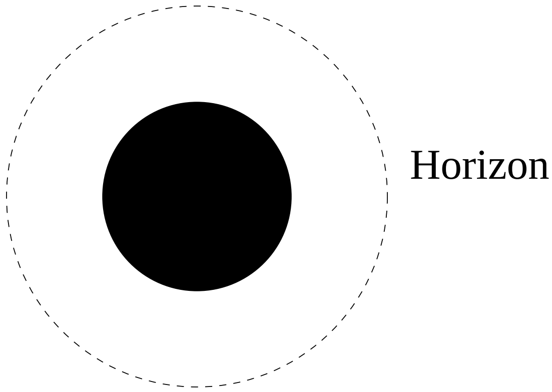
BLACK HOLES IN STRING THEORY

Black holes and their importance

Black holes are objects of very large mass.

They are described as classical solutions of the equations of motion of general theory of relativity.

Their gravitational attraction is so large that even light cannot escape a black hole.



A given black hole is surrounded by an imaginary surface such that no object inside the surface can ever escape to the outside world.

This surface is called the event horizon.

In quantum theory this picture of the black hole gets modified.

Due to uncertainty principle, it may happen that a particle-antiparticle pair is produced out of the vacuum for a short duration.

Analogy: In a harmonic oscillator the coordinate does not sit at the origin but has some quantum fluctuations.

In a normal situation such fluctuations change the zero point energy without causing much trouble.

In the case of a black hole, if a particle-antiparticle pair is produced near the horizon with large energy in each, then it may happen that one of them falls into the black hole while the other escapes.

From the point of view of the external observer it would appear that the black hole is radiating particle /antiparticle.

Non-trivial result:

1. This radiation looks like a black body radiation with a given temperature.

$\rightarrow$ Hawking temperature T_H

2. In its interaction with matter, a black hole behaves like a thermodynamic object with temperature T_H and some fixed entropy.

$\rightarrow$ Bekenstein-Hawking entropy S_{BH}

We shall work in units where the

Boltzmann's constant $k_B = 1$

velocity of light $c = 1$

Planck's constant $\hbar = 1$

In these units both entropy and temperature are given by simple formulæ:

$$S_{BH} = \frac{A}{4G_N}, \qquad T_H = \left(\frac{\partial S_{BH}}{\partial M}\right)^{-1}$$

A: Area of the event horizon

G_N: Newton's gravitational constant

M: mass of the black hole

Black hole thermodynamics:

First law: $dM = T_H dS_{BH}$

M: Mass of the black hole

Second Law: $dS_{BH} + dS_{rest} \geq 0$

For an ordinary Schwarzschild black hole of mass M, we have

$$S_{BH} = 4\pi G_N M^2 \qquad T_H = \frac{1}{8\pi G_N M}$$

The Hawking temperature of an astronomical black hole, calculated using this formula, is extremely tiny and unobservable in a present day experiment.

For example, for a solar mass black hole T_H is of order 10^{-7}K

Nevertheless the thermal nature of the black hole causes some theoretical problems.

Some concepts in statistical mechanics:

Microscopic description: corresponds to a description of a system in terms of the states of the fundamental constituents of the system, *e.g.* atoms or molecules for a normal matter.

This is difficult to achieve for a large system.

Macroscopic description: corresponds to a description of a system in terms of some crude properties which can be easily measured.

e.g. pressure, temperature, energy, entropy, volume etc.

Microstate: a state which has a microscopic description $\rightarrow$ also known as a pure state

Macrostate: a state which only has a macroscopic description

Normally there are many microstates associated with a given macrostate.

$\rightarrow$ an ensemble of states

We call such a macrostate to be in a mixed state.

Such a state does not have a definite wave-function.

Instead its wave-function can be that of any one of the microstates of the system.

$$\langle \mathcal{O} \rangle = \frac{1}{N} \sum_{microstates} \langle microstate | \mathcal{O} | microstate \rangle$$

For a thermodynamic system we do not know the exact microscopic description.

Thus such a system is in a mixed state or macrostate.

