String Theory

And

Gauge Theories

[String Symposium Of The National Academy Of Sciences, India, At H.R.I., Allahabad, Oct 1-2]

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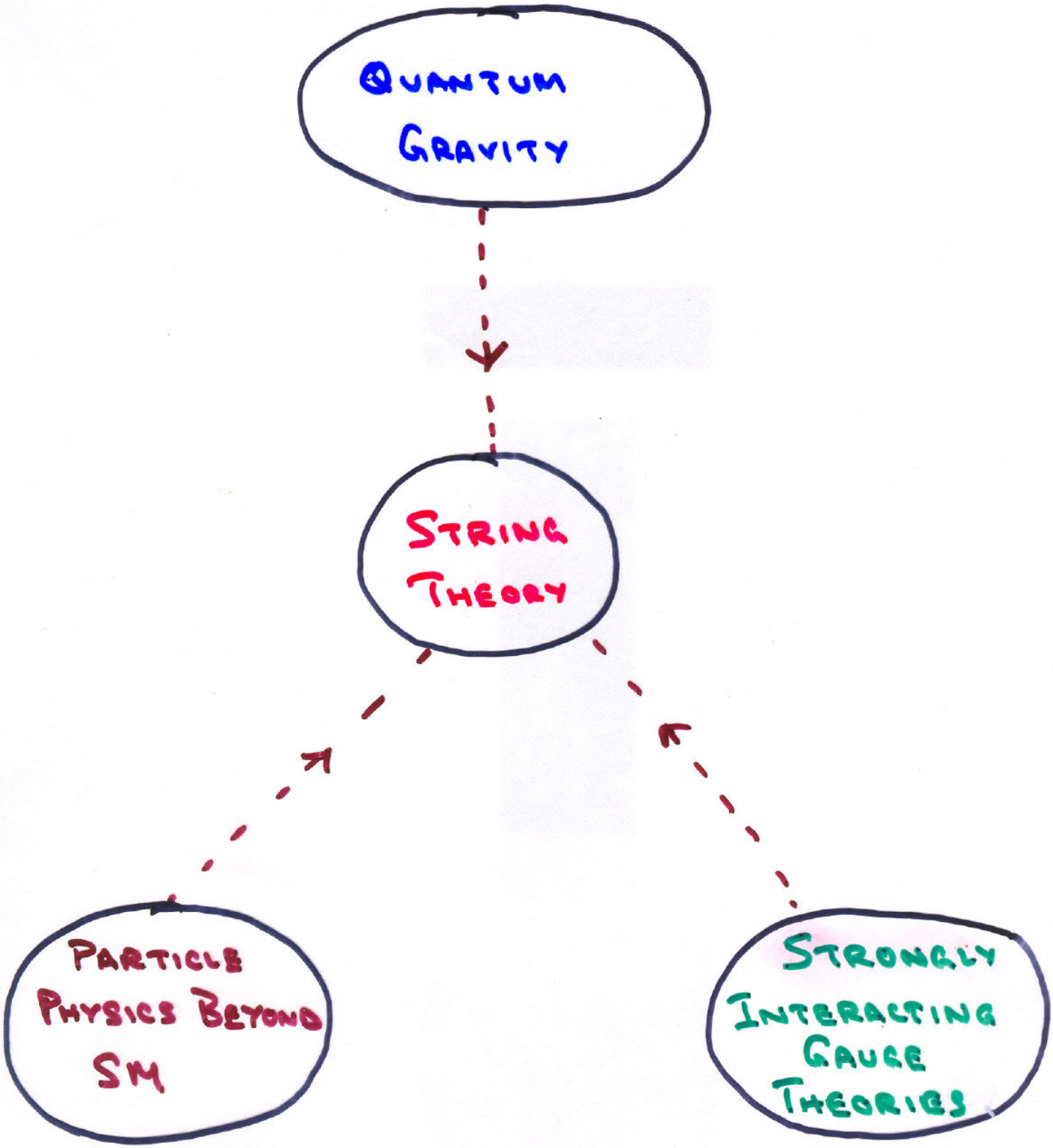

