Introduction to Strings: Part II

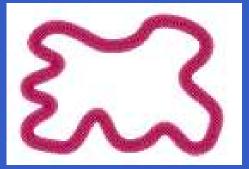
Largely an open-ended story...

Debashis Ghoshal

Harish-Chandra Research Institute Allahabad

Recap

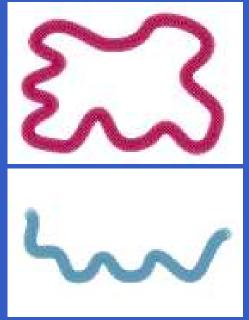
[animated] Closed string



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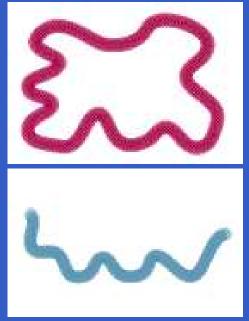
[animated] Closed string



[animated] Open string

- Open strings ... need boundary conditions
 - Neumann condition
 - Dirichlet condition

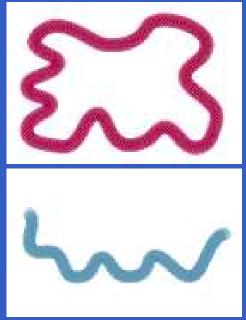
[animated] Closed string



[animated] Open string

- Open strings ... need boundary conditions
 - Neumann condition
 - Dirichlet condition
- Spectrum of open strings

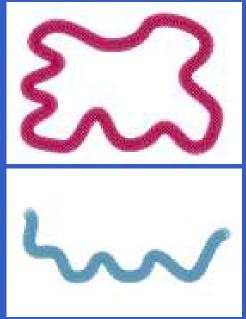
[animated] Closed string



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- Open strings ... need boundary conditions
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- Spectrum of open strings
- D-branes

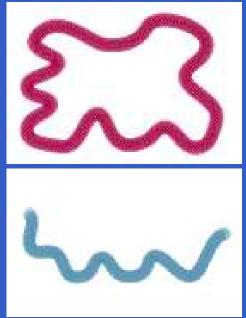
[animated] Closed string



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- Open strings ... need boundary conditions
 - Neumann condition
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- Spectrum of open strings
- D-branes
- Open string interactions

[animated] Closed string



[animated] Open string

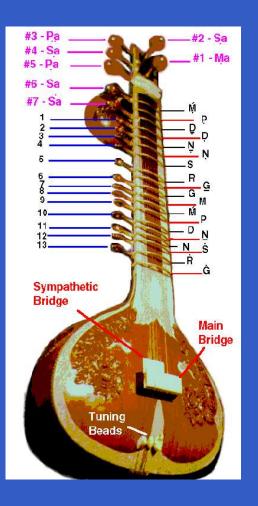
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 - Neumann conditionDirichlet condition
- Spectrum of open strings
- D-branes
- Open string interactions
- A larger picture?

Fundamental constituent: strings of size $\sim 10^{-33}$ cm.

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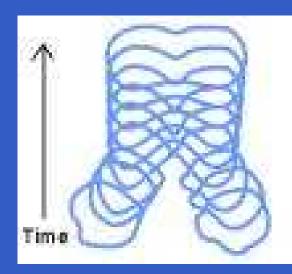
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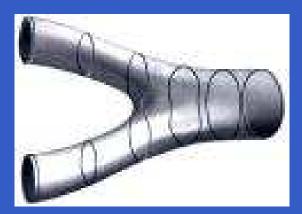
Fundamental constituent: strings of size $\sim 10^{-33}$ cm. Normal modes of vibration of the strings are perceived as particles. gravitons, electrons,... The music of the strings!



One string can split into two or two strings can join into one:

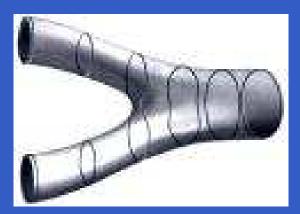


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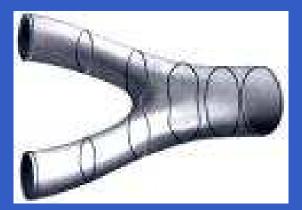
One string can split into two or two strings can join into one: Interaction

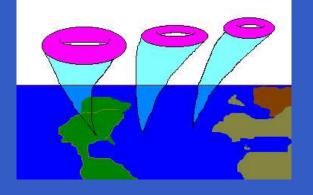
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Front, back Left, right Could there really be something more? Another dimension? Fascinating!	

