String/M theory Compactification and Phenomenology

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Lecture Notes
Part I

Course Outline

0. Introduction and Motivation
1. (Brief) Review of Standard Model
2. Superstring / M theory in Ten and Eleven Dimensions
3. Supersymmetric Compactifications to 4d.
4. Ricci Flat manifolds and Special Holonomy
5. Low energy Spectrum Calculations
    - a) Heterotic
    - b) G_2 M theory
6. Generic Low energy Lagrangian
7. Generic Predictions for Dark Matter
8 Outlook and (Big) Open Questions

Introduction and Motivation

- Even though there is much we still do not understand about our Universe, it's truly remarkable how much we do know
- "Knowledge" ≅ how well observation fits the Standard Models of Particle Physics and Cosmology
- The Standard Model of Particles is a very simple mathematical model which fits beautifully an enormous amount of data/observations/experiments.

Many Open Questions: $\hbar = c = 1$

- Quantum Numbers: There are 45 fermions and 45 anti-fermions in the SM, transforming in a sum of representations of $G_{SM} = SU(3)\times SU(2)\times U(1)$. What is the origin of this structure? $(\text{charge}(e^+) = \text{charge}(p))$?

- Mass Problems:
  - Fermions: their masses span 12 orders of magnitude. Why? ← addressed by strings
- Bosons: $M_{Higgs} \sim M_{W^\pm} \sim M_Z \sim O(100)$ GeV
  This has to be tuned (quadratically) to high precision in the SM.

- What is the origin of the electroweak scale ($M_{EW} \sim O(100)$ GeV) and what stabilises it?
- Neutrinos have a mass. What is its origin?
- Baryon Asymmetry: why is there so much more matter than anti-matter?
- Dark Matter: what is it? It is not part of the Standard Model. It's properties are unknown: spin, mass interactions... ???

However, the Standard Big Bang Cosmology which is just: the SM plus non-interacting Cold Dark Matter plus Dark Energy (sometimes called Λ-CDM) in a background

$$\underline{ds^2 = -dt^2 + a^2(t)\, d\underset{\sim}{x}^2}$$

evolving according to G.R. fits all appropriate data remarkably well.

i.e. our world is described by a coupled set of Einstein's equations (cosmo)
Maxwell's equations (γ)
Dirac equation (fermions)
Yang-Mills equation ($W^{\pm}, Z, g$)
Klein-Gordon equation (Higgs)

✱

The physics of superstring / M theory at low energies (below M_{pl} and M_{st}) are also described by

- Einstein's equations
- Dirac equation
- Maxwell / Yang-Mills eq^{n}s
- Klein-Gordon Equation

Superstring/M theory seems to unify these equations into a single framework that, in a sense, "extends" quantum field theory. ☺ 👍!

This is one of the main reasons we study string/M theory as a framework for physics beyond the Standard Model.

- One goal of these lectures is to show that it is possible to extract some quite general ("generic") conclusions ("predictions") from the string/M theory framework. Some of these predictions can be *directly tested*.

- Since the theory is intrinsically ten and eleven dimensional we must first develop "*string compactification*" i.e. study solutions with compact extra dimensions and how to describe their physics.

- We will use techniques of geometry (differential and algebraic), harmonic analysis on manifolds, group theory and special holonomy to obtain the basic results.

- Assume that the students have studied:
  - General Relativity and basic differential geometry
  - Basic group theory of simple Lie groups
  - Quantum Field theory
  - Some supersymmetry.

1) Standard Model (Brief) Review

Most QFT's we work with are based on Lagrangian densities, $\mathcal{L}$.

Each particle corresponds to a field of the same (ψ) name, ψ. $\mathcal{L}$ depends on all the ψ's and their derivatives.

Usually ψ and $\partial_\mu \psi$ are considered independent functions in $\mathcal{L}$.

In this chapter, restrict to particles with spin ≤ 1.

Then, assuming also Lorentz invariance, $\mathcal{L}$ has the following generic form:

$$\mathcal{L} = \left.\begin{array}{l} -\frac{1}{4g^2}F_{\mu\nu}^2 - (D_\mu\phi)^\dagger(D^\mu\phi) + i\bar\Psi \not{D}\psi \\ + \; m_\Psi\bar\Psi\psi \; - m^2\phi^\dagger\phi - \lambda(\phi^\dagger\phi)^2 \\ \qquad + \; y\,\phi\,\bar\Psi\psi \end{array}\right\} \text{dim} \leq 4$$

$+$ higher order terms, e.g.

$$\left(\ldots \frac{1}{m}F_{\mu\nu}\bar\Psi\gamma^{\mu\nu}\psi + \frac{1}{M^2}|\bar\Psi\psi|^2 + \ldots \right)$$

$[A_\mu] = [\phi] = M \qquad [\psi] = M^{3/2}$ $\qquad y, g, \lambda$ dimensionless couplings

Note: the first term is defined by gauge symmetry $\underline{G}$.

- The Ψ's and ϕ's are defined by $\underline{\text{rep}^{\underline{n}}\text{'s}}$ of $G \times$ Lorentz.
- A given mass term for fermions $\overline{\Psi}\Psi$ may be zero by gauge invariance, since $\overline{\Psi}\Psi = \overline{\Psi}_L\Psi_R + \overline{\Psi}_R\Psi_L$ and Ψ_L and Ψ_R can have different reps under G. (true in nature!)
- $D_\mu \equiv \partial_\mu + igT^aA^a_\mu$ [T^a generate Lie Algebra of G. Lie(G)]

The Standard Model of Particle Physics

$$G_{SM} = SU(3) \times \underline{SU(2)} \times U(1)_Y$$

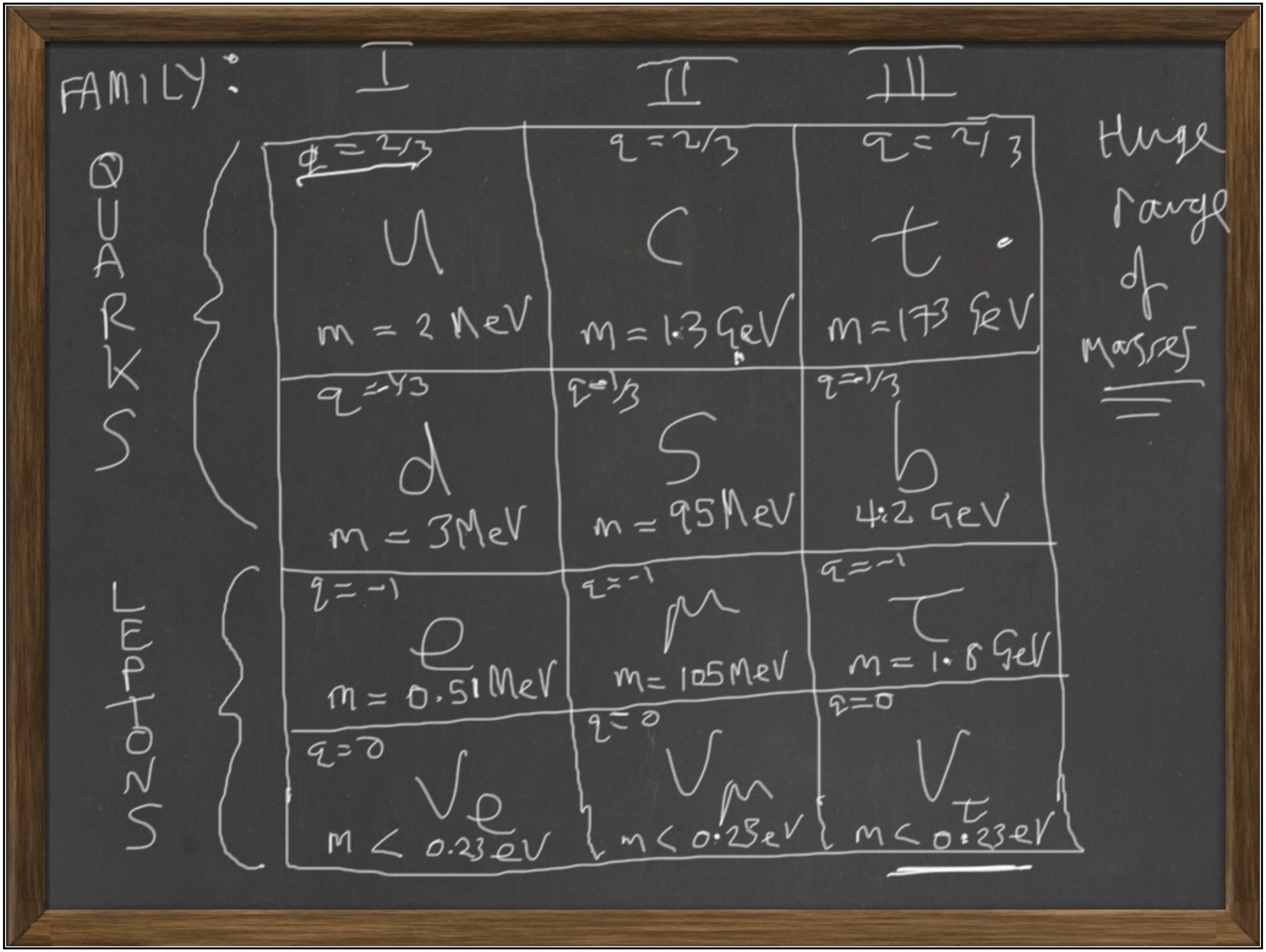
$$R_{fermion} = 3 \text{ copies of } .$$

$$(3,2)_{1/6} \oplus (\bar{3},1)_{-2/3} \oplus (\bar{3},1)_{1/3} \oplus (1,2)_{-1/2} \oplus (1,1)_{+1} \quad *$$

$$\begin{array}{cccccc}
\underline{I} & \begin{pmatrix} u_L \\ d_L \end{pmatrix} & \overline{u_R} & \overline{d_R} & \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix} & \overline{e_R} \\
\underline{II} & \begin{pmatrix} c_L \\ s_L \end{pmatrix} & \overline{c_R} & \overline{s_R} & \begin{pmatrix} \nu_{\mu L} \\ \mu_L \end{pmatrix} & \overline{\mu_R} \\
\underline{III} & \begin{pmatrix} t_L \\ b_L \end{pmatrix} & \overline{t_R} & \overline{b_R} & \begin{pmatrix} \nu_{\tau L} \\ \tau_L \end{pmatrix} & \overline{\tau_R}
\end{array}$$

Note: i) only L-handed fermions and <u>R-handed antifermions</u> couple to SU(2) gauge bosons.

ii) No R-handed neutrinos.