QUANTUM GRAVITY
STRING THEORY
PARTICLE PHYSICS BEYOND SM
STRONGLY INTERACTING GAUGE THEORIES

Outline Of The Talk

* Gauge Fields And Strings
* Strings In QCD?
* The Maldacena Conjecture
* Applications

Gauge Fields And Strings

Trace Back The Relation To Maxwell And Faraday:

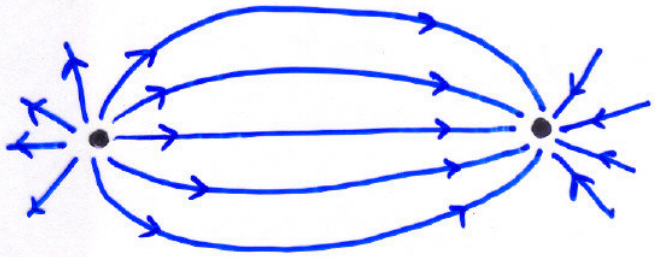
Maxwell

Localised Fields

$\vec{E}(t,\vec{x})$, $\vec{B}(t,\vec{x})$

Faraday

Delocalised Lines Of Flux



Two Contrasting Pictures Of Electromagnetism - The "Simplest" Gauge Theory.

In Essence, The Possibility Of These Two Different "Dual" Descriptions Underlies Relation Between Gauge Fields And Strings.

However, in electrodynamics, Maxwellian view prevailed - We write equations for $\vec{E}, \vec{B}$ rather than for lines of flux.

Why?

Field description more appropriate since quantum fluctuations are small in electrodynamics ($\alpha \approx \frac{1}{137} \ll 1$).

Can talk of a quantum field exciting individual quanta - photons, which are weakly interacting.

At the same time flux lines are spread out all over space - they lack "rigidity".

But field description not necessarily appropriate for all gauge theories.

E.g. QCD - Theory of the strong interactions.

HERE QTM. EFFECTS ARE ALL IMPORTANT

$\Leftrightarrow$ QUANTUM FIELD EXCITING A LOCALISE

QUANTUM - THE GLUON - IS A USELESS DESCRIPTI

MAXWELLIAN PICTURE NOT APPROPRIATE.

PHYSICAL STATES IN QCD COMPRISE OF MANY QUANTA STRONGLY BOUND TOGETHER INTO A COLLECTIVE EXCITATION.

HOW DO WE DESCRIBE THE DYNAMICS OF THESE COLLECTIVE EXCITATIONS? (GLUEBALLS, MESONS, BARYONS etc...)

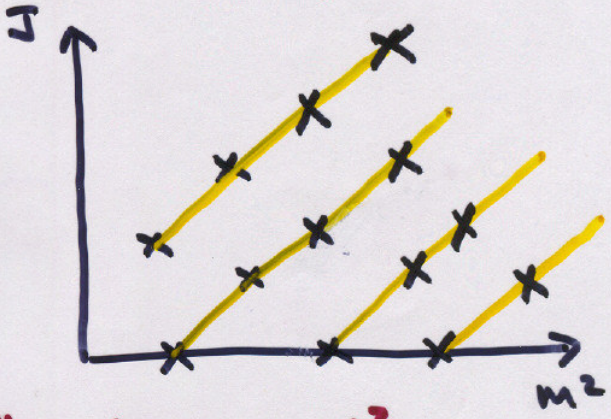
COULD A FARADAY LIKE DESCRIPTION IN TERMS OF FLUX LINES HELP?

Strings In QCD ?

Many Hints Of A String Description Of QCD.

* QCD has mesons - bound states of a quark and an antiquark. (e.g. $\pi^+ = (u\bar{d})$)

Their Masses Obey Striking Pattern



$$M^2 \approx J T_{ot} + \text{Const.}$$

T_{ot} → "Tension" $\approx (300\,\text{MeV})^2$

J → Angular momentum / Spin

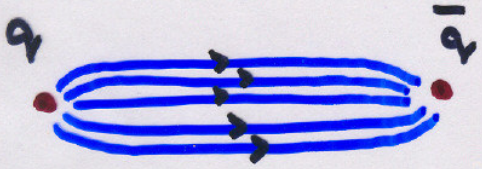
Note Close Similarity To The Spectrum Of The Relativistic String.