One string can split into two or two strings can join into one: Interaction String theory is consistent only in ten dimensions!



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The strings that we see around us are macroscopic nonrelativistic strings. Many of these are **musical strings**



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The strings that we see around us are macroscopic nonrelativistic strings. of these Many are and tied at the ends. These strings need boundary conditions. Fundamental open strings also need boundary conditions.

Two types of boundary conditions are natural:

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Dirichlet: $\partial_t X^i = 0$

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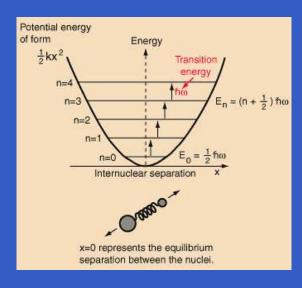
Boundary conditions are independent at each end

point.

Recall:Ontheclosedstringleft-movingandright-movingmodes are independent.

Recall: On the closed string left-moving and right-moving modes are independent. For Open strings: reflection at the end-points relate these modes.

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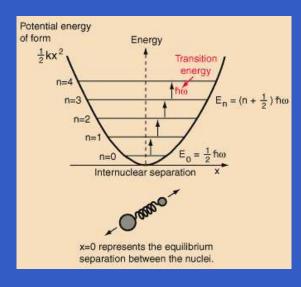
nite number of simple harmonic oscillators

 x_n^i

Recall: On the closed string right-moving left-moving and modes are independent. For Open strings: reflection at the end-points relate these modes. Each coordinate x^{i} is a collection of infi- These are no longer independent:

$$x_n^i = \pm \tilde{x}_n^i$$

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Each coordinate x^i is a collection of infi-

nite number of simple harmonic oscillators

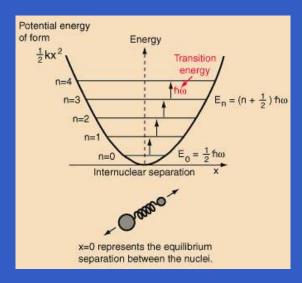
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vacuum state $|0\rangle$

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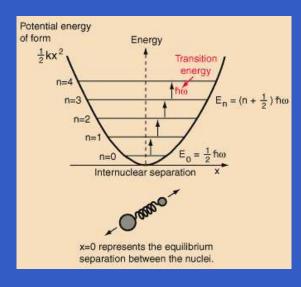
Each coordinate x^i is a collection of infi-

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nite number of simple harmonic oscillators vacuum state carrying momenta

$$\left|k\right\rangle = \prod \left(a_{0}^{i}\right)^{\dagger} \left|0\right\rangle$$



Each coordinate x^i is a collection of infi-

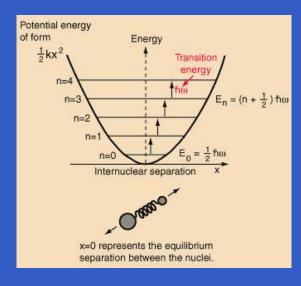
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$$\frac{x_n^i = \pm \tilde{x}_n^i}{\left(a_1^i\right)^\dagger \left|k\right\rangle}$$

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nite number of simple harmonic oscillators

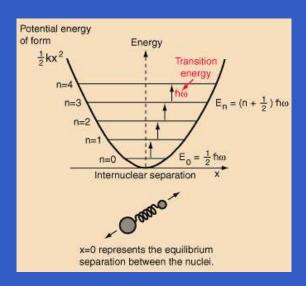
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$$(a_2^i)^{\dagger} |k\rangle, \quad (a_1^i)^{\dagger} (a_1^j)^{\dagger} |k\rangle$$

 $x^i = +\tilde{x}^i$

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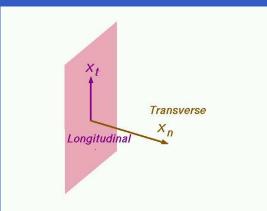
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 $(a_1^i)_{II}^{\dagger}|k\rangle$

The extra indices I, J refer to the endpoints.



