# Standard model of particle physics

Plan:

1. A qualitative description of the physics of elementary particles.

2. Theoretical foundation of the standard model of particle physics.

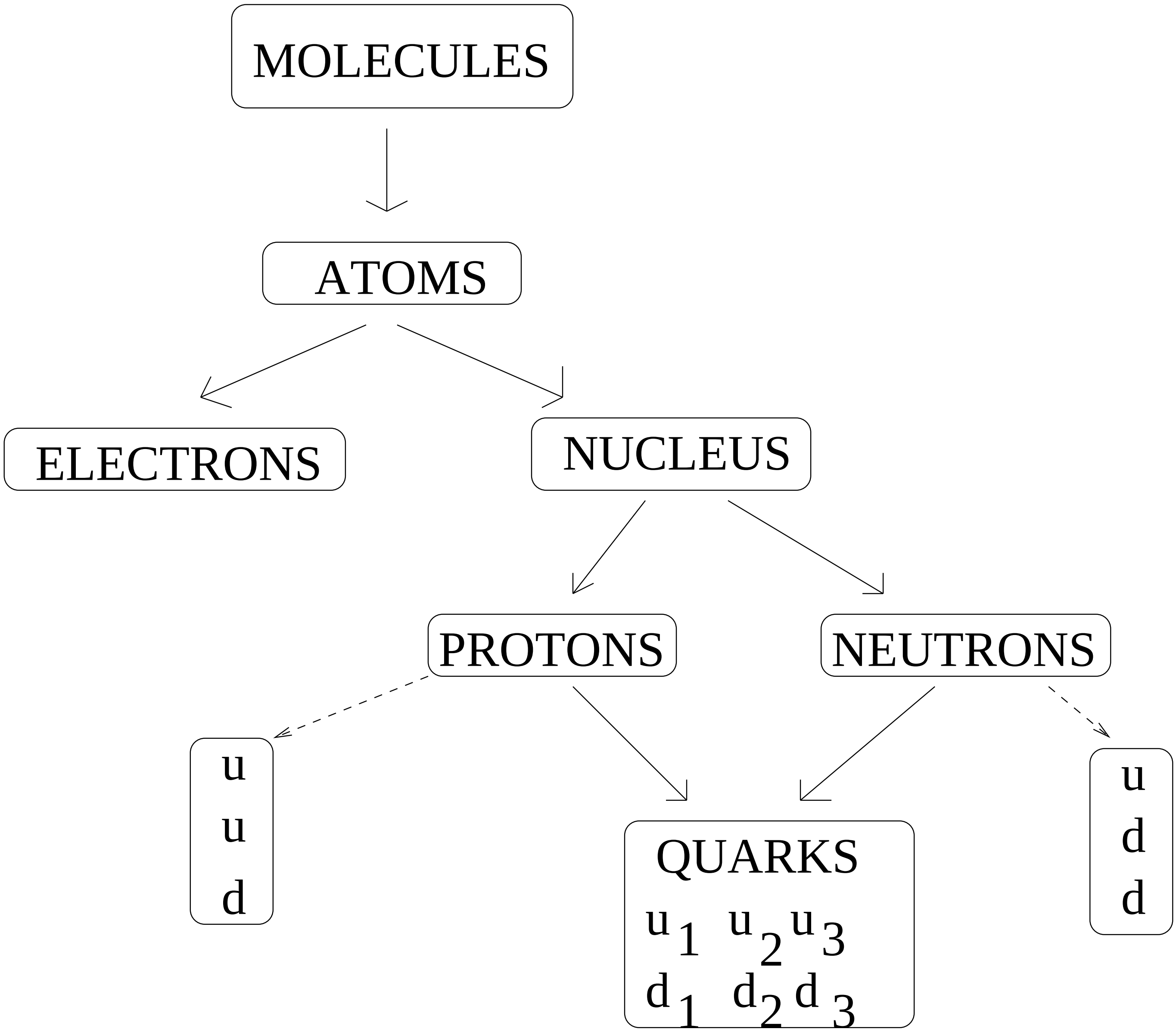
3. Limitations of the standard model

4. Beyond the standard model

The key question:

**What are we and everything around us made of?**

I shall begin by giving a bird's eye view of our current understanding of the ultimate constituents of matter.