FAMILY:	I	II	III
QUARKS	$q = 2/3$ u $m = 2$ MeV	$q = 2/3$ c $m = 1.3$ GeV	$q = 2/3$ t $m = 173$ GeV
	$q = -1/3$ d $m = 3$ MeV	$q = -1/3$ s $m = 95$ MeV	$q = -1/3$ b 4.2 GeV
LEPTONS	$q = -1$ e $m = 0.51$ MeV	$q = -1$ μ $m = 105$ MeV	$q = -1$ τ $m = 1.8$ GeV
	$q = 0$ ν_e $m < 0.23$ eV	$q = 0$ ν_μ $m < 0.23$ eV	$q = 0$ ν_τ $m < 0.23$ eV

Huge range of masses

$$\begin{array}{ccccccccc} (3,2)_{1/6} & \oplus & (\bar{3},1)_{-2/3} & \oplus & (\bar{3},1)_{1/3} & \oplus & (1,2)_{-1/2} & \oplus & (1,1)_{+1} \\ \begin{pmatrix} u_L \\ d_L \end{pmatrix} & & \bar{u}_R & & \bar{d}_R & & \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix} & & \bar{e}_R \end{array}$$

$\boxed{\text{*Mass is forbidden by } G_{SM} :}$

$\bar{u}_R u_L$, $\bar{d}_R d_L$, $\bar{e}_R e_L$ are not gauge invariant. But if we introduce a scalar field $\phi \in \underline{(1,2)_{1/2}}$ then

$\underline{\phi \bar{u}_R u_L}$, $\underline{\phi^\dagger \bar{d}_R d_L}$, $\underline{\phi \bar{e}_R e_L}$ contain gauge invariant terms.

$\psi = \begin{pmatrix} u_L \\ u_R \end{pmatrix}$

So, if $\langle \phi \rangle \neq 0$, all fermions except neutrinos become massive.

$$\underset{\sim}{\mathcal{L}_{\text{yukawa}}} \sim y_u \phi \bar{u}_R u_L + y_d \phi^\dagger \bar{d}_R d_L + y_e \phi \bar{e}_R e_L + \text{h.c.}$$

So $\underline{m_u} = \underline{y_u} \langle \underline{\phi} \rangle$ $\qquad$ $\underline{m_e} = \underline{y_e} \langle \underline{\phi} \rangle$ etc.

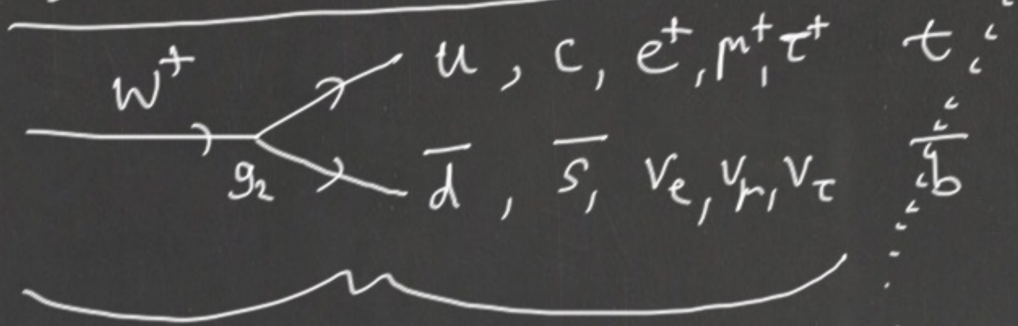
Since $M_{\text{top-quark}} \sim 172\,\text{GeV}$ and $m_e \sim \frac{1}{2}\,\text{MeV}$

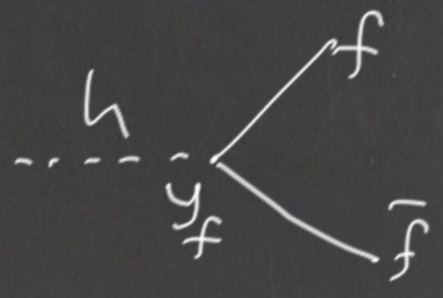
$$\frac{y_t}{y_e} \sim O(10^6). \quad \text{Why?}$$

$$\boxed{\text{Ex: Calculate } \frac{\Gamma(h \to b\bar{b})}{\Gamma(h \to \tau\bar{\tau})}}$$

h = Higgs.

Some SM vertices





forbidden as $m_t > M_W$

W- decays: 3 lepton channels

3+3 hadron channels

↑ 3 u's

9 in total.

$$\therefore \; Br(W \to \text{hadrons}) \cong \frac{3+3}{9} = 2/3$$

$$Br(W^+ \to e^+ \nu) \cong 1/9$$

W- branching fractions

- All L-handed fermions are SU(2) doublets

  $\rightarrow$ from $i\bar{\Psi}\gamma^\mu(\partial_\mu + i\frac{g}{2}\underline{W^a_\mu\sigma_a})\Psi_L$.

- the W-vertices are $\underline{\underline{\text{universal}}}$ ie the same for all fermions

- W^+ can decay into

$$\overbrace{\begin{pmatrix}e^+\\ \nu_e\end{pmatrix}\begin{pmatrix}\mu^+\\ \nu_\mu\end{pmatrix}\begin{pmatrix}\tau^+\\ \nu_\tau\end{pmatrix}}^{\text{lepton}}\overset{\text{hadronic}}{\begin{pmatrix}c\\ \bar{s}\end{pmatrix}\begin{pmatrix}u\\ \bar{d}\end{pmatrix}}$$

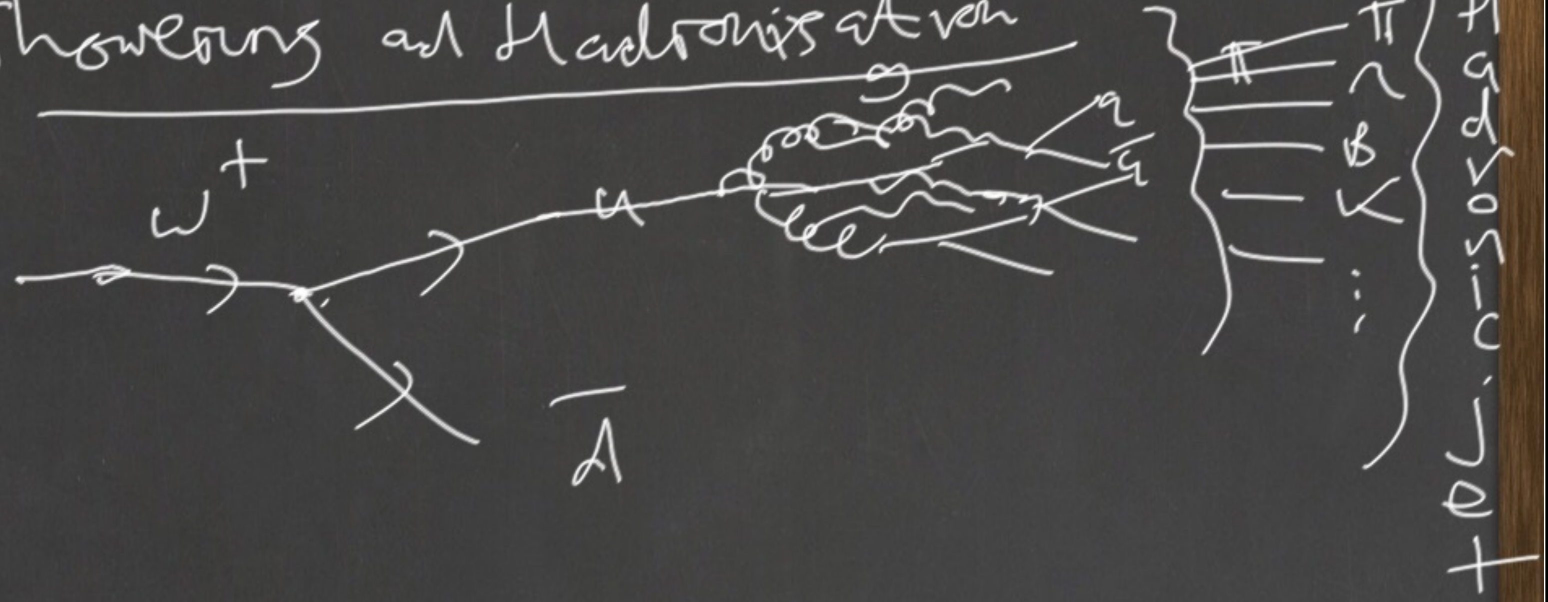
plus CKM suppressed.