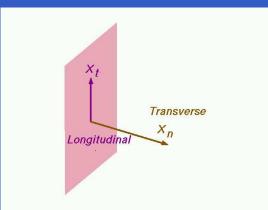
Open string can carry momentum only along the (hyper-)plane:

Neumann b.c. on X^i_{\parallel}

Dirichlet b.c. on X^i_{\perp}

String end-points are on the hyperplane

 $X^i_{\perp} = \text{constant}$



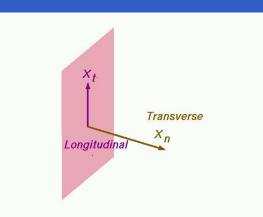
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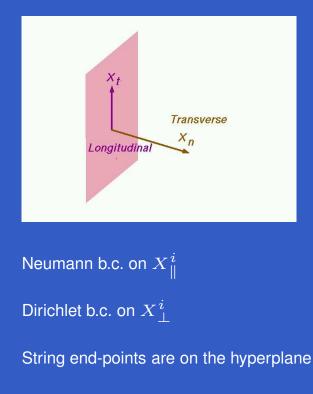
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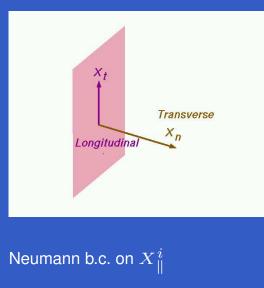
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Open string can carry momentum only along the (hyper-)plane: $k \equiv k_{\parallel}$ Longitudinal excitations $\left(a_{1}^{\parallel i}\right)_{IJ}^{\dagger} |k_{\parallel}\rangle$ Gauge bosons.



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Open string can carry momentum only along the (hyper-)plane: $k \equiv k_{\parallel}$ Longitudinal excitations $\left(a_{1}^{\parallel i}\right)^{\intercal}_{I}|k_{\parallel}\rangle$ Gauge bosons. Transverse excitations $(a_1^{\perp i})_{II}^{\dagger} |k_{\parallel}\rangle$ Scalar fields. Both live on the hyper-plane.

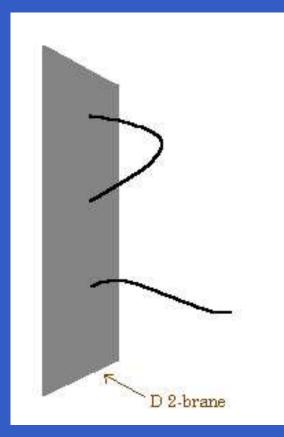


Dirichlet b.c. on X^{i}_{\perp}

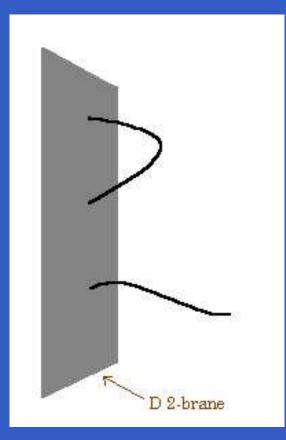
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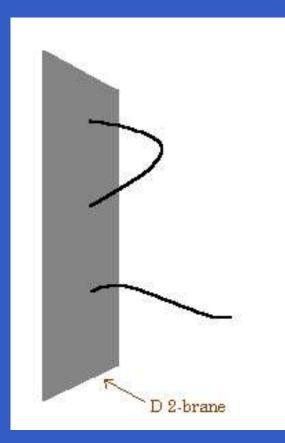
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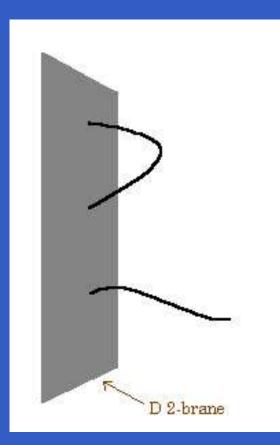
Dirichlet b.c: *End points* of the string are fixed at all times,



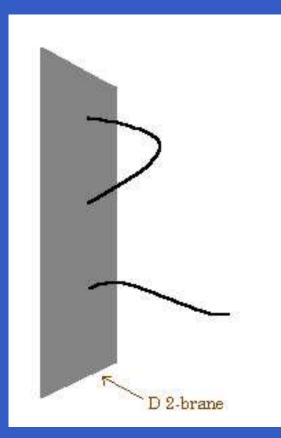
Dirichlet b.c: *End points* of the string are fixed at all times, but the *bulk* of the string can be anywhere.



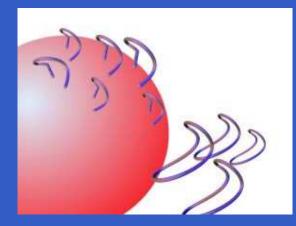
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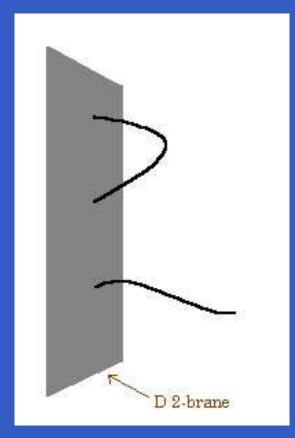


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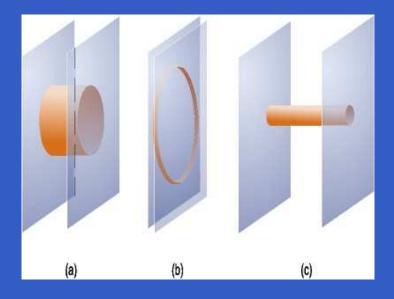


Can be any (hyper-)surface.