For such a system we define:

Statistical entropy: $S_{stat} = ln\Omega$

Ω= total number of microstates corresponding to a given macrostate

Temperature: $T = \left(\frac{\partial S}{\partial E}\right)^{-1}$

This gives a statistical interpretation of entropy and temperature.

Now consider the following thought experiment.

Ordinary matter in pure quantum state

$\downarrow$ gravitational collapse

black hole

$\downarrow$ hawking radiation

Thermal state of ordinary matter

Thus the final state should be regarded as a mixed state for which we do not know the wave-function exactly.

Puzzle: How can a pure state, for which we know the wave-function exactly, evolve to a mixed state for which we do not know the wave-function?

Does it imply loss of information and hence violation of quantum mechanics?

A possible resolution: a given black hole solution has a large number of microstates associated with it.

All of these states are represented by the same black hole solution.

Thus the description of the intermediate state of the system as a black hole is inadequate if we are trying to follow the evolution of the exact quantum state.

Would explain why the final state 'appears to be thermal.'

Unfortunately, within the conventional formulation of gravity based on the general theory of relativity, there is no explanation of how these microstates arise.

If string theory is to provide a successful quantum theory of gravity, then it must provide a resolution of this puzzle.

Recall some facts about string theory:

1. A string can be viewed as a collection of infinite number of harmonic oscillators.

Thus even a single string can exist in infinite number of quantum states.

2. One of these quantum states describes the graviton, – the mediator of gravitational interaction.

3. If string theory is the right theory of nature, then some of the other states will describe some of the elementary particles we observe in nature.

4. However most of these states are extremely massive and beyond the reach of present day experiments.

These massive states can provide important theoretical tool in understanding black hole physics.

Define $\Omega(M, \vec{Q})$: number of elementary string states with a given mass M and charges $\vec{Q} = (Q_1, Q_2, \ldots)$.

Ω can be calculated by standard method of quantum string theory.

One finds that Ω grows very rapidly with mass.

Thus it seems natural to define a 'statistical entropy' associated with elementary string states:

$$S_{stat}(M, \vec{Q}) = \ln \Omega(M, \vec{Q})$$

On the other hand, for large mass we expect that these states will have very strong gravitational field, and become black holes.

In this case we can associate an entropy to these black holes through the relation:

$$S_{BH}(M, \vec{Q}) = \frac{A}{4G_N}$$

as in the case of ordinary gravity.

A: Area of the event horizon of a black hole of mass M and charges $\vec{Q}$.

Question: Is $S_{BH} = S_{stat}$?

If the answer is yes, then it would provide a microscopic explanation of black hole entropy in string theory.

We shall consider a specific string theory.

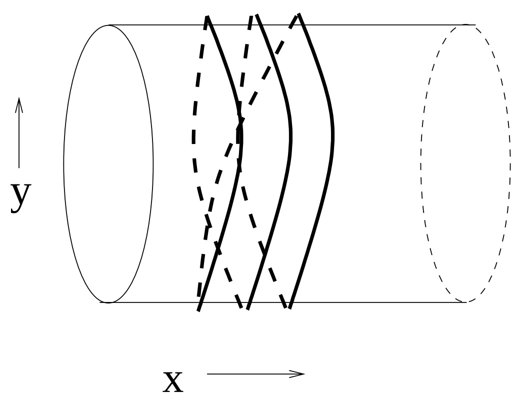
Take heterotic string theory with periodic boundary condition on six of the directions.

$\leftrightarrow$ compact space is product of six circles

Pick any one of these directions and call this direction y.

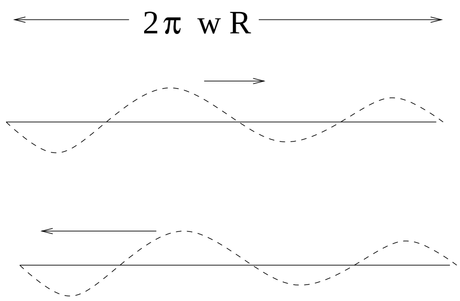
R: radius of this circle

Take a string and wind it w times along the y-circle.