MOLECULES
ATOMS
ELECTRONS
NUCLEUS
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QUARKS
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In order to understand the various properties of matter, it is not enough to know the constituents of matter.

We also need to know how these constituents interact with each other, that is, what kind of force they exert on each other.

We observe two kinds of interaction in everyday life

1. Gravitational Interaction:

   Responsible for earth's gravity, motion of planets etc.

2. Electromagnetic Interaction:

   Responsible for force due to a magnet, lightning etc.

In order to describe the interaction between the elementary particles, we need to include two other kinds of interactions

1. Strong interaction:

   Responsible for binding the quarks inside a proton or a neutron, and the protons and neutrons together inside a nucleus.

2. Weak interaction:

   Responsible for radioactive $\beta$-decays of nuclei.

It turns out that in studying the physics of elementary particles, we can ignore gravity to a very good approximation.

For example, one can compare the electrostatic force between two protons with the gravitational force between two protons at rest.

Result:

$$\frac{\text{Grav. Force}}{\text{Elec. Force}} = \frac{G_N m_p^2/r^2}{e_p^2/r^2} \sim 10^{-36}$$

$G_N$: Newton's Constant ($6.67 \times 10^{-8}$ cm$^3$/gm sec$^2$)

$m_p$: proton mass ($1.67 \times 10^{-24}$ gm)

$e_p$: proton charge ($4.8 \times 10^{-10}$ e.s.u.)

Similarly all other forces are also much larger than the gravitational force.

For this reason we shall ignore the effect of gravitational force in our discussion of elementary particles.

However we should remember that the theory is not complete without gravity.

Gravitational force is small but not zero.

**Summary of our discussion so far:**

In order to understand the basic constituents of matter we need to understand:

1. What are the elementary particles?

   Quarks, electron, . . .

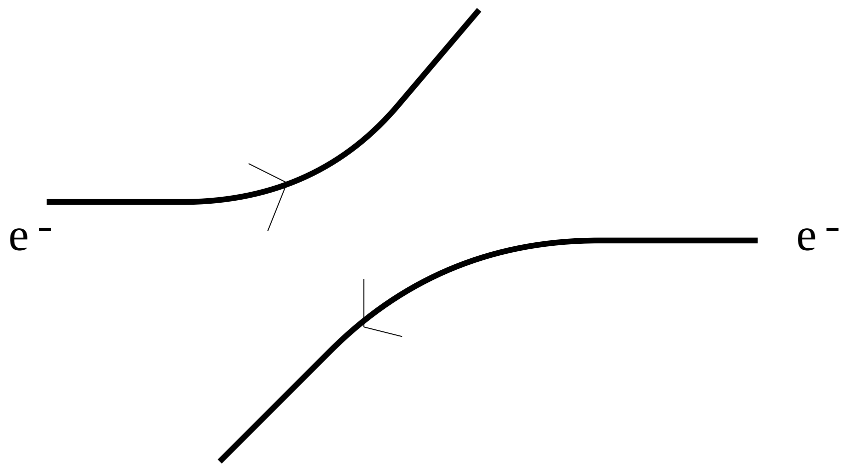
2. What are the forces between these particles?

   Strong, Weak, Electromagnetic, Gravitational, . . .

   Of these the gravitational force is much smaller than the other forces and can be ignored in studying the dynamics of elementary particles.

With the help of quantum theory one can give a unified description of elementary particles, and the interaction among the elementary particles.