$M_N^2 = 2\frac{N}{\alpha'} + \text{Const.}$ (But the tension here is very different from the Planck scale tension we had earlier.)

Suggests The Possibility Of Describing Bound States In QCD As String Excitations

* Qualitative Picture Of "Confinement" In QCD. Based On String-Like Flux Tubes.

Quark-Anti-Quark Pairs In Mesons Are Always Confined. How?



Faraday Lines Of Flux Bunched Up Into A Tube Rather Than Spread Out.

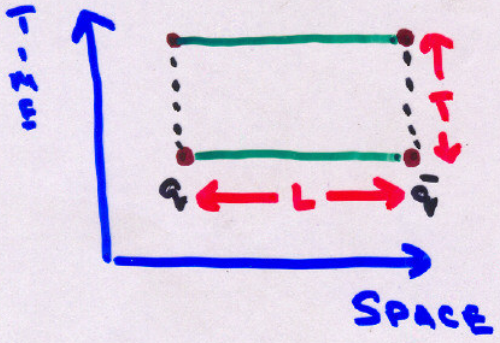
Like Behaviour Of Magnetic Fields In Superconductors (Meissner Effect)

Presence Of Chromoelectric Fields Energetically Disfavoured.

Hence Lines Of Flux Confined To Narrow Tube Between q And $\bar{q}$.

- String Like Object w/ Energy Proportional To Length.

✱ Quantitative Test For The Presence Of String.



Amplitude For Time Evolution Of $q\bar{q}$ Pair Given By

$$\langle \mathrm{Tr}[P e^{i\oint A dx}] \rangle \sim e^{-E(L)T}$$

Approximate Estimation Shows

$$E(L) \propto L$$

Suggests That The Configuration That Dominates Is String-Like (has constant tension)

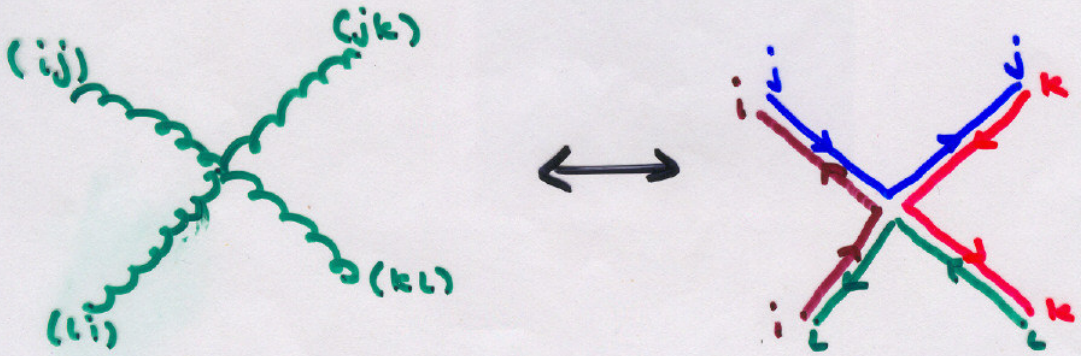
[Note That A Weakly Interacting Coulomb Behaviour $\Rightarrow E(L) \propto \frac{1}{L}$.

Signifies That In QCD Many, Many Quanta Are Bound Up Into A String-Like Configuration.