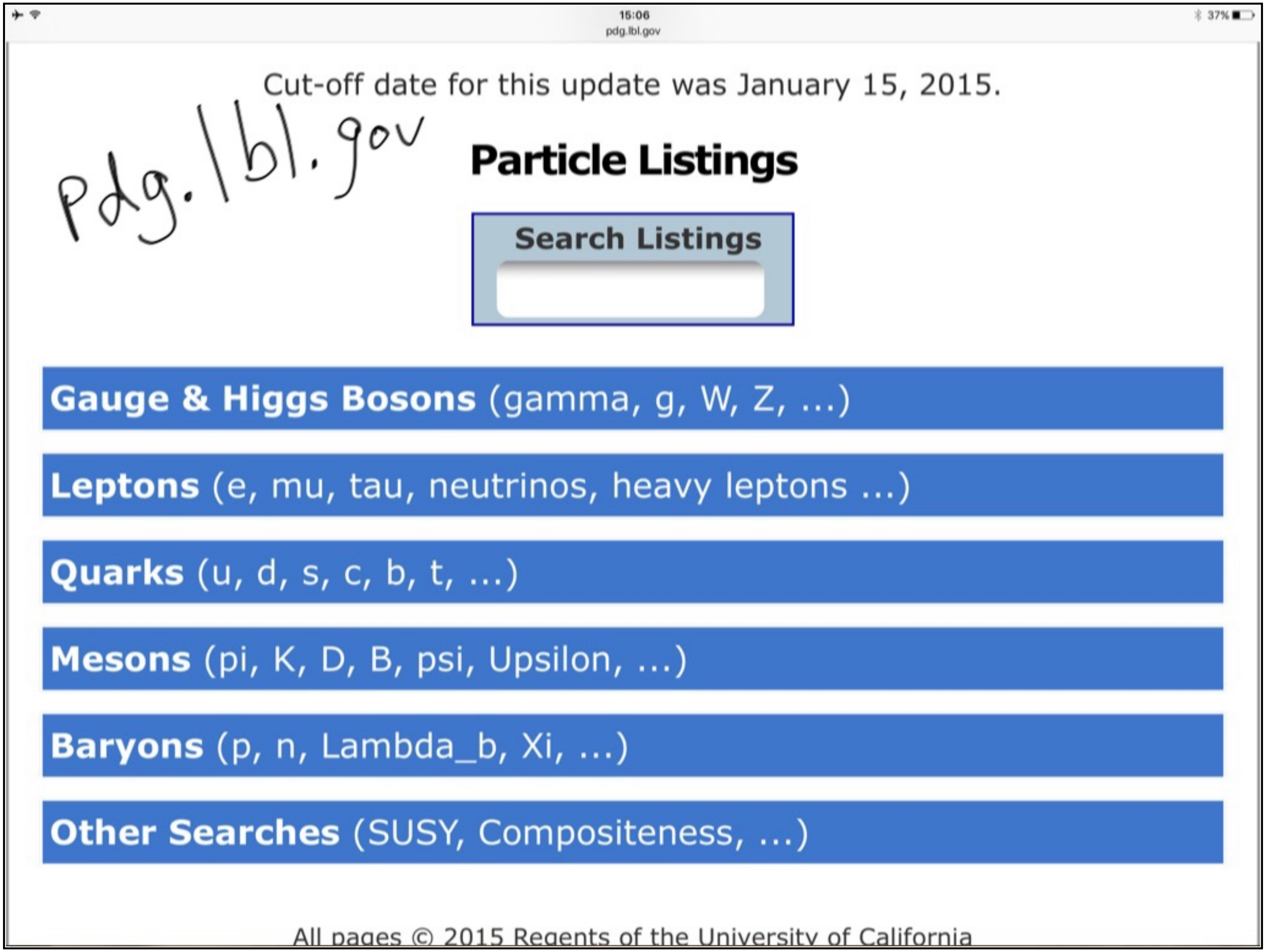
(not $\begin{pmatrix}t\\ b\end{pmatrix}$ as $m_t > m_W$)

- Since $\underline{\underline{M_W \gg m_c}}$

$$\Gamma(W^+\to e^+\nu_e) = \Gamma(W^+\to\mu^+\nu_\mu) = \Gamma(W^+\to\tau^+\nu_\tau) = \frac{1}{3}\Gamma(W^+\to c\bar{s}) \Rightarrow \boxed{\begin{aligned} &Br(W^+\to e^+\nu_e) = 1/9\,. \\ &Br(W^+\to \text{hadrons}) = 2/3\,. \end{aligned}}$$

Showering and Hadronisation





Cut-off date for this update was January 15, 2015.

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Particle Listings

Gauge & Higgs Bosons (gamma, g, W, Z, ...)

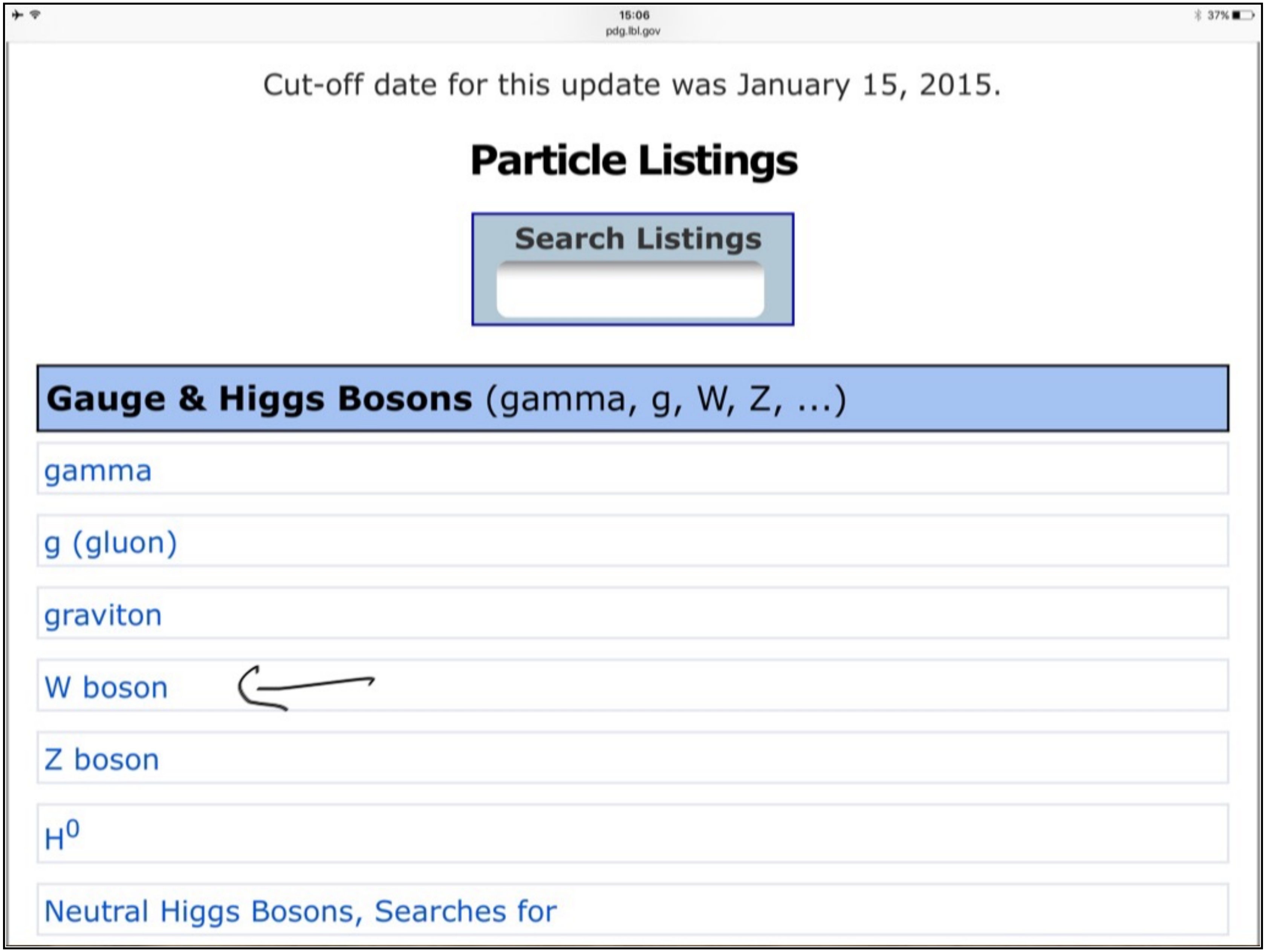
Leptons (e, mu, tau, neutrinos, heavy leptons ...)

Quarks (u, d, s, c, b, t, ...)

Mesons (pi, K, D, B, psi, Upsilon, ...)

Baryons (p, n, Lambda_b, Xi, ...)

Other Searches (SUSY, Compositeness, ...)



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Particle Listings

Gauge & Higgs Bosons (gamma, g, W, Z, ...)

gamma

g (gluon)

