATION CAN BE DERIVED FROM

$$S = \frac{1}{8\pi G_N} \int d^4x \sqrt{-\det g}\; R(x) \rightarrow \text{EINSTEIN-HILBERT ACTION}$$

THE EFFECT OF GRAVITY IS MIMICKED BY CURVED SPACE-TIME.

MASS GENERATES GRAVITATIONAL POTENTIAL

$\Rightarrow$ MASS GENERATES CURVATURE OF SPACE-TIME

## SOLUTION TO EINSTEIN EQUATION:

ANOLOGY: ELECTROSTATICS AROUND A POINT CHARGE

$$\nabla^2 \phi_e(x) = q\,\delta^3(\vec{x}) \qquad q \rightarrow \text{ELECTRIC CHARGE}$$

1) IGNORE $\delta$-FN SOURCE & SOLVE $\nabla^2 \phi_e(x) = 0$

SOLUTION: $\phi_e(x) = C/r \qquad r = \sqrt{x^2+y^2+z^2}$

WE HAVE USED SPHERICAL SYMMETRY OF THE PROBLEM

2) DETERMINE ELECTRIC FLUX AT INFINITY

$$\vec{E} = -\vec{\nabla}\phi_e = C/r^2$$

3) USE GAUSS' LAW

$$\int \vec{E}\cdot d\vec{s} = \int d^3x\, q\,\delta^3(\vec{x}) = q$$

$$\int \vec{E}\cdot d\vec{s} = C\int \frac{1}{r^2} r^2 d\Omega = 4\pi C$$

$$\Rightarrow C = \frac{q}{4\pi} \quad \& \quad \phi_e(x) = \frac{q}{4\pi r}$$

GRAVITATIONAL FIELD DUE TO A POINT PARTICLE OF MASS 'M'.

1) IGNORE THE SOURCE:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 0 \Rightarrow R_{\mu\nu} = 0$$

2) SPHERICAL SYMMETRY DUE TO POINT MASS

$\Rightarrow$ SOLUTION OF THE FORM

$$d\tau^2 = -g_1(r)\,dt^2 + g_2(r)\,dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)$$

USING THIS FORM OF THE SOLUTION EVALUATE ALL COMPONENTS OF $R_{\mu\nu}$.

$$\Rightarrow g_1(r) = \frac{1}{g_2(r)} = 1 + \frac{1}{cr}$$

3) USE GAUSS' LAW

$$\Rightarrow c_0 = -\frac{1}{2G_N M}$$

$$\Rightarrow d\tau^2 = -\left(1 - \frac{2G_N M}{r}\right)dt^2 + \frac{1}{\left(1 - \frac{2G_N M}{r}\right)}dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)$$

THE SCHWARZSCHILD SOLUTION

- METRIC IS SINGULAR AT $r = 2G_N M$.
  - → ARTIFACT OF CHOICE OF COORDINATES
  - → SURFACE AT $r = 2G_N M$ IS CALLED <u>HORIZON</u>
- METRIC HAS A GENUINE SINGULARITY AT $r = 0$.

  - COMPONENTS OF RIEMANN TENSOR DIVERGE AT $r = 0$.
  - BLACK HOLE SOLUTION

SCHWARZSCHILD SOLUTION IS PARAMETRIZED BY ITS MASS.

MORE GENERAL BLACK HOLE SOLUTIONS ARE PARAMETRIZED BY MASS $M$, ELECTRIC CHARGE $q_e$, MAGNETIC CHARGE $q_m$ AND ANGULAR MOMENTUM $J$.

# GRAVITY AND QUANTUM THEORY

AS WE SAW IN ASHOKE'S LECTURE THAT THE STANDARD MODEL DESCRIBES ALL THE INTERACTIONS OF ELEMENTARY PARTICLES, EXCEPT GRAVITY, IN TERMS OF A QUANTUM THEORY.

GRAVITY ON THE OTHER HAND WORKS QUITE WELL AS A CLASSICAL THEORY.

WHY DON'T WE COMBINE CLASSICAL GRAVITY WITH QUANTUM THEORY OF STANDARD MODEL?

CLAIM: RESULTING THEORY WILL CONTRADICT WITH UNCERTAINTY PRINCIPLE

MEASURE GRAVITATIONAL FIELD GENERATED BY AN ELECTRON. SINCE GRAVITY IS CLASSICAL WE CAN MEASURE $g_{\mu\nu}$ & ALL ITS TIME DERIVATIVES TO ARBITRARY ACCURACY. CAN BE USED TO COMPUTE POSITION & MOMENTUM OF THE ELECTRON TO ARBITRARY ACCURACY. CONTRADICTION WITH UNCERTAINTY PRINCIPLE

THUS COMBINING CLASSICAL GRAVITY WITH QUANTUM THEORY OF STANDARD MODEL IS INCONSISTANT.