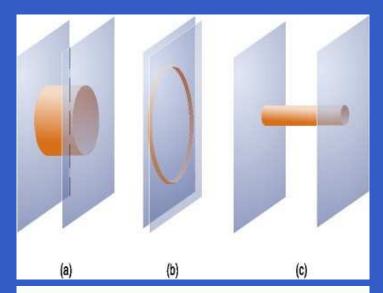
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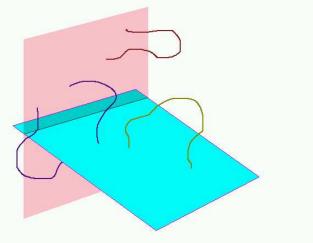


D-branes look like delecis/impurilies in space.

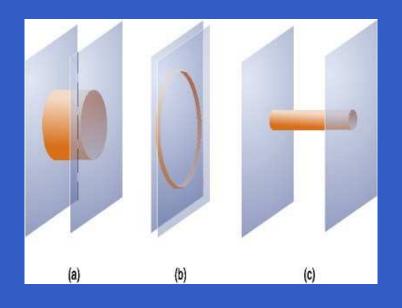


D-branes look like deiecis/impurities in space. There may be more than one D-brane,

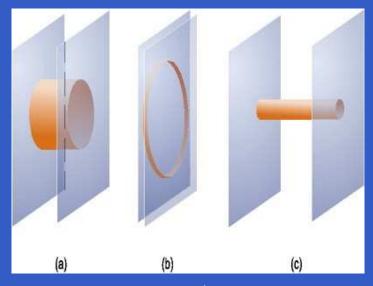




D-branes look like deiecis/impurities in space. There may be more than one D-brane, aligned arbitrarily — can intersect each other.



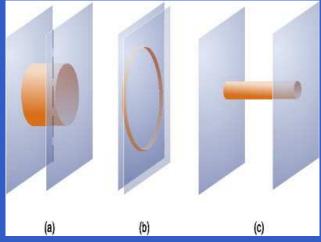
D-branes look like defects/impurities in space. There may be more than one D-brane, aligned arbitrarily — can intersect each other. Energy of the strings between branes depend on the streching,



 $\begin{aligned} A_{IJ}^{i}(\mathbf{k}) &= (a^{i})_{IJ}^{\dagger} |\mathbf{k}\rangle \\ \text{can become massless} \\ E^{2} &= |\mathbf{k}|^{2} \end{aligned}$

D-branes look like delects/impurities in space. There may be more than one D-brane, aligned arbitrarily — can intersect each other. Energy of the strings be-

tween branes depend on the streching, *i.e.* on the distance between the branes.



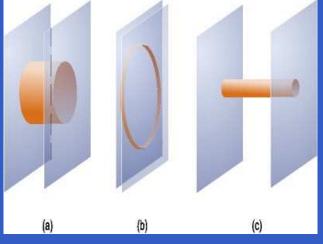
 $A_{II}^{i}(\mathbf{k}) = (a^{i})_{II}^{\dagger} |\mathbf{k}\rangle$ come from excitations of a string starting and ending on the same Dbrane.

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less

$$E^2 = \left| \mathbf{k} \right|^2$$



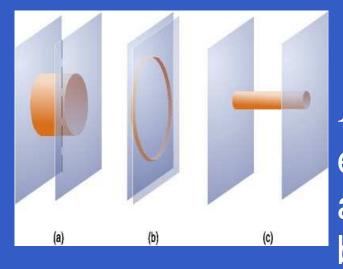
 $A_{IJ}^{i}(\mathbf{k})=\left(a^{i}
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angle$

can become mass-

 $A_{II}^{i}(\mathbf{k}) = (a^{i})_{II}^{\dagger} |\mathbf{k}\rangle$ come from excitations of a string starting and ending on the same Dbrane. It is always massless.

less

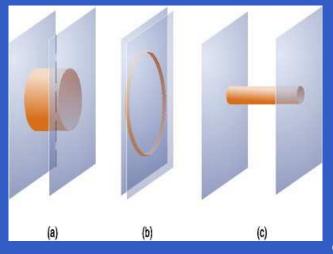
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 $A_{II}^{i}(\mathbf{k}) = (a^{i})_{II}^{\dagger} |\mathbf{k}\rangle$ come from excitations of a string starting and ending on the same Dbrane. These are like photons — satisfy Maxwell eqns plus corrections.