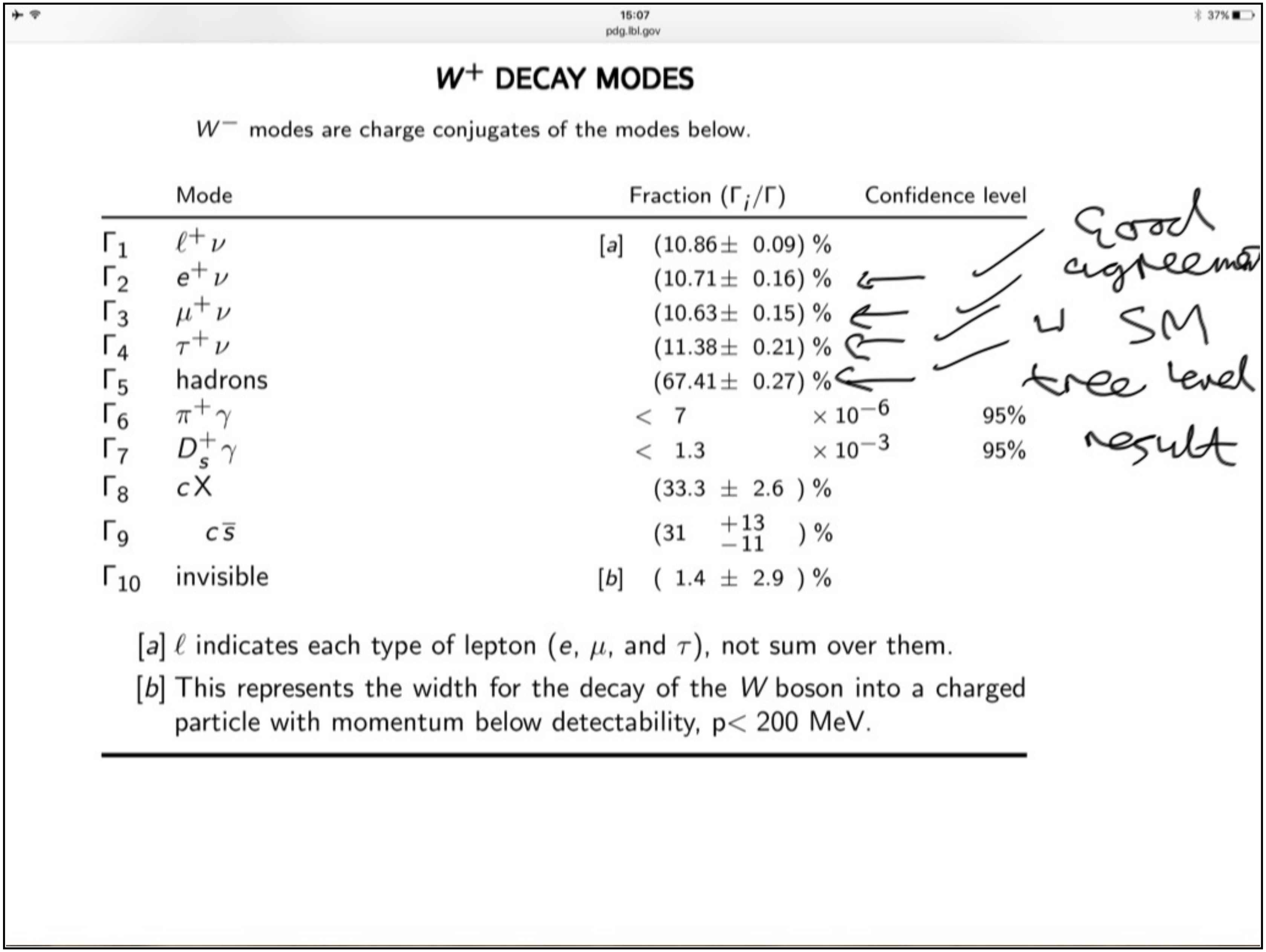
graviton

W boson

Z boson

H^0

Neutral Higgs Bosons, Searches for



W^+ DECAY MODES

W^- modes are charge conjugates of the modes below.

	Mode		Fraction (Γ_i/Γ)	Confidence level
Γ_1	$\ell^+\nu$	[a]	(10.86 ± 0.09) %	
Γ_2	$e^+\nu$		(10.71 ± 0.16) %	
Γ_3	$\mu^+\nu$		(10.63 ± 0.15) %	
Γ_4	$\tau^+\nu$		(11.38 ± 0.21) %	
Γ_5	hadrons		(67.41 ± 0.27) %	
Γ_6	$\pi^+\gamma$		$< 7 \times 10^{-6}$	95%
Γ_7	$D_s^+\gamma$		$< 1.3 \times 10^{-3}$	95%
Γ_8	cX		(33.3 ± 2.6) %	
Γ_9	$c\overline{s}$		$(31\ ^{+13}_{-11})$ %	
Γ_{10}	invisible	[b]	(1.4 ± 2.9) %	

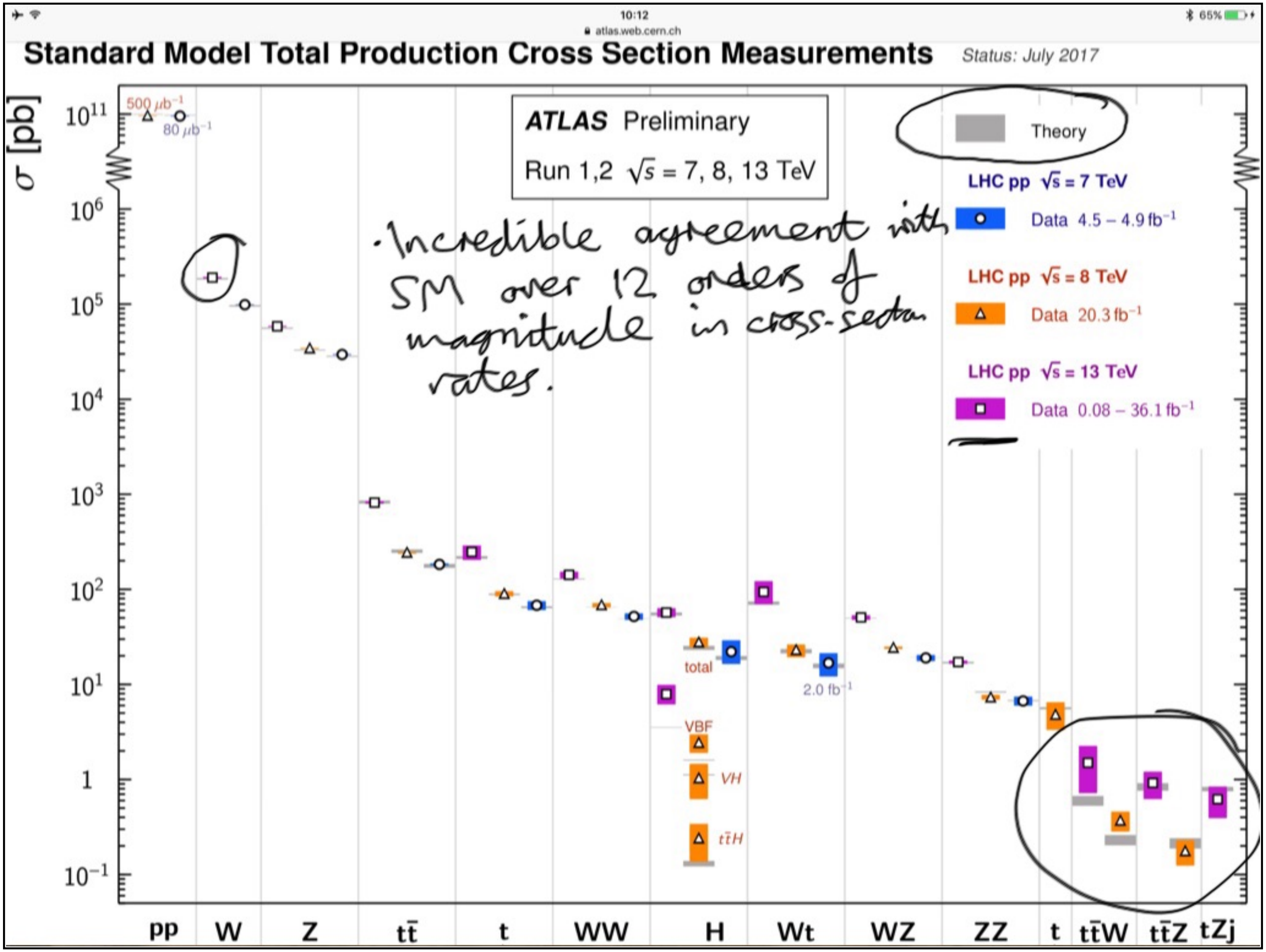
[a] ℓ indicates each type of lepton (e, μ, and τ), not sum over them.

[b] This represents the width for the decay of the W boson into a charged particle with momentum below detectability, p< 200 MeV.