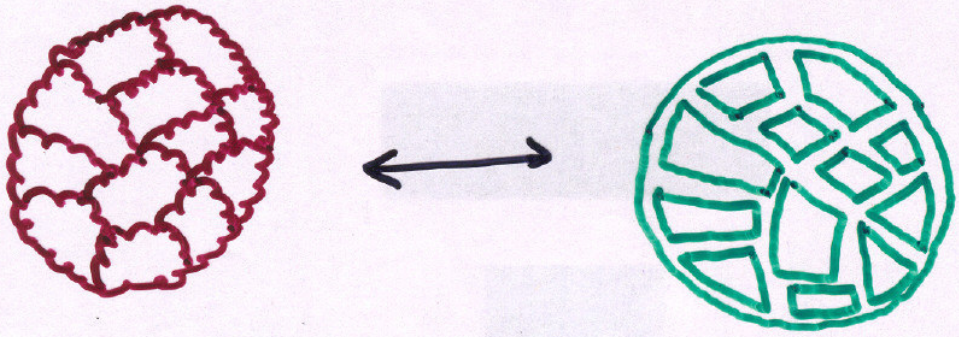
* 't Hooft's Picture Of String Worldsheets In SU(N) Gauge Theory.

Idea: Look At Feynman Diagrams In U(N) Gauge Theory. Gluon Is $N \times N$ Matrix $(A_m)_{ij}$ $(i,j = 1, \ldots, N)$

Keep Track Of Matrix Indices In A Diagram - via double line repn.



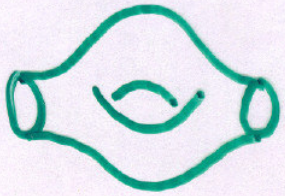
Each Feynman Diagram Can Be Viewed As A Patched Up Two Dim. Surface.



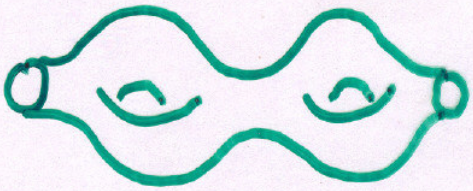
"Fishnet" Approximation To The Worldsheet Of A String?

In Fact, Analogy To String Becomes Better When N (of SU(N)) Becomes Larger.

Can Show That Surfaces Corresp. To String Splitting + Joining Weighted By $\frac{1}{N^2} \sim g_s^2$



$\propto \frac{1}{N^2}$



$\propto \frac{1}{N^4}$

etc.

So, Many Indications That A String Description Underlies Gauge Theories.

But Many Problems Arose In Getting The Details To Work.

For Example, Scattering Amplitude For Two Strings Off Each Other.

In Flat Space $\propto e^{-E^2\alpha'}$ For High Energies (compared to String Tension)

But can calculate this accurately in QCD and find $\propto \frac{1}{E^{c_1}}$.

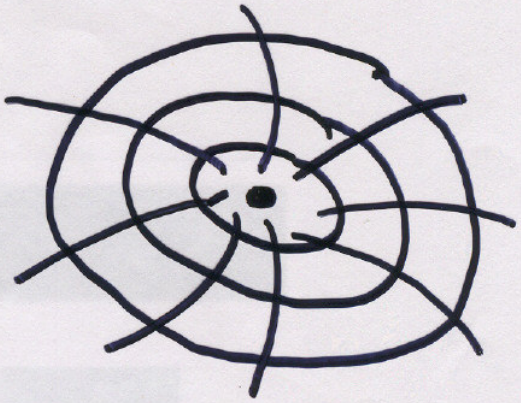
Very Different Behaviour

- Difficult To Reconcile

The Maldacena (or AdS/CFT) Conjecture

* Breakthrough via A Concrete Example – involving a Supersymmetric version of QCD ($\mathcal{N}=4$ Super Yang-Mills).
* Showed That Stumbling Block Was In Preconception That The Equations Of Motion Of Dual String Are Those In Flat 4-D Space. Dynamics Actually In Curved Higher Dimensional Space.
* Made The Connection Between Strings Underlying Gauge Theory w/ Strings Describing Gravity !!!
* Connection Between Gauge Theory + Gravity Arising From Two Contrasting Pictures Of D-Branes.

Two Pictures Of D-Branes



(transverse geometry)

D3 Branes In Flat Space Time w/ Yang-Mills Fields (3+1 Dim.)

→ U(N) Supersymm. Yang-Mills Theory At Low Energy.