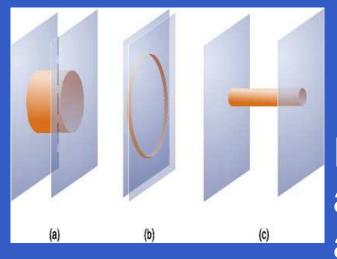
$$A_{IJ}^{i}(\mathbf{k}) = (a^{i})_{IJ}^{i} |\mathbf{k}\rangle$$

can become mas
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 r^{2} $|\mathbf{k}|^{2}$



If there are two D-branes, there are two types of photons A_{11} and A_{22} .

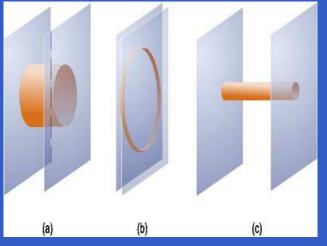
 $\begin{array}{l} A_{IJ}^{i}(\mathbf{k})=(a^{i})_{IJ}^{\dagger}\left|\mathbf{k}\right\rangle \\ \text{can become mass-}\\ \text{less}\\ E^{2}=\left|\mathbf{k}\right|^{2} \end{array}$



If there are two D-branes, there are two types of photons A_{11} and A_{22} . If they coincide, A_{12} and A_{21} also become massless.

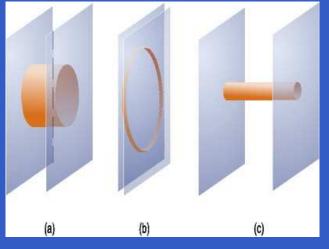
 $A^i_{IJ}(\mathbf{k}) = (a^i)^{\dagger}_{IJ} |\mathbf{k}\rangle$ can become massless

$$E^2 = \left| \mathbf{k} \right|^2$$



Now all *four* fields A_{11}, A_{12}, A_{21} and A_{22} are massless.

 $A_{IJ}^{i}(\mathbf{k}) = (a^{i})_{IJ}^{\dagger} |\mathbf{k}\rangle$ can become massless $E^{2} = |\mathbf{k}|^{2}$



Now all *four* fields A_{11}, A_{12}, A_{21} and A_{22} are massless. Also massless are *transverse scalar* fields X_{IJ}^{\perp} .

 $A_{IJ}^{i}(\mathbf{k}) = (a^{i})_{IJ}^{\dagger} |\mathbf{k}\rangle$ can become mass-less

$$E^2 = \left| \mathbf{k} \right|^2$$

ThisisaU(2)non-abelian gauge theory.

This is a U(2)non-abelian gauge theory. N coincident D-branes give U(N) non-abelian gauge theory. $U(N) \sim U(1) \times SU(N)$

This is a U(2) non-abelian gauge theory. N coincident D-branes give U(N) non-abelian gauge theory. U(N) ~ U(1)×SU(N) For superstrings one has supersymmetric non-abelian U(N) gauge theory.

The world of the end

The *dynamics* of open strings come from the end-points.

The world of the end

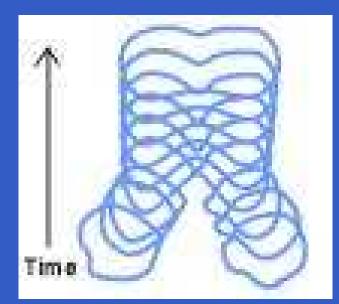


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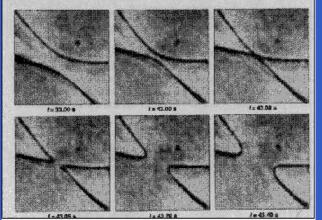


The *dynamics* of open strings come from the endpoints. This may be how we look to the big green giants living in higher dimensions!



String scattering

Closed strings interact by splitting and joining.



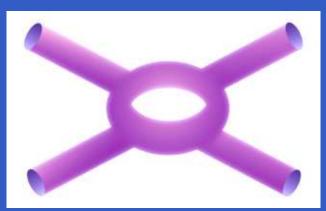
Closed strings interact by splitting and joining.