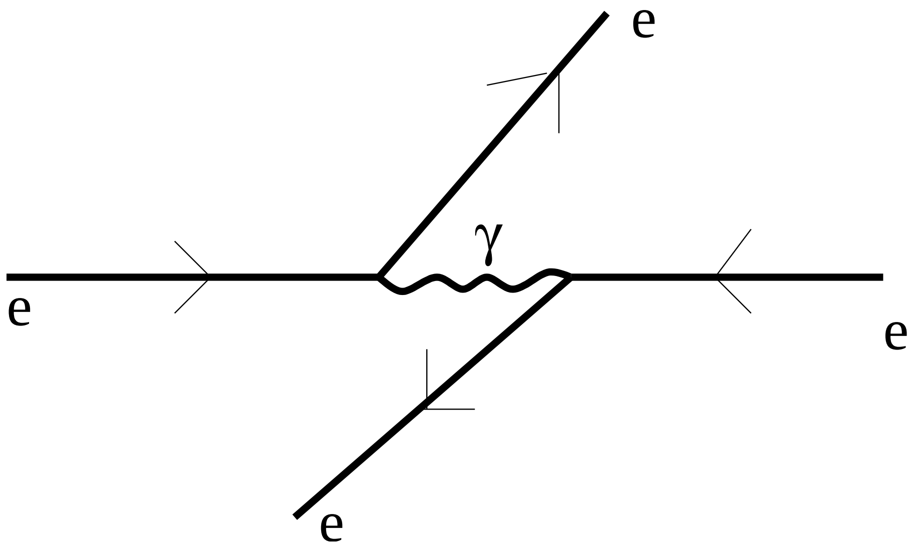
Consider for example the electromagnetic interaction between two electrons when they pass each other.




Due to the interaction, each particle gets deflected from their original trajectory.

In quantum theory, one provides a different explanation of the same phenomenon.

In quantum theory, the deflection takes place because the two electrons exchange a new particle, called photon, while passing near each other.




The photon is capable of carrying some amount of energy and momentum from the first electron to the second electron, thereby causing this deflection.

We call the photon the mediator of electromagnetic interaction.

In this language, we can describe an interaction by specifying the particle(s) which mediate the interaction.

Thus for example:

- Strong interaction is mediated by eight different particles known as gluons

  These particles are all electrically neutral.

- Weak interaction is mediated by three particles, denoted by $W^+$, $W^-$ and $Z$.

  $W^+$ and $W^-$ carry $+1$ and -1 unit of electric charge

  $Z$ is neutral.

We must add these particles, as well as the photon, to our list of elementary particles.

But this still does not exhaust the list of all elementary particles.

One often finds new elementary particles in cosmic rays, during the decay of unstable particles (*e.g.* the neutron), or when one makes two elementary / composite particles collide with each other.

Also, mathematical reasoning shows that for every particle there is an anti-particle which carries charge opposite to that of the particle.

All of these new elementary particles must also be added to the list.

## LIST OF KNOWN ELEMENTARY PARTICLES

### QUARKS:

$u^1, u^2, u^3 \qquad d^1, d^2, d^3 \qquad c^1, c^2, c^3$

$s^1, s^2, s^3 \qquad t^1, t^2, t^3 \qquad b^1, b^2, b^3$

### LEPTONS

$(e, \nu_e) \qquad (\mu, \nu_\mu), \qquad (\tau, \nu_\tau)$

### MEDIATORS

gluons: $g_1, \ldots g_8$ Photon: $\gamma$

$W^+$, $W^-$, $Z$

These, together with the anti-particles of quarks and leptons, constitute the full list of elementary particles which have been observed so far.

It turns out that this information can be incorporated into a sound mathematical theory, known as the

**Standard Model**

So far standard model has succeeded in explaining almost all observed experimental results.

Particle content of the standard model:

QUARKS:

$u^1, u^2, u^3$ $\quad$ $d^1, d^2, d^3$ $\quad$ $c^1, c^2, c^3$

$s^1, s^2, s^3$ $\quad$ $t^1, t^2, t^3$ $\quad$ $b^1, b^2, b^3$

LEPTONS

$(e, \nu_e)$ $\quad$ $(\mu, \nu_\mu)$, $\quad$ $(\tau, \nu_\tau)$

GAUGE BOSONS

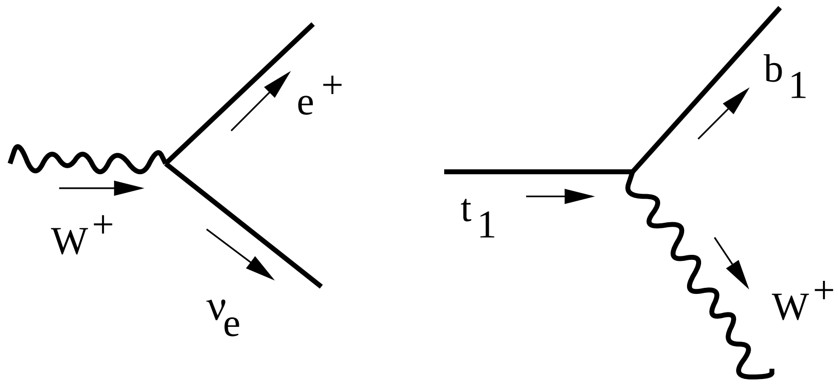
gluons: $g_1, \ldots g_8$ $\quad$ Photon: $\gamma$

$W^+$, $W^-$, $\quad$ $Z$

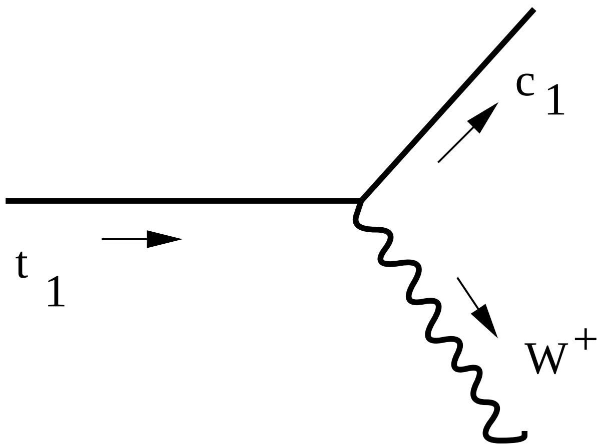
HIGGS $\phi$

The standard model not only gives us a specific list of elementary particles, but also tells us what kind of processes various particles can go through.

Examples of allowed processes:

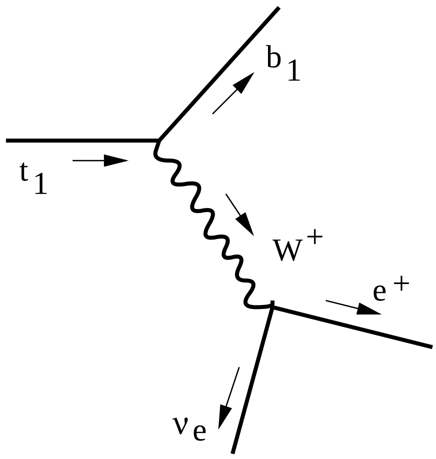



Example of a disallowed process:



Once we know the basic allowed processes, we can build up complicated processes with the help of these elementary processes.

Example of a composite allowed process:




Standard model tells us how to calculate the probability amplitude for each such process.

This can then be compared with experimental observations.

One can explain almost all of the present day experimental results with the help of the standard model.

With the help of this theory, we can also predict the result of any new experiment involving elementary particles.

Many such predictions have been experimentally verified.

Example:

- The masses of $W^+$, $W^-$ and $Z$ particles were predicted long before they were discovered at the accelerator at CERN, Geneva.

- The existence of $t$-quark was predicted long before it was discovered.

In fact the Higgs particle, $\phi$, has not yet been discovered.

But mathematical consistency of the theory demands that such a particle must be found.

**Some key features of the standard model**

1. Quarks and leptons are fermions and carry spin $\frac{1}{2}\hbar$.

The mediators or the gauge bosons are bosons and carry spin $\hbar$.

The Higgs particle is also a boson and carries spin 0 (prediction of the standard model).

2. Even though quarks are listed as elementary particles, it is not possible to get an isolated quark.

The strong interaction is so strong that it always binds a quark with other quarks or antiquarks.

This phenomenon is known as **confinement**.

In contrast, the leptons (electron, muon, neutrinos etc.) do not have strong interaction and hence can be isolated.

3. Consider a fast moving quark or lepton.

We can consider two types of states.

a) The component of the spin along the direction of motion is $\frac{1}{2}\hbar$ (right handed).

b) The component of the spin along the direction of motion is $-\frac{1}{2}\hbar$ (left handed).