WE NEED QUANTUM THEORY OF GRAVITY SO AS TO COMBINE IT WITH STANDARD MODEL.

TO CARRY OUT QUANTIZATION, SPLIT THE LAGRANGIAN INTO QUADRATIC PART AND A PART INVOLVING HIGHER ORDER TERMS

THE GRAVITY LAGRANGIAN CONTAINS $g_{\mu\nu}$, ITS DERIVATIVES AS WELL AS ITS INVERSE $g^{\mu\nu}$.

THE LAGRANGIAN IS NON-LINEAR

EXPAND $g_{\mu\nu}(x)$ AROUND $\eta_{\mu\nu}$.

$$g_{\mu\nu}(x) = \eta_{\mu\nu} + k\, h_{\mu\nu}(x) \qquad k^2 = 8\pi G_N$$

$$g^{\mu\nu}(x) \doteq \eta^{\mu\nu} - k\, h^{\mu\nu} + \cdots$$

WRITE LAGRANGIAN IN TERMS OF $h_{\mu\nu}(x)$

$$S = \int d^4x \sqrt{-\det g}\; \mathcal{L} \qquad \mathcal{L} = R(x)$$

$$\sqrt{-\det g}\, R(x) = \frac{1}{2}\partial_\mu h_{\rho\sigma}\partial^\mu h^{\rho\sigma} + k\, h^{\lambda\alpha}\partial_\lambda h_{\rho\sigma}\partial_\alpha h^{\rho\sigma} + k^2 O(h^4)$$

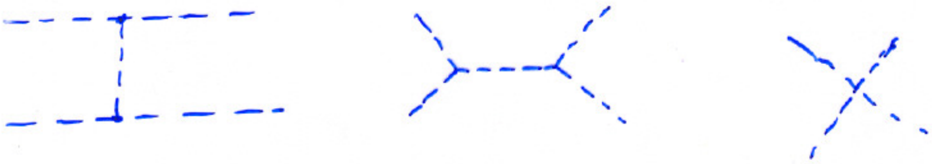
$$h^{\mu\nu} = \eta^{\mu\rho}\eta^{\nu\sigma} h_{\rho\sigma}$$

THIS LAGRANGIAN IS NON-POLYNOMIAL IN $h_{\mu\nu} \Rightarrow$ IT CONTAINS ALL POWERS OF $h$.

FEYNMAN DIAGRAMS:

PROPAGATOR 

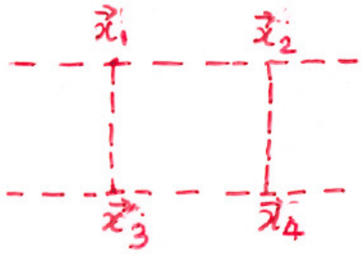
INTERACTIONS 



ETC.

ETC

CONSIDER THE DIAGRAM




THIS DIAGRAM IS DIVERGENT.

THE DIVERGENCE OCCURS WHEN

$\vec{x}_1 = \vec{x}_2 = \vec{x}_3 = \vec{x}_4$

UNLIKE IN THE STANDARD MODEL, THE DIVERGENCES IN THIS THEORY CANNOT BE REMOVED WITHOUT ADDING INFINITE NO. OF TERMS TO THE LAGRANGIAN

MAY BE THERE IS NO THEORY OF PURE GRAVITY!

MAY BE GRAVITY COMES PACKAGED WITH ITS OWN MATTER CONTENT.

SUPERGRAVITY (A SUPERSYMMETRIC VERSION OF ORDINARY GRAVITY) THEORY IS ONE SUCH ATTEMPT OF THEORY OF GRAVITY WITH MATTER.

SUPERGRAVITY DOES LITTLE BETTER THAN PURE GRAVITY BUT IT ALSO SUFFERS FROM THE PROBLEM OF UNREMOVABLE DIVERGENCES.

SINCE DIVERGENCES OCCUR WHEN ALL $x_i$'S BECOME COINCIDENT, IT LOOKS LIKE AN ARTIFACT OF POINT PARTICLE THEORIES.

A THEORY OF EXTENDED OBJECTS MAY NOT HAVE SUCH SEVERE DIVERGENCES.

AS WE WILL SEE LATER, INDEED, THE STRING THEORY ENCOUNTERS NO SUCH DIVERGENCES.