Interation of strings in nematic liquid crystal

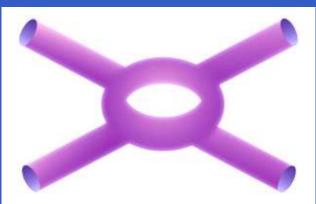
Closed strings interact by splitting and joining. In a typical process: 2 strings $\xrightarrow{\text{join}}$ 1 string

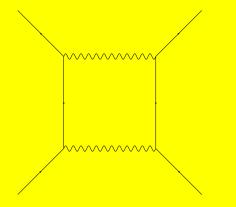
Closed strings interact by splitting and joining. In a typical process: $2 \text{ strings} \xrightarrow{\text{join}} 1 \text{ string} \xrightarrow{\text{split}} 2 \text{ string}$

Closed strings interact by splitting and joining. In a typical process: 2 strings \xrightarrow{join} 1 string \xrightarrow{split} 2 string \xrightarrow{join} 1 string

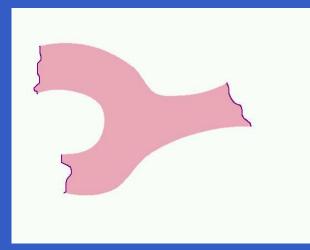


Closed strings interact by splitting and joining. In a typical process: 2 strings \xrightarrow{join} 1 string \xrightarrow{split} 2 string \xrightarrow{join} 1 string \xrightarrow{split} 2 string



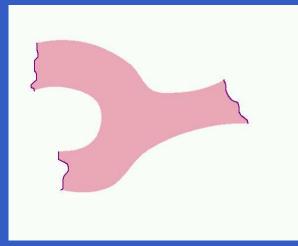


Closed strings interact by splitting and joining. In a typical process: 2 strings \xrightarrow{join} 1 string $\xrightarrow{\text{split}}$ 2 string \xrightarrow{join} 1 string $\xrightarrow{\text{split}}$ 2 string Small string tension ($\alpha' \rightarrow 0$) limit.



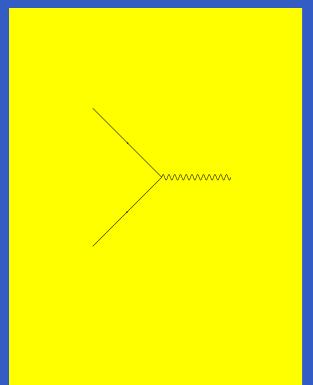
Open strings interact by splitting and joining at the end points.

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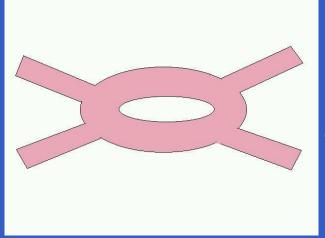


Open strings interact by splitting and joining at the end points. Gives the correct interaction for supersymmetric gauge theories

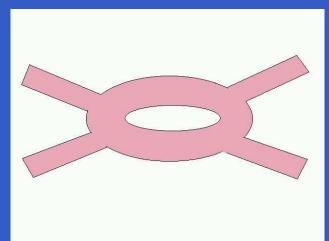
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Open strings interact by splitting and joining at the end points. Gives the correct interaction for supersymmetric gauge theories in the small string tension ($\alpha' \rightarrow 0$) limit.

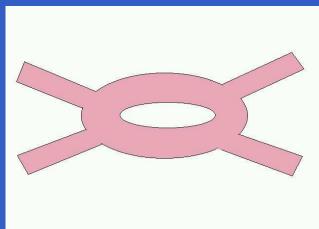


Open strings interact by splitting and joining at the end points. A typical 2 strings \rightarrow 2 strings process.

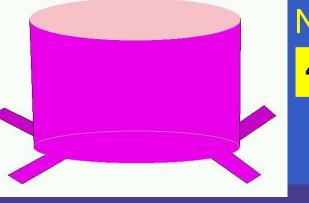


Open strings interact by splitting and joining at the end points. A typical 2 strings \rightarrow 2 strings process.

Now there is another way to see this:

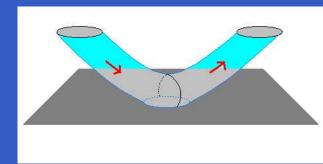


Open strings interact by splitting and joining at the end points. A typical 2 strings \rightarrow 2 strings process. Now there is another way to see this: 4 open strings \rightarrow 1 closed string

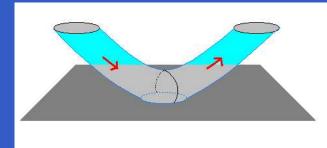


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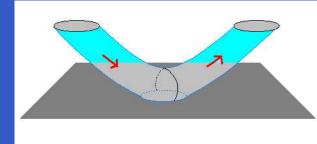
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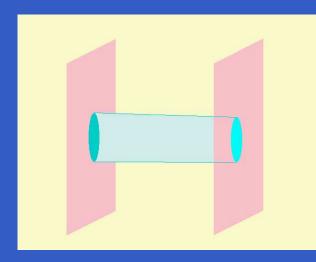


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process involving a D-brane. D-branes can *exchange* closed strings.