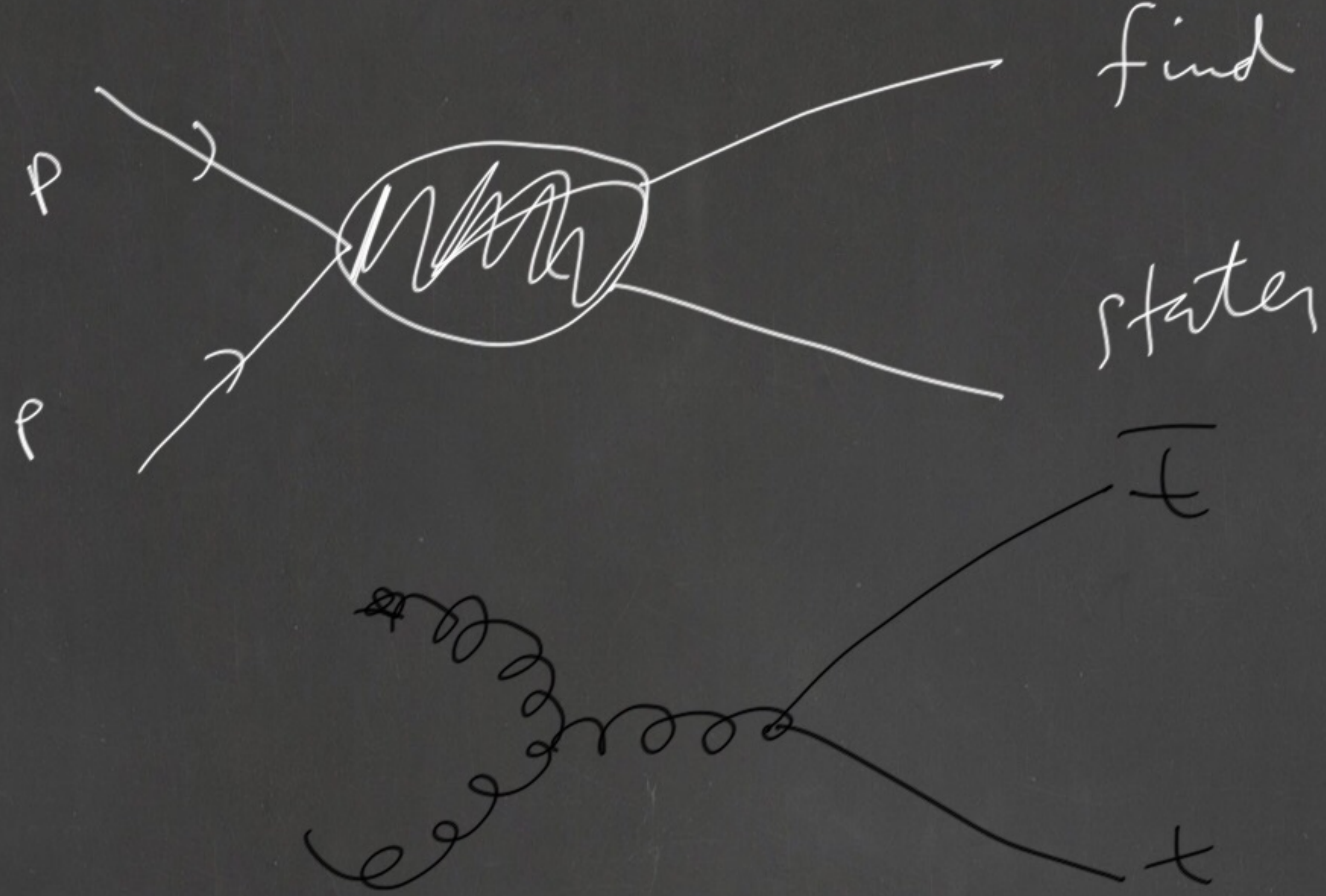
Good
agreement
w SM
tree level
result

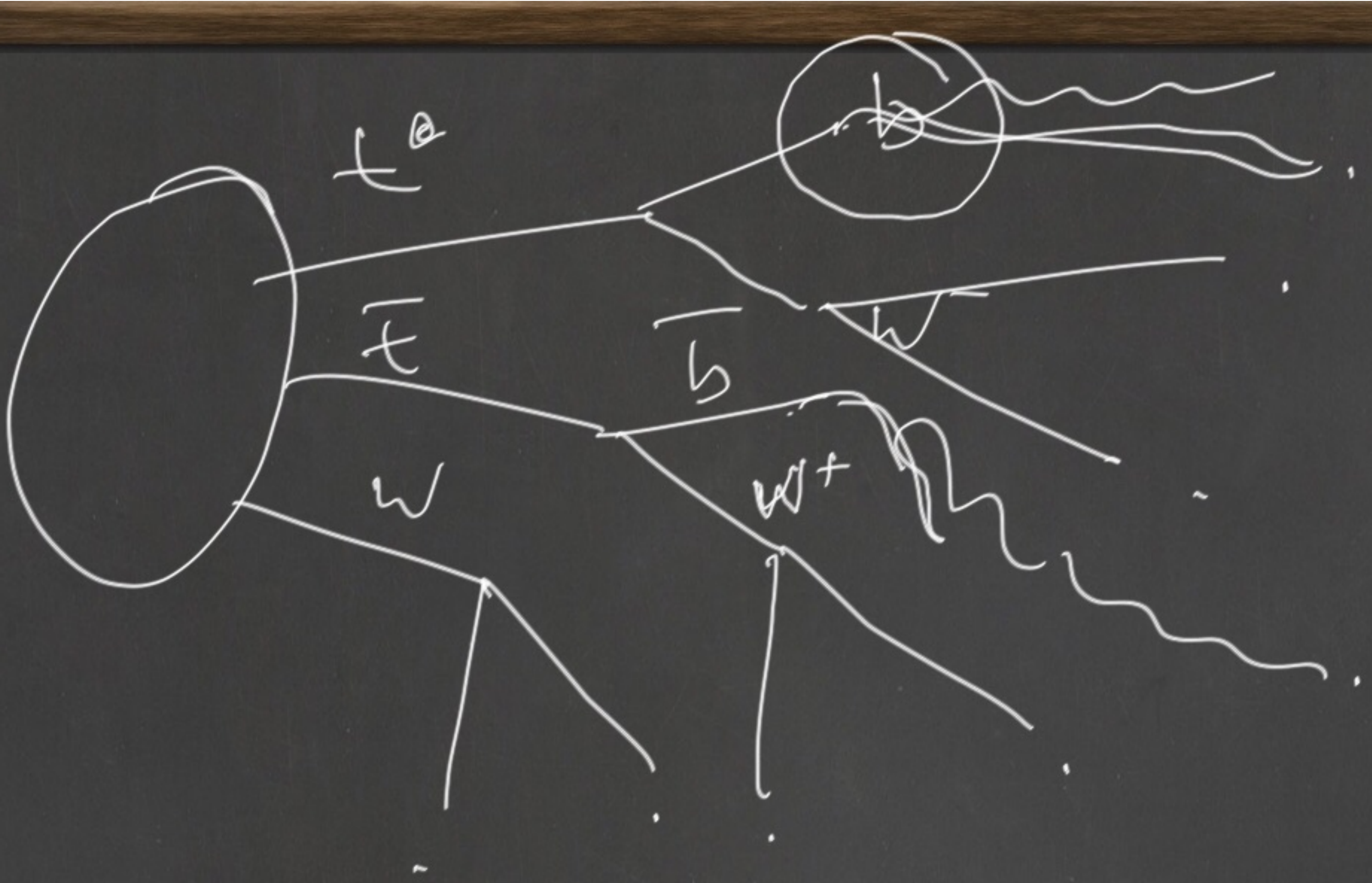
Q: If $m_t = 45\,\text{GeV}$, what would the SM predict for $Br(W^{\pm} \to \dots)$?

If $SU(3) \to SU(4)$,
3 lepton but 8 hadron chanells
inconsistent w observation
$\Rightarrow$ Observations test fundamental structure of the SM.



· Incredible agreement with SM over 12 orders of magnitude in cross-section rates.


p
p
find
states
$\bar{t}$
t


t
b
$\bar{t}$
$\bar{b}$
W^-
W
W^+

This understanding of the behaviour of matter is truly a remarkable achievement.

But where does the SM come from? What explains its features?

Generic Predictions

Q: What are the generic predictions of Quantum Field Theory?

Generic Predictions

Q: What are the generic predictions of Quantum Field Theory?

A: This is not a good question!

There are infinitely many QFT's with varying field contents, symmetries, interactions, even dimensions.

BUT....

A better question might be

Q: In $d=4$, what are the generic predictions of QFT's with non-Abelian gauge symmetry, chiral fermions, $(R_L \neq R_R)$ hierarchical Yukawas and spontaneous symmetry breaking?

spins ≤ 1.

A: Hierarchical fermion masses, a rich pattern of 3-body fermion decays and massive boson decays.

This is <u>independent</u> of the mass scale of symmetry breaking.

Note: if this question had been posed and answered, say, in 1971, the subsequent discovery of c, τ, b, t, W, Z, h and their rich spectrum of decays would have been a verification of those generic predictions.

In that sense, these are useful, robust, generic predictions which reflect some perhaps "more fundamental qualities" of the Standard Model.

We will

a) show that these are also quite generic features of 4d solutions of string/M theory.

b) try to extract similar, additional generic predictions from 4d compactified string/M theory.

2. Superstring and M theories in Ten and Eleven Dimensions.

Spacetime, Low Energy Physics

I will assume that the students have studied the basic quantisation of world sheet superstrings in flat spacetime, $(\mathbb{R}^{9,1}, \underset{\substack{\uparrow \\ \text{Minkowski} \\ \text{metric.}}}{\eta})$

Spacetime $\overset{\substack{9+1 \\ \shortparallel}}{\text{Dimension}}$ fixed by superconformal invariance.

Spectrum in $D = 9+1$ consists of a finite number of zero mass particles with spins ≤ 2 and infinitely many massive modes with masses $\gtrsim m_{st}$ and high spins.

"Most" of the massive modes are unstable and can decay into zero modes?

So, we focus on zero modes.

These include a spin 2 particle which is identified as a graviton, (so this is a theory of gravity).

In fact, in flat spacetime, the effective Lagrangian density is that of a supergravity theory.

There are five superstring theory in 9+1 D.

Fields

Type IIA	g, B, ϕ, $\underbrace{C^1, C^3}_{RR}$, $\underbrace{\psi^\alpha_\mu, \psi^{\dot\alpha}_\mu}_{\text{gravitino}}$, $\underbrace{\lambda^\alpha, \lambda^{\dot\alpha}}_{\text{dilatino}}$ metric, 2-form B-field, dilaton
Type IIB	g, B, ϕ, C^2, C^4_+, $\underline{\psi^\alpha_\mu}$, $\underline{\tilde\psi^\alpha_\mu}$, $\underline{\lambda^\alpha}$, $\underline{\tilde\lambda^\alpha}$
Heterotic $E_8 \times E_8$	g, B, ϕ, $\underline{A}^a_\mu$ $E_8\times E_8$ gauge, ψ^α_μ, λ^a λ
Heterotic $\frac{Spin(32)}{\mathbb{Z}_2}$ $\updownarrow$	g B ϕ A^a_μ $SO(32)$ gauge "
Type I	g B ϕ " "

At energies low compared to M_{st}, each is described by an effective SUPERGRAVITY theory in 9+1 dimensions.

Solutions of the SUPERGRAVITY theory are solutions of the corresponding string theory at $g_s \to 0$ and $\alpha' \to 0$ $(M_{st} \to \infty)$

M theory ($g_s \to \infty$)

- M theory can be defined as the $g_s \to \infty$ limit of Type IIA string theory.
- M theory is defined in 10+1 dimensions, the length of the 11th dimension growing as $R \sim g_s^{2/3}$
- Low energy description (g, C^3, Ψ_μ) $d=11$ supergravity

Eleven Dimensional Supergravity

(Cremmer, Julia, Scherk; 1978).

$$\mathcal{L} \sim M_{11}^{9} \Big(\sqrt{-g}R - \frac{1}{2} dC^3 \wedge * dC^3 - \frac{1}{6} C^3 \wedge dC^3 \wedge dC^3 - \overline{\Psi}_M \Gamma^{MNL} D_L \Psi_N + \dots \Big)$$

$\nearrow$ 11d Planck Length

SUSY: $\delta_\eta \Psi_M = \underline{D_M \eta} + \text{"} \cancel{G}_M \cdot \eta \text{"}$ $\leftarrow *$

$$\delta_\eta C_{IJK} = -\frac{\sqrt{2}}{8} \overline{\eta} \Gamma_{[IJ} \Psi_{K]}$$

$$\delta_\eta e_I{}^m = \frac{1}{2} \overline{\eta} \Gamma^m \Psi_I$$

$G = dC$

$\cancel{G}_I = \frac{\sqrt{2}}{288} \left(\Gamma_I{}^{JKLM} - 8 \delta_I^J \Gamma^{KLM} \right) \eta \, \partial_{[J} C_{KLM]}$ $\leftarrow$

Euler-Lagrange equations,
schematically, with $\langle \tilde{\psi}_I \rangle = 0$,

$$R_{MN} - \frac{1}{2} g_{MN} R = T_{MN}(C) \quad \leftarrow$$

$$\left(T_{MN} \sim G_M{}^{\cdots} G_{N\ldots} - g_{MN} G^2 \right)$$

$$d * G = \frac{1}{2} G \wedge G \quad . \text{ 8-form}$$

Bianchi $dG = 0$.