These states have the same strong and electromagnetic interaction.

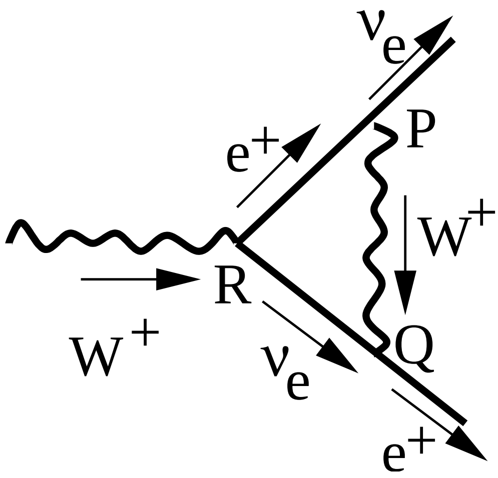
However only left-handed states participate in weak interaction.

We say that weak interaction is **chiral** or that it violates parity symmetry.

In fact neutrinos are always left-handed.

We say that neutrinos are **chiral fermions**.

4. Consider a composite diagram with a closed loop.




Calculation of the probability amplitude requires integration over the space-time coordinates $P$, $Q$, $R$ of the interaction vertices.

The integrand typically becomes infinite when two or more interaction vertices coincide.

In some cases the integral diverges when the interaction vertices which form part of a loop come close to each other.

$\rightarrow$ ultraviolet (short distance) divergence.

How do we deal with these infinities?

Suppose $\alpha$, $\beta$, $\gamma$, $\cdots$ are various parameters used to describe the theory.

$A_1$, $A_2$, $A_3$, $\cdots$ are various experimentally measurable quantities which we can calculate from the theory.

Then each $A_i$ is a function of $\alpha$, $\beta$, $\gamma$ etc.

$$A_i = f_i(\alpha, \beta, \gamma, \cdots)$$

$f_i$ are calculable in the standard model.

But due to ultraviolet divergence they are infinite.

$$A_i = f_i(\alpha, \beta, \gamma, \cdots)$$

Note, however that the functions $f_i$ are not directly measurable since $\alpha$, $\beta$, $\gamma$ etc. are not directly measurable.

The best we can do to test the theory is to use these functions to eliminate $\alpha$, $\beta$, $\gamma$, $\cdots$ and find relations among the $A_i$'s.

These relations are in principle amenable to experimental tests.

Are these relations finite?

In the standard model they are finite.

$\rightarrow$ the standard model is renormalizable.

This is the way we solve the ultraviolet divergence problem in the standard model.

Note: Gravity is not renormalizable.

**Theoretical foundation of the standard model**:

We shall follow the following order:

1) Electromagnetic interaction

(origin of photon)

2) Strong and weak interaction

(origin of gluons, $W^+$, $W^-$ and $Z$ bosons)

3) Quarks and Leptons

4) Higgs particle

I shall deliberately simplify some aspects of the model; so not everything that I'll say will be completely correct.

Conventions:

$x^0 = t$: time coordinate

$\vec{x} = (x^1, x^2, x^3)$: space-coordinates

We shall choose our units so that

1) the velocity of light $c = 1$

2) The Planck's constant $\hbar = 1$.

Thus spin $s \to$ angular momentum $s\hbar$.

Indices are raised and lowered by

$$\eta_{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

**Electromagnetic interaction:**

Classical electrodynamics is described by a 4-vector field

$$\{A_\mu\} = (A_0, \vec{A})$$

Define

$$F_{\mu\nu} = \frac{\partial A_\nu}{\partial x^\mu} - \frac{\partial A_\mu}{\partial x^\nu}$$

Then

$$E_1 = F_{01}, \quad E_2 = F_{02}, \quad E_3 = F_{03}$$

$$B_1 = F_{23}, \quad B_2 = F_{31}, \quad B_3 = F_{12}$$

Maxwell's equations in space in the absence of charges

$$\sum_{\mu=0}^{3} \frac{\partial F^{\mu\nu}}{\partial x^{\mu}} = 0 \qquad \text{for} \quad \nu = 0, 1, 2, 3$$

These equations can be derived from an action

$$-\frac{1}{4} \int dt \int d^3x \sum_{\mu,\nu=0}^{3} F_{\mu\nu} F^{\mu\nu}$$

using the usual variational principle.