We can open up the string and view it as a long string of length $2\pi Rw$ with periodic boundary condition.

As the string oscillates there can be both left-handed and right-handed waves propagating on the string.



If both kinds of oscillations are present on the string then they may collide and annihilate into radiation.

Such states of the string are unstable.

We shall consider those states of the string which carry only left-moving oscillations.

Such states of the string are stable and called BPS states.

Such a BPS string is characterized by two quantum numbers.

1. The winding number w.

2. Total momentum along the y direction carried by the oscillations.

$$n/R, \qquad n = \text{integer}$$

Question: What is the total number of BPS states $\Omega(n,w)$ characterized by the quantum numbers n, w?

$\leftrightarrow$ how many ways can we get total momentum n/R out of various oscillation modes carrying momentum in units of $1/(wR)$?

For large n, w

$$\Omega(n,w) \sim \exp(4\pi\sqrt{nw})$$

Thus

$$S_{stat} = \ln \Omega(n,w) \simeq 4\pi\sqrt{nw}$$

$$S_{stat} = 4\pi\sqrt{nw}$$

Question: If we construct a black hole solution carrying the same mass and charges, and calculate its entropy S_{BH}, does it reproduce S_{stat}?

Is

$$S_{stat} = S_{BH}?$$

First attempt:

Since string theory describes gravity, the dynamics of string theory at long distance scale is described by a classical field theory that includes general theory of relativity.

This classical field theory is called supergravity theory.

The classical equations of motion of the supergravity theory admits a BPS black hole solution carrying charge quantum numbers n and w.

What is the Bekenstein-Hawking entropy associated with this black hole?

It turns out that this black hole has zero area event horizon.

Thus $S_{BH} = A/(4G_N) = 0$!

$\rightarrow$ apparent disagreement with $S_{stat} = 4\pi\sqrt{nw}$.

However we need to be a little more careful.

Reduction of string theory to supergravity is only an approximation.

There are two kinds of corrections to this approximation.

1. Stringy corrections:

Strings are not point particles.

They appear to be point particles only at large distances.

At short distance string theory is expected to deviate from supergravity, which describes a theory of point particles.

2. Quantum effects:

String theory is a quantum theory.

Thus classical solutions in string theory will get corrected due to quantum effects.

What is the effect of stringy corrections and/or quantum corrections on the black hole solution?

It turns out that the parameter that effectively controls the quantum effects is small ($\sim 1/\sqrt{nw}$) near the black hole.

Thus we can ignore the effect of quantum corrections on the black hole solution and focus only on the stringy corrections.

A detailed analysis based on the symmetries of the classical string theory gives

$$S_{BH} = a\sqrt{nw}$$

after taking into account stringy corrections.

a: a universal constant whose value depends on the detailed structure of the stringy corrections.

Compare this with the statistical entropy:

$$S_{stat} = 4\pi\sqrt{nw}$$

Thus S_{BH} and S_{stat} has the same dependence on n and w.

However at this stage the parameter a is undetermined due to computational difficulties.

How can we do better?

Try to identify black hole solutions which are

- BPS states

- Have non-vanishing area of the event horizon even without stringy corrections.

It turns out that there are indeed such black holes present in the theory, but they do not carry the same quantum numbers as elementary string states.

Instead they can be regarded as a configuration of D-branes.

A D-p-brane is a p-dimensional extended object.

Quantum excitations on a D-brane are open strings whose ends are forced to lie on the D-brane.

D-branes carry mass per unit volume as well as charges.

Strategy:

1) Identify a BPS black hole with non-vanishing area of the event horizon and calculate S_{BH} from this area.

2) Identify the D-brane configuration carrying the same quantum numbers as this black hole and calculate S_{stat} by computing the degeneracy of these states.

3) Compare the two answers.

The analysis is simplest in type IIB string theory with periodic boundary conditions in 5 directions.

We denote this five dimensional compact space by T^5.

We consider a D-brane configuration containing

1) Q_5 D-5-branes along T^5.

2) Q_1 D-1-branes wrapped on one of the circles S^1 of T^5.

3) n units of momentum along S^1.