Branes also appear

$E_8 \times E_8$ Heterotic Supergravity in 10 dimensions

$(g, B, \phi, A_\mu^{a \in \mathfrak{L}(E_8 \times E_8)}, \lambda^a, \psi_\mu)$ (Chapline Manton 83)

weyl-gravitino

$$\mathcal{L} \sim \frac{M_{st}^8}{e^{2\phi}}\left(\underline{\sqrt{-g}}R - \frac{1}{M_{st}^2}\left(d\phi \wedge * d\phi + H \wedge * H + \mathrm{tr} F^a \wedge * F^a\right)\right) + \dots$$

Bianchi $\boxed{dH = \mathrm{tr} F \wedge F - \mathrm{tr} \hat{R} \wedge \hat{R}}$

$\hat{R}$: curvature 2-form $\hat{R} = d\omega + \omega \wedge \omega$
(Riemann)

so: $H = dB + \underline{\omega^3_{CS}(A)} - \underline{\omega^3_{CS}}$ (spin connection)

Equations of motion, heterotic
($\psi_I = \lambda^a = 0$)

$$R_{MN} - \frac{1}{2} g_{MN} R = T_{MN}(A, B, \phi) \leftarrow$$

$$d * H = 0$$

$$d * d\phi = (H \wedge * H + d\phi \wedge * d\phi + F \wedge * F) + \dots$$

Susy:

$$\delta \psi_M = D_M \eta + \frac{\sqrt{2}}{32} e^{2\phi} (\gamma_M \cancel{H} - 12 H_M) \cdot \eta \leftarrow$$

$$\delta \lambda = \sqrt{2}\, \cancel{\phi}\, \eta + \frac{e^{2\phi}}{8} \cancel{H} \cdot \eta$$

$$\delta \lambda^a = -\frac{1}{4} e^{\phi} \cancel{F} \cdot \eta \leftarrow$$

Our starting point will be solutions of these 10d and 11d supergravities with six or seven (extra) compact dimensions.

So $M^{9,1} = Z_6 \times \mathbb{R}^{3,1}$

↑ small, compact ↓

$M^{10,1} = X_6 \times \mathbb{R}^{3,1}$

3. - Supersymmetric Compactifications to Four Dimensions

- Begin with a manifold,

$$M^{9,1} = Z_6 \times \mathbb{R}^{3,1}$$

or

$$M^{10,1} = X_7 \times \mathbb{R}^{3,1}$$

- We need to solve the equations of motion and Bianchi ident, on this manifold.

Natural to consider a product metric on $M^{9,1} = Z_6 \times \mathbb{R}^{3,1}$:

$$\longrightarrow ds^2 = g(Z) + \eta_{3,1}$$

$$= g_{ij}(y)\, dy^i dy^j + \eta_{\mu\nu}\, dx^\mu dx^\nu$$

Minkowski metric

y^i - coordinates on Z_6 $\left(\begin{array}{c} \text{compact} \\ \partial Z = \phi \end{array} \right)$

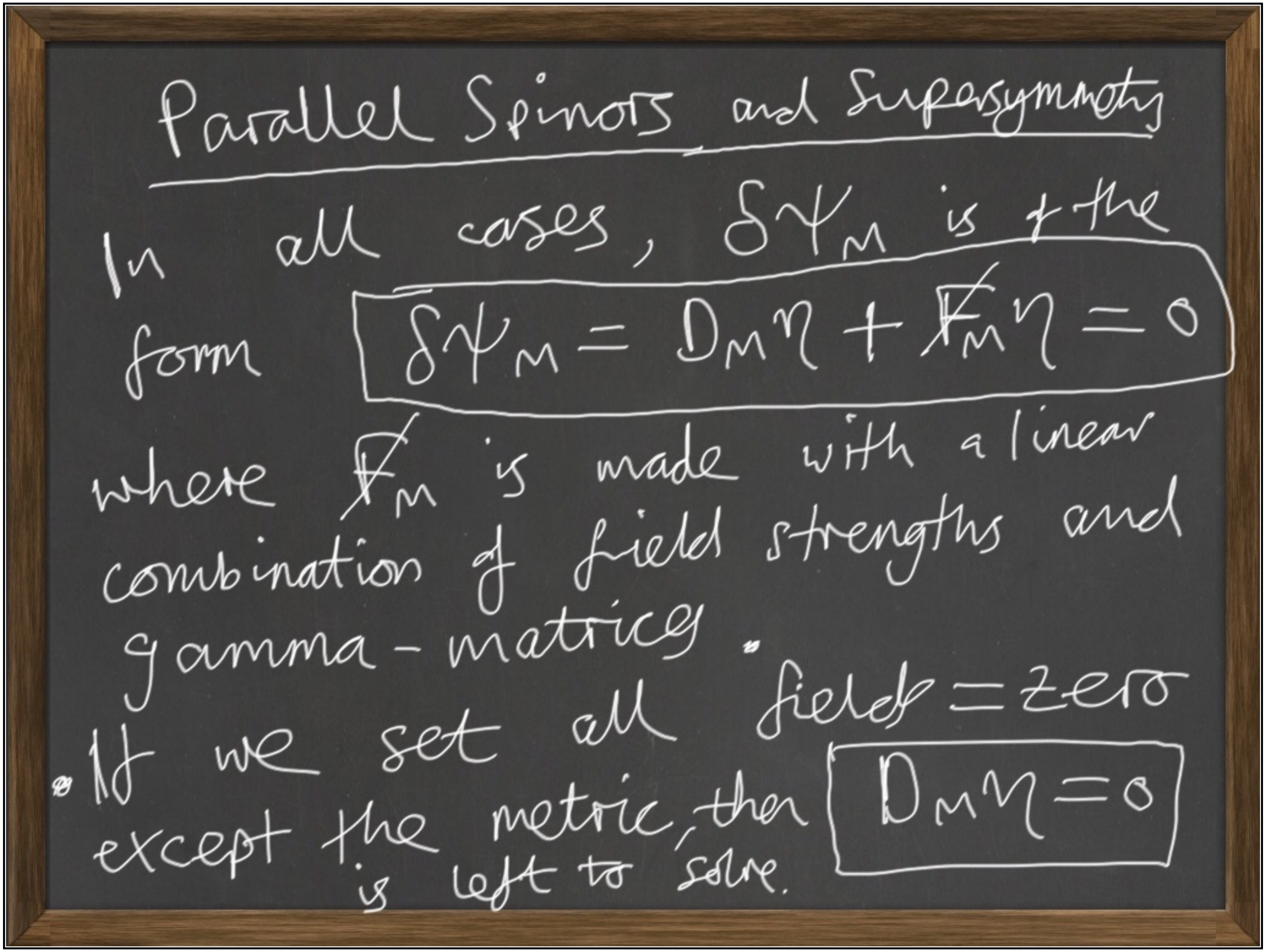
$$\frac{1}{16\pi G_{10}} \int_{Z_6} \sqrt{-g}\, R = \underbrace{\boxed{\frac{\mathrm{Vol}(Z_6, g)}{16\pi G_{10}}}}_{\boxed{\frac{1}{16\pi G_N}}} \int \left(\sqrt{-\eta}\, R_4 + \cdots + \underline{\sqrt{-g_6}\, R_6} + \cdots \right)$$

$+ \cdots \quad \cdots$

Further, since $d=10$ or 11 theories have spacetime supersymmetry, natural to consider solutions preserving (some) supersymmetry.

So
$$\delta \Psi_M = 0$$
$$\delta \lambda^a = 0$$

- This is also motivated by low energy supersymmetry as a solution to the hierarchy problem (why the SM scale is stable and small).



Parallel Spinors and Supersymmetry

In all cases, $\delta\psi_M$ is of the form $\boxed{\delta\psi_M = D_M\eta + \not{F}_M\eta = 0}$

where $\not{F}_M$ is made with a linear combination of field strengths and gamma-matrices.

- If we set all fields = zero except the metric, then $\boxed{D_M\eta = 0}$ is left to solve.

So we need to find a manifold Z_6 or X_7, with a metric g, which admits a spinor η which is covariantly constant wrt the Levi-Civita spin connection

$$D_M \eta = 0$$

These spinors are called parallel, since they are invariant under parallel transport.

Parallel fields and Holonomy

Existence of a parallel field strongly restricts the metric.

In fact, a generic (Riemannian) metric admits NO parallel fields except g_{MN} and $\epsilon_{M^1 \ldots M^N}$ (if oriented).

- **Holonomy**

(M^n, g) Riemannian mfld, (oriented)

consider all loops based at a pt $\in M$.

Then parallel transport around each loop generates an element of $SO(n)$. The set of such elements is the

- Holonomy group of (M, g).

$= \{$ set of all parallel transports $\}$.

Generically $\text{Hol}(g) \cong SO(n)$.

Our interest is in metrics with parallel spinors.

Berger (50's) classified possible
Holonomy groups of Riemannian metrics.
List of possibilities.