**Gauge symmetry**

$$F_{\mu\nu} = \frac{\partial A_\nu}{\partial x^\mu} - \frac{\partial A_\mu}{\partial x^\nu}$$

$A_\mu$ and $A_\mu + \frac{\partial \Lambda(x)}{\partial x^\mu}$ give same $F_{\mu\nu}$ and hence same $(\vec{E}, \vec{B})$ for any function $\Lambda(x)$ of $x^0, x^1, x^2, x^3$.

Thus these two configurations describe the same physical situation.

We say that these two configurations are **gauge equivalent**.

So far the theory we have described is a classical theory of electromagnetic field.

We can quantize it using standard procedure and find the eigenstates and eigenvalues of the Hamiltonian.

Typically these eigenstates are characterized by total energy, total momentum, total angular momentum and other quantum numbers.

One finds that among all the eigenstates there is a special class of states

$$|E, \vec{p}, s\rangle$$

characterized by

1. energy $E$,

2. momentum $\vec{p}$ and

3. component of spin along $\vec{p} = s$:

such that

$$E = |\vec{p}| \qquad \text{and} \qquad s = \pm 1$$

$\rightarrow$ describes states of a massless particle of spin 1 (compare with $E = \sqrt{\vec{p}^2 + m^2}$)

This state is called the photon.

The quantum theory also contains many other states, but they can be viewed as states containing multiple photons.

The theory of strong and weak interactions are based on an extension of electrodynamics, known as **non-abelian gauge theories** or **Yang-Mills theories**.

We shall first outline formulation of non-abelian gauge theories and then consider the specific cases of strong and weak interactions.

The main feature of these theories is that unlike electrodynamics which has a single vector field $A_\mu$, now we have

multiple vector fields.

Instead of describing a general non-abelian gauge theory, we shall focus on a special class of theories known as $SU(N)$ gauge theories.

For $SU(N)$ gauge theory, the vector fields can be regarded as components of

an $N \times N$ hermitian matrix with zero trace.

Example: $SU(2)$ gauge theory

$$\mathbf{A}_\mu = \begin{pmatrix} S_\mu & Q_\mu + iR_\mu \\ Q_\mu - iR_\mu & -S_\mu \end{pmatrix}$$

Thus there are 3 independent vector fields, each with four components

$$S_\mu, \quad Q_\mu, \quad R_\mu$$

An $SU(N)$ gauge theory will have $N^2 - 1$ independent vector fields.

These are known as gauge fields.

Field strengths: $N \times N$ matrix

$$\mathbf{F}_{\mu\nu} = \frac{\partial \mathbf{A}_\nu}{\partial x_\mu} - \frac{\partial \mathbf{A}_\mu}{\partial x_\nu} + ig(\mathbf{A}_\mu \mathbf{A}_\nu - \mathbf{A}_\nu \mathbf{A}_\mu)$$

Note: For electrodynamics the last term is trivially zero since

$$A_\mu A_\nu = A_\nu A_\mu$$

$g$: A parameter of the theory known as the coupling constant

Action

$$-\frac{1}{4} \int dt \int d^3x \sum_{\mu,\nu=0}^{3} Tr\,(\mathbf{F}_{\mu\nu}\mathbf{F}^{\mu\nu})$$

Gauge transformation:

$$\mathbf{A}_\mu \to U(x)\mathbf{A}_\mu U(x)^{-1} + \frac{i}{g}\frac{\partial U(x)}{\partial x^\mu}U(x)^{-1}$$

$U(x)$ is an $N \times N$ unitary matrix valued function with determinant 1.

Quantization of an SU(N) gauge theory in $g \to 0$ limit

$\to$ $(N^2 - 1)$ different massless spin 1 particles.

The theory of strong interaction is based on an $SU(3)$ gauge theory.