D-branes interact

gravitationally.

D-branes have gravitational interaction



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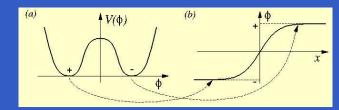
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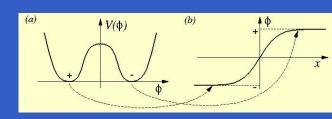
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D-branes have gravitational interaction — they have mass. To be precise: Tension = mass/(hyper-)volume Tension of D-1-brane = D-string is: $T_{D1} = \frac{1}{2\pi\alpha' g_s} \gg \frac{1}{2\pi\alpha'} = T_{\text{string}}$ for small g_s . D-branes are *not* rigid defects in spacetime.

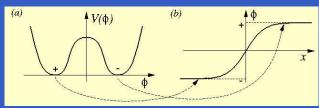
D-branes are black hole -like solutions



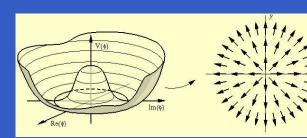
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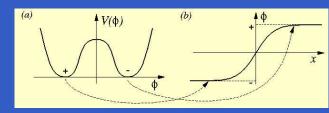
D-branes are black hole -like solutions of *closed string* equations of motion. They carry mass



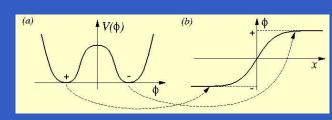
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D-branes are black hole -like solutions of *closed string* equations of motion. They carry mass and sometimes charge. If a D-brane carries a conserved charge, it is stable, else it is unstable.

A D-branes carrying *positive* charge is stable.



Another one with *negative* charge is also stable — an anti-D-brane.

But ... the two together: a brane-antibrane pair is *unstable*.

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No net force between two stable D-branes

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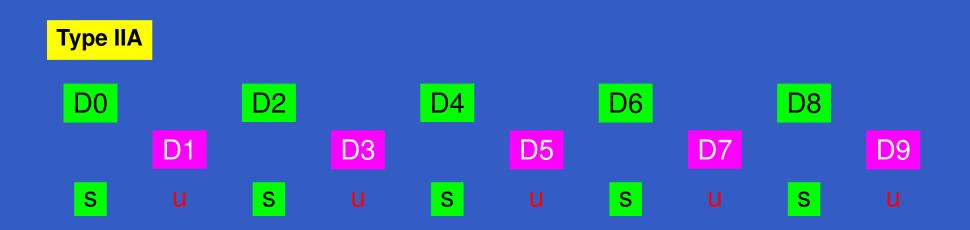
No net force between two stable D-branes gravitational and Coulomb force cancel.

D-brane spectrum

Type IIA

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D-brane spectrum



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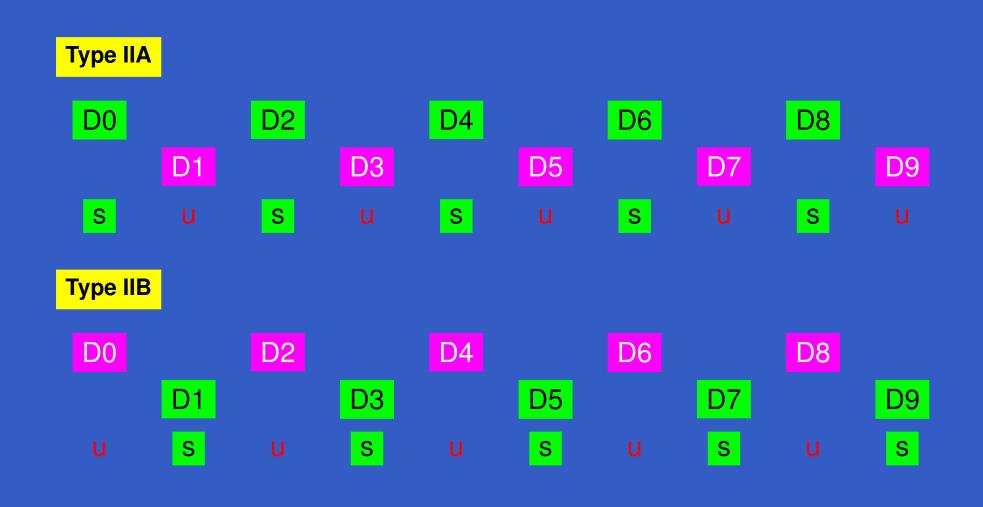
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D-brane spectrum