Assume M is irreducible and
g nonsymmetric ie $\nabla_M R_{IJKL} \neq 0$.

dims	Hol(g)	oriented irreducible non-symmetric
N	$SO(N)$	Riemannian
$N=2K$	$U(K)$	Kähler
$N=2K$	$SU(K)$	Calabi-Yau
$N=4K$	$Sp(K)\cdot Sp(1)$	Quaternion Kähler
$N=4K$	$Sp(K)$	HyperKähler
7	G_2	Exceptional Holonomy
8	$Spin(7)$	

Note:

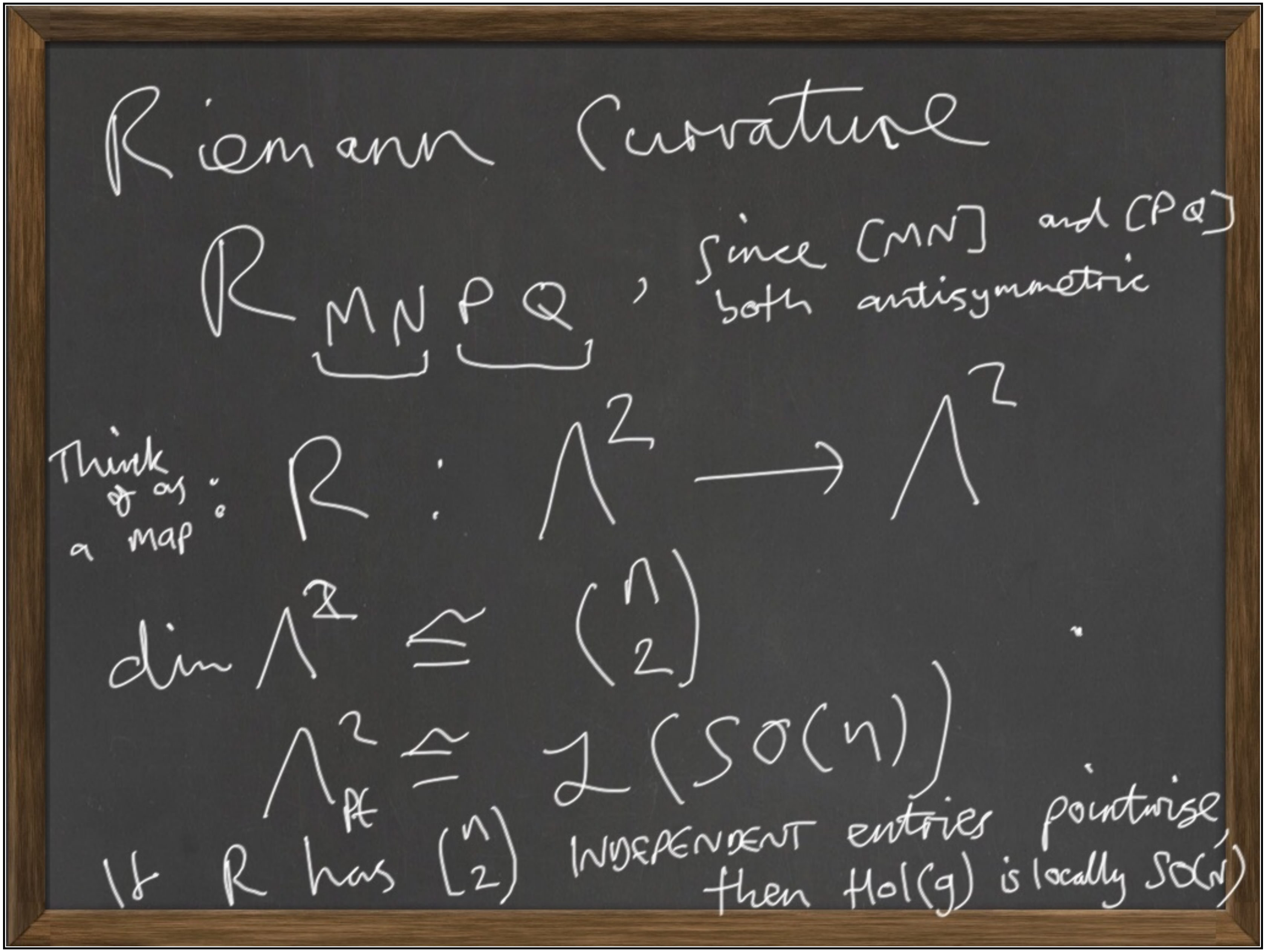
- $SU(k) \subset U(k)$

  $Sp(k) \cdot Sp(1) \not\subset U(2k)$

- $Sp(k) \subset SU(2k)$

- Examples can be constructed for all entries in the table.
- All $\mathrm{Hol}(g) \neq SO(N)$ in dim N manifolds, admit parallel fields in addition to g and $dVol$.

  $'' \epsilon_{\mu_1 \ldots \mu_N}$
- This can be understood group theoretically.



Riemann Curvature

$$R_{\underbrace{MN}\underbrace{PQ}} ,$$ since [MN] and [PQ] both antisymmetric

Think of as a map: $R : \Lambda^2 \longrightarrow \Lambda^2$

$$\dim \Lambda^2 \cong \binom{n}{2}$$

$$\Lambda^2_{pt} \cong \mathcal{L}(SO(n))$$

If R has $\binom{n}{2}$ INDEPENDENT entries pointwise, then $Hol(g)$ is locally $SO(n)$

e.g. Kähler geometry, $\mathrm{Hol}(g) = U(k)$

Consider $\Lambda^*(M^{2k}) \equiv$ Set of all p-forms for all p

At a pt $\in M$, $T_{pt}M^{2k} \equiv \mathbb{R}^{2k}$

$\cong \mathbb{C}^k$

$$z_1 = x_1 + i x_2$$
$$z_2 = x_3 + i x_4$$
$$\vdots$$

$$U(k) \subset SO(2k)$$

$$\Lambda^2(\mathbb{R}^{2k}) \cong \mathcal{L}(SO(2k))$$

$$\cong \Lambda^{2,0}(\mathbb{C}^k) \oplus \Lambda^{0,2}(\mathbb{C}^k) \oplus \underbrace{\Lambda^{1,1}(\mathbb{C}^k)}$$

$$\mathrm{adj}(SO(2k))\Big| \cong \binom{k}{2} + \overline{\binom{k}{2}} + \underbrace{k \otimes \overline{k}}_{\mathrm{adj}(\mathcal{L}(U(k)))}$$

$$K \otimes \overline{K} = \underbrace{1} + \underbrace{(K^2 - 1)}$$

$$\mathfrak{L}(U(K)) \cong \mathfrak{L}(U(1)) \oplus \mathfrak{L}(SU(K))$$

$\Lambda^2(\mathbb{R}^{2K})$ contains a <u>singlet</u> under $U(K)$ transformations.

$\Rightarrow$ Every Kähler manifold has a "singlet" 2-form, ω_g.

$$\iff \quad \underline{\underline{D_\mu}}\, \omega_g = 0$$

↑

connection of g.

Calabi-Yau $SU(k)$ holonomy

This is a subset of Kähler

additionally

$\Lambda^k(\mathbb{R}^{2k})$ contains two singlets ...

$$\cong \underbrace{\Lambda^{k,0}} + \Lambda^{k-1,1} + \dots \Lambda^{1,k-1} + \Lambda^{0,k}$$

$$\epsilon_{i_1 \dots i_k} \cong \text{Holomorphic volume form}$$

$$\approx dz_1 \wedge dz_2 \wedge \dots dz_k$$

Parallel Spinors

$$\mathrm{Spin}(N) \longrightarrow \mathrm{Hol}(M^N, g)$$

$$\left.\begin{array}{lll}
\mathrm{Spin}(2K) & \longrightarrow & \underset{2}{SU(K)} \\
\mathrm{Spin}(4K) & \longrightarrow & \underset{K+1}{Sp(K)} \\
\mathrm{Spin}(7) & \longrightarrow & G_2 \quad 1 \\
\mathrm{Spin}(8) & \longrightarrow & \underset{1}{\mathrm{Spin}(7)}
\end{array}\right\} \text{PARALLEL SPINORS}$$

eg

$$\mathrm{Spin}(8) \longrightarrow SU(4)$$

$$\begin{array}{c} 8_s \\ 8_c \end{array} \longrightarrow \begin{array}{c} 1+1+\underline{6} \\ 4+\overline{4} \end{array}$$

$$\mathrm{Spin}(6) \longrightarrow SU(3)$$

$$\begin{array}{c} 4 \\ \overline{4} \end{array} \longrightarrow \begin{array}{c} 1+\underline{3} \\ 1+\overline{3} \end{array}$$

$$\mathrm{Spin}(7) \longrightarrow G_2$$

$$8 \longrightarrow 1+7$$

Theorem:

A manifold with $Hol(g)$ $= Sp(k), SU(k), G_2$ or $Spin(7)$ admits parallel spinors and therefore provides a solution of the supergravity eq^{n}s, which is supersymmetric.

Theorem: Let (M,g) be an orientable, spin Riemannian manifold of dimension n and N the # of linearly independent parallel spinors on M. If n is even, $N_\pm$ = dim space of parallel spinors of +ve or −ve chirality.

If $N > 0$ and $\pi_1(M) = 0$, **exactly one** of the following is true:

1) $n = 4m$ and $\mathrm{Hol}(g) = SU(2m)$, $N_+ = 2, N_- = 0$
2) $n = 4m$ and $\mathrm{Hol}(g) = Sp(m)$, $N_+ = m+1, N_- = 0$
3) $n = 4m+2$ and $\mathrm{Hol}(g) = SU(2m+1)$, $N_+ = 1, N_- = 1$
4) $n = 7$ and $\mathrm{Hol}(g) = G_2$, $N = 1$
5) $n = 8$ and $\mathrm{Hol}(g) = Spin(7)$, $N_+ = 1, N_- = 0$

Calabi-Yau Geometry, SU(K)

Let (M^{2K}, g) be a Calabi-Yau manifold

i.e. $\mathrm{Hol}(g) = SU(k)$.

Then at each $T_{Pt}M^{2K} \cong \mathbb{C}^K$ we have local coordinates $(z_1, z_2 \ldots z_K)$ and an isomorphism

Kähler: $\omega(Pt) \cong -\frac{i}{2}\sum_i dz_i \wedge d\bar{z}_i = dx_1 \wedge dx_2 + dx_3 \wedge dx_4 + dx_{2K-1} \wedge dx_K$

Hol. volume: $\Omega^{K,0}(Pt) \cong dz_1 \wedge dz_2 \wedge \ldots dz_K$

Calabi-Yau $\longleftrightarrow$ Parallel (K,0)-form Ω and (1,1)-form ω

4. Ricci Flat Manifolds and Special Holonomy

- Riemannian manifolds with parallel spinors have zero Ricci tensor.
- Supersymmetry ⟶ equations of motion

$$D_M \eta = 0$$

$$\Rightarrow [D_M, D_N]\eta = \frac{1}{4} R_{MNPQ}\Gamma^{PQ}\eta = 0$$

Now, contract with Γ^N and use Γ-algebra + Bianchi to show

$$R_{MN} = 0$$

Note: $R_{MNPQ}\Gamma^{PQ}\eta = 0$ ⊛

says that the action of R_{MNPQ} on η is zero.

Since $R_{MNPQ}\Gamma^{PQ}$ takes values in $\mathcal{L}(SO(N))$, by exponentiating "$e^{R_{MNPQ}\Gamma^{PQ}}$", we get elements of Spin(N) for each 2-plane tangent to M and ⊛ becomes exactly $\boxed{g\cdot\eta=\eta}$ i.e. a singlet of the Holonomy group.