$\rightarrow$ contains gauge fields of the form $\mathbf{B}_\mu$, where for each $\mu$, $\mathbf{B}_\mu$ is a $3 \times 3$ traceless hermitian matrix.

$\rightarrow$ $3^2 - 1 = 8$ different spin 1 particles.

$\rightarrow$ 8 gluons

The theory of weak interaction is based on an $SU(2)$ gauge theory.

$\rightarrow$ contains gauge fields of the form $\mathbf{C}_\mu$, where for each $\mu$, $\mathbf{C}_\mu$ is a $2 \times 2$ traceless hermitian matrix.

$\rightarrow$ $2^2 - 1 = 3$ different spin 1 particles.

$\rightarrow$ $W^+$, $W^-$ and $Z$ particles.

**Quarks and Leptons: spin half fermions.**

In the absence of any interaction, quarks and leptons are described by Dirac equation for a free relaivistic spin half particle.

For the $k$-th fermion (quark or lepton) of mass $m^{(k)}$ we introduce a Dirac field $\psi^{(k)}$ satisfying the Dirac equation:

$$i\sum_{\beta}(\gamma^{\mu})_{\alpha\beta}\frac{\partial}{\partial x^{\mu}}\psi_{\beta}^{(k)}+m^{(k)}\psi_{\alpha}^{(k)}=0$$

$\alpha, \beta$: spinor indices, $\gamma^{\mu}$: Dirac $\gamma$-matrices

For neutrinos we need additional constraint:

$$\sum_{\beta}(1-\gamma_5)_{\alpha\beta}\psi_{\beta}=0, \qquad \gamma_5=i\gamma_0\gamma_1\gamma_2\gamma_3$$

This projects out the right-handed component.

**Coupling to gauge fields:**

In electrodynamics the interaction between electromagnetic field and the electron gives rise to two modifications.

1. The Maxwell's equation acquires a new term:

$$\sum_{\nu=0}^{3} \frac{\partial F^{\mu\nu}}{\partial x^{\mu}} = J^{\nu} \qquad \text{for} \quad \nu = 0, 1, 2, 3$$

$(J^0, \vec{J})$: electric charge density / current produced by the electron.

2. The Diract equation is also modified:

$$i \sum_{\beta} \gamma^{\mu}_{\alpha\beta} \left( \frac{\partial}{\partial x^{\mu}} \psi_{\beta} - ieA_{\mu}\psi_{\beta} \right) + m\psi_{\alpha} = 0$$

$e$: electric charge of the electron.

Coupling of quarks / leptons to strong and weak interaction gauge fields follow the same pattern.

Example: Coupling of a quark to strong gauge field $\mathbf{B}_\mu$:

Define $\psi^{(1)}$, $\psi^{(2)}$ and $\psi^{(3)}$ to be the Dirac field associated with the three components $q^1$, $q^2$ and $q^3$ of the quark $q$.

Then the Dirac equations for these fields take the form:

$$i \sum_\beta \gamma^\mu_{\alpha\beta} \left[ \frac{\partial}{\partial x^\mu} \begin{pmatrix} \psi^{(1)}_\beta \\ \psi^{(2)}_\beta \\ \psi^{(3)}_\beta \end{pmatrix} - i g_s \mathbf{B}_\mu \begin{pmatrix} \psi^{(1)}_\beta \\ \psi^{(2)}_\beta \\ \psi^{(3)}_\beta \end{pmatrix} \right] + m \begin{pmatrix} \psi^{(1)}_\alpha \\ \psi^{(2)}_\alpha \\ \psi^{(3)}_\alpha \end{pmatrix} = 0$$

$g_s$: coupling constant of strong interaction

Another example: Coupling of weak gauge field $\mathbf{C}_\mu$ to electron and electron neutrino.

Let $\psi^{(e)}$ and $\psi^{(\nu)}$ denote the Dirac fields associated with the electron and the electron neutrino.