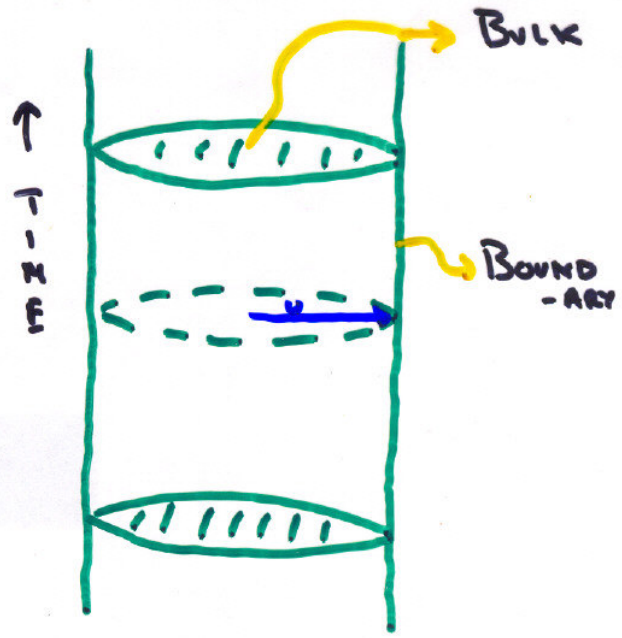
Black Hole-Like Curved Geometry w/ Gravitational Fields (in 9+1 dim.)

$$ds^2 = \left(1+\frac{R^4}{r^4}\right)^{-1/2}(-dt^2+dx_1^2+dx_2^2+dx_3^2) + \left(1+\frac{R^4}{r^4}\right)^{1/2}(dr^2+r^2 d\Omega_5^2)$$

* Consistency Of String Theory Requires These Two Completely Different Descriptions Be Equivalent. ⇒ The Surprising Connection Between Gauge Theory And Gravity.

More Precisely, IIB Closed Strings On $AdS_5 \times S^5$

(geometry near core of black hole description)



$\equiv$ $\mathcal{N}=4$ Super Yang-Mills On Boundary Of AdS_5.

AdS_5 Geometry: $ds^2 = \frac{R^2}{u^2}\left[-dt^2 + dx_1^2 + dx_2^2 + dx_3^2 + du^2\right]$

$(0 < u < \infty)$

$u=0$ Is The Boundary Of $AdS_5 \rightarrow$ 4 Dim. Spacetime On Which The Gauge Theory Lives (the directions of the D3 brane worldvolume)

Thus, "Holographic" Relation Between A String Theory In The "Bulk" And A Gauge Theory On The "Boundary".

How Can A Theory Of Gauge Fields Be The Same As A Theory w/ Gravity?

Answer: They Are "Dual" Descriptions.

* When A Classical General Relativity Description Of AdS, Geometry Is Valid, Then The Gauge Theory Is Strongly Interacting ("non-perturbative")

* When The Gauge Theory Is Weakly Interacting, The Gravity Description Requires Stringy Corrections Since It Is At Planck Scale.

* Thus A Duality Between Open String (gauge theory) And Closed String (gravity) Descriptions

Applications

* Maldacena's Conjecture Generalised To More Realistic Gauge Theories. But Would Like To Construct Dual For Real Life QCD!
* Modifications Of Maldacena's Conjecture Provides Models For Mesons etc. Which Are Good Fits To Experimental Results.
* Dual Gravity Description Of Gauge Theories At Finite Temperature Give Good Fit To Results From Strongly Interacting Quark-Gluon Plasma Found At RHIC.

Summary

* There is a possibility that gauge theories can be described in terms of Faraday lines of flux
* Many hints that in strongly interacting theories like QCD. These lines of flux behave like relativistic strings.
* These hints (especially the 't Hooft idea) are realised in Maldacena's conjecture
* Makes the connection between strings in gauge theories and strings describing gravity.
* The connection made via two dual pictures of D-branes.
* Helps one to learn about gravity from gauge theory and about gauge theory from gravity !!