We can calculate the number of BPS quantum states of this D-brane configuration.

The calculation is a little bit more complicated that the corresponding calculation for elementary string states.

The result is:

$$S_{stat} = 2\pi\sqrt{Q_1 Q_5 n}$$

Now consider the supergravity theory that describes this string theory at large distance scale.

Can we find a black hole solution in this theory that carries the same mass and charges as the D-brane configuration described earlier?

Yes we can.

The black hole solution:

the non-compact coordinates: t, r, θ, ϕ, ψ

Non-zero metric components:

$$g_{tt} = -\lambda^{-2/3}, \quad g_{rr} = \lambda^{1/3}, \quad g_{\theta\theta} = r^2 \lambda^{1/3},$$

$$g_{\phi\phi} = r^2 \lambda^{1/3} \sin^2\theta, \quad g_{\psi\psi} = r^2 \lambda^{1/3} \cos^2\theta$$

where

$$\lambda = (1 + r_1^2/r^2)(1 + r_5^2/r^2)(1 + r_n^2/r^2)$$

$$r_1^2 = gQ_1/V, \qquad r_5^2 = gQ_5, \quad r_n^2 = 2g^2 n/R^2 V$$

$(2\pi)^4 V$: Volume of T^4 $\qquad$ R: Radius of S^1

g: String coupling constant

T^4: product of four circles transverse to the D1-brane direction.

From this solution we can calculate the area of the event horizon and hence S_{BH}.

Result:

$$S_{BH} = (2\pi)\sqrt{Q_1 Q_5 n}$$

Exact agreement with S_{stat}

A brief history of subsequent developments:

1. This result was soon generalized to other black holes including black holes in heterotic string theory with periodic boundary condition along six directions.

The black holes for which this procedure works are those which carry both electric and magnetic charges.

On the other hand elementary string states discussed earlier carry only electric charges.

If in the expression for the black hole entropy we set the magnetic charges to zero, we get

$$S_{BH} = 0$$

$\rightarrow$ back to the original problem.

2. This however is not the end of the story.

For black holes with large electric and magnetic charges, the stringy corrections are small.

Nevertheless they can modify the result by a small amount as long as the charges are finite.

Can we calculate these corrections?

These have been calculated by taking into account a special class of stringy effects.

Corrections to S_{BH} agree with the 'finite size' corrections to S_{stat}.

3. Given the corrected formula for S_{BH}, we can now set the magnetic charges to zero in the formula, and calculate S_{BH} for black holes which carry only electrc charges.

Result: $$S_{BH} = 4\pi\sqrt{nw}$$

in agreement with the result for S_{stat} for large n, w!

Thus we have come around a full circle.

Even though the initial attempt focussed on relating black holes to elementary string states, this had to wait till other apparently more complicated problems were solved first.

Only after solving these apparently more difficult problems, we can solve the original problem as a special case.

Other related developments:

So far we have talked about stable (BPS) states.

The corresponding black holes have zero temperature and hence do not Hawking radiate.

We can repeat the analysis for black holes which are nearly stable.

These have small but non-zero Hawking temperature.

It turns out that for this black holes we have again

$$S_{stat} = S_{BH}$$

Furthermore one can compute the rate of decay of the D-brane configuration using the rules of ordinary quantum mechanics.

This agrees with the hawking radiation rate from the corresponding black hole!

Lesson:

The main lesson to be learned from this analysis is that in string theory black holes behave like ordinary quantum states.

In fact there is no real distinction between elementary 'particles' and black holes.

When the particles are light it is more appropriate to think of them as ordinary elementary particles.

When the particles are very heavy it is more appropriate to describe them as black holes.

On the other hand duality symmetries in string theory tell us that in string theory there is no fundamental distinction between elementary particles and composite particles.

The same physical particle may appear to be elementary in one description and composite of two or more elementary particles in another description.

Duality symmetries also relate elementary particles to D-branes, solitons etc.

Thus in string theory there seems to be complete democracy of all objects, – elementary particles, composite particles, D-branes, solitons, black holes ....

$\rightarrow$ a macroscopic description