Then the Dirac equation for these fields take the form:

$$i \sum_\beta \gamma^\mu_{\alpha\beta} \left[ \frac{\partial}{\partial x^\mu} \begin{pmatrix} \psi^{(\nu)}_\beta \\ \psi^{(e)}_\beta \end{pmatrix} \right.$$
$$\left. - i g_w \mathbf{C}_\mu \begin{pmatrix} \frac{1}{2}\{(1+\gamma_5)\psi^{(\nu)}\}_\beta \\ \frac{1}{2}\{(1+\gamma_5)\psi^{(e)}\}_\beta \end{pmatrix} \right] + \begin{pmatrix} m_\nu \, \psi^{(\nu)}_\alpha \\ m_e \, \psi^{(e)}_\alpha \end{pmatrix} = 0$$

$g_w$: coupling constant of weak interaction

$$
i\sum_\beta \gamma^\mu_{\alpha\beta}\left[\frac{\partial}{\partial x^\mu}\begin{pmatrix}\psi^{(\nu)}_\beta\\ \psi^{(e)}_\beta\end{pmatrix}\right.
$$

$$
\left.-ig_w\mathbf{C}_\mu\begin{pmatrix}\frac{1}{2}\{(1+\gamma_5)\psi^{(\nu)}\}_\beta\\ \frac{1}{2}\{(1+\gamma_5)\psi^{(e)}\}_\beta\end{pmatrix}\right]+\begin{pmatrix}m_\nu\,\psi^{(\nu)}_\alpha\\ m_e\,\psi^{(e)}_\alpha\end{pmatrix}=0
$$

1. Due to the presence of the $(1+\gamma_5)$ factor only left-handed fermions couple to the weak interaction vector fields.

2. Weak interaction mixes the Dirac equations for electron and electron neutrino $(\nu_e, e)$.

Similarly it mixes $(\nu_\mu, \mu)$, $(\nu_\tau, \tau)$

and also (u,d), (c, s), (t, b)

$\rightarrow$ three generations of quarks and leptons.

**The Higgs particle:**

We have seen that quantization of an $SU(N)$ gauge theory in the zero coupling limit gives rise to $(N^2 - 1)$ massless spin 1 particle.

Thus strong interaction should have 8 massless vector fields (gluons).

This is indeed true and the massless nature of the gluons is responsible for the 'strongness' of strong interaction and confinement for non-zero coupling.

According to the same analysis weak interaction should have 3 massless vector fields ($W^+$, $W^-$, $Z$).

If this were true then weak intearction should also be strong.

$\rightarrow$ not true.

Resolution: $W^+$, $W^-$ and $Z$ must be massive.

This is achieved by a mechanism known as the Higgs mechanism (spontaneous symmetry breaking).

The price we have to pay is that the final list of particles has an additional spin zero bosonic particle.

$\rightarrow$ the Higgs particle.

**Limitations of the standard model:**

1. It does not include gravity.

Gravitational interaction, however small it may be, is present in nature and a theory is not complete if it cannot describe gravity.

2. According to the standard model, neutrinos are massless.

But recent experiments show that neutrinos have small but non-zero mass.

This can be incorporated in the theory via a slight modification of the standard model.

3. The Higgs particle, and consequently the origin of the masses of $W^+$, $W^-$ and $Z$ in the standard model is somehow mysterious.

According to general arguments these masses should have been much larger than what we observe in experiment.

Only a very fine tuning of the parameters of the theory can keep these masses small.

$\rightarrow$ looks somewhat unnatural.

Due to these reasons many people have tried to contstruct theories 'beyond the standard model'.

1. Supersymmetric models:

In these models for every fermionic particle in the standard model there is also a bosonic particle with similar properties and vice versa.

These models partially solve the naturalness problem.

2. Grand unified theories:

In these models the strong, weak and electromagnetic gauge fields come from components of a single big matrix ($5 \times 5$, $10 \times 10$ etc.)

3. Supersymmetric grand unified theories:

These theories combine 1 and 2.

None of these theories include gravity.

4. String theory:

Combines 1 and 2 and also includes gravity.

On the experimental side, we expect to get new results around 2008 from an accelerator at CERN, Geneva that is currently been built.

This experiment is expected to discover the Higgs particle, and (if we are lucky) may test some of the ideas involved in the theories beyond the standard model.