D-brane spectrum

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D-branes & T-duality

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Recall from closed strings Momenta $\stackrel{T-duality}{\leftrightarrow}$ Winding

D-branes & T-duality

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D-branes & T-duality

Recall from closed strings Momenta $\xrightarrow{T-duality}$ Winding For open strings: Neumann b.c. $\xrightarrow{T-duality}$ Dirichlet b.c.

$$D-m-brane = \begin{cases} D-(m-1)-brane \\ D-(m+1)-brane \end{cases}$$

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T-duality relates

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Type IIA string $\stackrel{T-duality}{\leftrightarrow}$ Type IIB string There are other relations...

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Type IIA string $\stackrel{\text{T-duality}}{\leftrightarrow}$ Type IIB string There are other relations... At large values of g_s , D-branes become light $T_{D1} = 1/2\pi \alpha' g_s$ and fundamental strings $(T_{P1} = 1/2\pi \alpha')$ become heavy.

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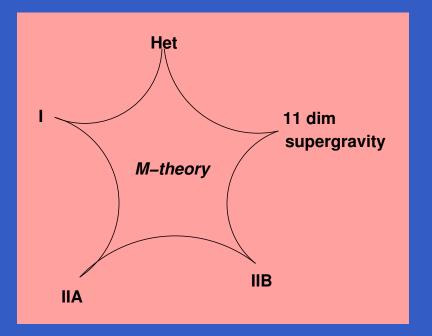
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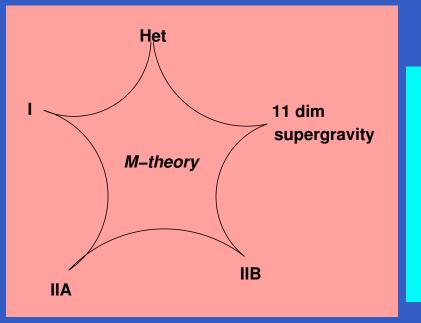
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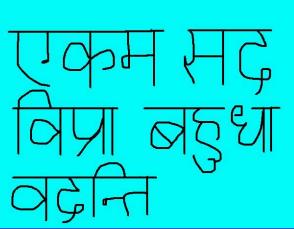
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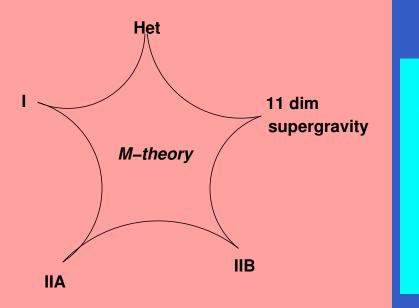
Sometimes Graviton $\stackrel{g_s \to 1/g_s}{\leftrightarrow}$ D-brane

All these symmetries hint at a web of relations



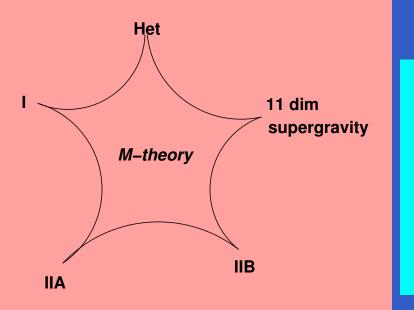






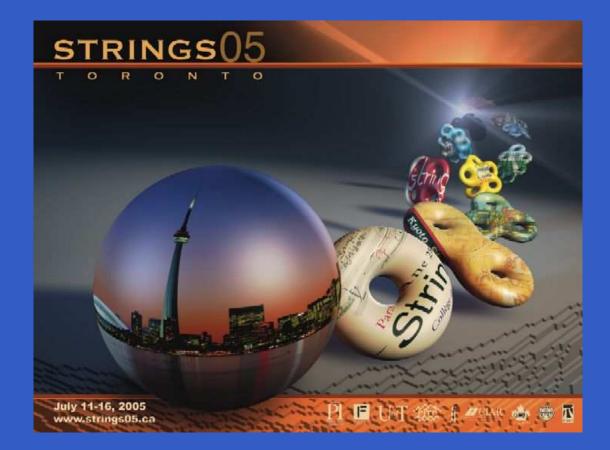


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Strings and poster art



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