The Structure of Ricci Flat Manifolds

* Splitting Theorem for Ricci flat Manifolds

(Cheeger-Gromoll, Fischer-Wolf, S. Bochner, E. Hopf]

Suppose (M, g) is a compact, Ricci flat Riemannian manifold. Then M is locally isometric to $X \times T^m$, where $\pi_1(X) = 0$ and $g = g_X + g_{T^m}$ with g_X Ricci flat and g_{T^m} flat.

Globally $M = \dfrac{X \times T^m}{\Gamma}$ with Γ finite.

To give an idea why this is true, (Bochner, '48)
consider a harmonic 1-form α.

Then, $\Delta \alpha \equiv (d d^* + d^* d)\alpha = 0$

In components, $\Delta \alpha_i = - g^{jk} D_j D_k \alpha_i + R_i{}^j \alpha_j$.

Now, consider $\int_M g^{jk} D_k D_j (\alpha^i \alpha_i) = 0$ as is total derivative.

$$\therefore \int g^{jk} D_k (\alpha_i D_j \alpha^i + \alpha^i D_j \alpha_i) = 0$$

$$= 2 \int_M (\alpha^i g^{jk} D_k D_j \alpha_i + D_j \alpha_i D^j \alpha^i) = 0$$

$$\therefore \int \alpha^i \Delta \alpha_i = \int - \alpha^i g^{jk} D_k D_j \alpha_i + \alpha^i R_i^j \alpha_j$$

$$= \int (D_j \alpha_i D^j \alpha^i + R_{ij} \alpha^i \alpha^j] = 0$$

Hence if $R_{ij} = 0$

$$D_i \alpha_j = 0$$

ie α_j is parallel.

But if X is simply connected, there are no parallel 1-forms $\equiv$ parallel vector fields.

Hence the parallel vector fields correspond to the circle factors in

$$M \equiv \frac{X \times (S^1)^k}{\Gamma} .$$

In fact if $\Pi_1(M)$ finite, so there are no torus factors, the only thus far known Ricci flat manifolds have special holonomy (i.e. $SU(m)$, $Sp(m)$, G_2 or $Spin(7)$).

I conjecture that all Ricci flat simply connected manifolds are special holonomy.

i.e. Ricci flat $\Longleftrightarrow$ supersymmetry

(paper to appear soon)

5. Low energy spectrum calculations

5a. Heterotic on Calabi-Yau

5b. M-theory on G_2

5a. Heterotic on Calabi-Yau

$M^{9,1} = Z_6 \times \mathbb{R}^{3,1}$ $g = g_Z + \text{flat}.$

$\mathrm{Hol}(g_Z) \cong SU(3).$ $\mathrm{Ricci}(g_Z) = 0$

$\delta \Psi_M = 0$ ✓ solved by g_Z.

Need also: $\delta \lambda^a = e^{-2\phi} F^a_{ij} \Gamma^{ij} \eta = 0$

and $H = dB + \omega_{cs}(A) - \omega_{cs}(\text{spin connectn})$

$= 0$

Note: $F^a_{ij}\Gamma^{ij}\eta = 0$ is

v. similar to $R_{ijkl}\Gamma^{kl}\eta = 0$

Since $\mathrm{Hol}(g_Z) = SU(3) \subset SO(6)$

$$\Lambda^2(\mathbb{R}^6) \cong \Lambda^{2,0}(\mathbb{C}^3) + \Lambda^{0,2}(\mathbb{C}^3) + \mathcal{L}(U(1)) + \mathcal{L}(SU(3))$$

or $15 = \bar{3} + 3 + 1 + 8$

The condition that $F_{ij}\Gamma^{ij}\eta = 0$ is equivalent to saying that

$$F^a_{ij} \subset \mathcal{L}(SU(3)) \subset \Lambda^2$$

Geometrically

$$F \cdot \eta = 0$$

$$\Leftrightarrow \quad F^{0,2} = F^{2,0} = 0 \qquad \text{F-term}$$

$$F_{m\bar{n}}\,\omega^{m\bar{n}} = 0 \qquad \text{D-term}$$

Hermitian Yang-Mills equations (HYM)

(Donaldson, Uhlenbeck-Yau):

HYM connection $\Longleftrightarrow$ Stable hol. bundle

- Bianchi Identity.
- $dH = \mathrm{tr}\, F \wedge F - \mathrm{tr}\, \hat{R} \wedge \hat{R}$

In our case, require, $H = 0$ identically

$$H = dB + \omega_{cs}(A) - \omega_{cs}\left(\begin{smallmatrix}\text{spin}\\\text{conn}\end{smallmatrix}\right)$$

set $dB = 0$ so $\omega_{cs}(A) = \omega_{cs}\left(\begin{smallmatrix}\text{spn}\\\text{conn}\end{smallmatrix}\right)$

So choose $A =$ spin connection

Note: if $A = \omega^{spin}$

$$F = d\omega^{spin} + \omega^{spin} \wedge \omega^{spin}$$

$$= \hat{R}$$ curvature 2-form

$$F_{MN} = R_{MNPQ} e^P \wedge e^Q$$

$R_{MNPQ} \Gamma^{PQ} \eta = 0$, implies

ω^{spin} is HYM so

$$\delta \lambda^a = 0 .$$

Low energy physics

- Fix a background (A, g)
- Perturb background $(A+\delta A, g+\delta g)$

  $(0+\delta\phi, \underline{B}+\delta B)$

- Solve Eq^{n}s of motion to first order in $\delta\Psi$

- Consider g_Z and $g_Z + \delta g$

$$R_{ij}(g_Z) = 0 \Rightarrow R_{ij}(\lambda^2 g_Z) = 0$$

At first order, get Lichnerowicz equation:

$$\Delta_L \delta g_{ij} \equiv \Delta \delta g_{ij} + 2 R_{\underset{\curvearrowright}{ik}jl} \delta g^{kl} = 0$$

Let R be diameter (Z)

Physics at $E \ll 1/R$ eg eigenmodes of $\Delta \sim \frac{1}{R^2}$

$$\delta g_{ij}(y,x)$$

$$\Rightarrow \boxed{\Delta_6 \delta g_{ij}(y,x) + 2 R_{ikjl}\, \delta g^{kl} = \Box_4 \delta g_{ij}}$$

So we restrict to zero modes ie $\Box_4 \delta g_{ij} = 0$

$\Delta_L \delta g_{ij} = 0$

to solve this we use "Holonomy".

$\delta g_{ij} \in S^2_0(\mathbb{R}^6)$ 20 dimensional space

$\underline{20}$ is an irrep of $SO(6)$, but not of $SU(3)$.

As an irrep of $SU(3)$

$\underline{20} \subset \Lambda^*$

$$SO(6) \supset SU(3)$$

$$\begin{array}{cccccc} \underline{20} & = & 6 & + \ \bar{6} & + & 8 \\ & & \cap & \cap & & \cap \\ & & \Lambda^{1,2} & \bar{\Lambda}^{2,1} & & \Lambda^{1,1} \end{array}$$

In fact

$$\boxed{\Delta_L \delta g_{ij} \Longleftrightarrow \Delta_p \delta h = 0}$$

$$\delta g_{ij} \begin{array}{l} \nearrow \Lambda^{1,1} \\ \searrow \Lambda^{2,1} + \Lambda^{1,2} \end{array}$$

$$\underline{\delta g}_{ij}\,,\ \underline{w}_{ij}\,,\ \Omega_{\underline{ijk}}\,,\ g_{ij}$$

See Candelas, Horowitz, Strominger Witten for details

Upshot:

Zero modes of δg_{ij} are in 1:1 correspondence with harmonic $[1,1]$ and $(2,1)$ forms.

So space of zero modes has dimension $h^{1,1}(Z) + 2h^{2,1}(Z)$

$h^{p,q}$ are Hodge numbers of Z.

So a metric of SU(3) holonomy has $h^{1,1} + 2h^{2,1}$ parameters which manifest themselves in the low energy Lagrangian as massless scalar fields, whose vev's correspond to changing the moduli (parameters) of the Calabi-Yau metric.

- Moduli Fields of string theory.

$$\begin{tikzcd}
g_{ij} \arrow[r] & h^{1,1} + 2h^{2,1} \arrow[d, bend right, leftrightarrow] & \text{Scalars} \\
B_{ij} \arrow[r] & h^{1,1} & \text{scalars} \\
\phi \arrow[r] & 1 & \text{scalar}
\end{tikzcd}$$

$$\underbrace{2h^{1,1} + 2h^{2,1}}_{\text{Calabi-Yau Moduli space}} + 1$$

Kähler

Complex Structures

Equivalent Formulation:

Kähler Moduli: $\omega_{g_z} + \underbrace{\delta\alpha_2}_{\text{small harmonic 2-form}} \equiv \omega_{g_z+\delta g}$

$\omega_{g+\delta g}$ is a <u>new</u> Kähler form.

ie $D_{g+\delta g}\,\omega_{g+\delta g} = 0$

Complex Moduli: $\Omega^{3,0}_{g_z} \to \Omega^{3,0}_{g_z} + \underbrace{\delta\beta_3}_{\text{small harmonic 3-form}}$

$\Omega^{3,0}_{g_z+\delta g_z}$ is also parallel in the new metric.

- Low energy theory is an $N=1$ Supergravity in $d=3+1$
- Typically scalars are complex in such theories
- We get $h^{1,1}$ complex scalars and $h^{2,1}$ " "
- $h^{1,1}$ moduli $T^{\alpha=1,\dots h^{1,1}} = \int_{\Sigma^2_\alpha}(\omega+iB)$
- $h^{2,1}$ $U^{\gamma=1,\dots,h^{2,1}} = \int_{\Sigma^3_\gamma}\Omega$

$B_{\mu\nu}$ in $d=3+1$

$$*_{3+1}\, dB = da$$

a is called universal axion

$$\delta B = d\lambda \longrightarrow a \to a + c.$$

$e^{-2\phi} + ia$ give another scalar.

In total $(h^{1,1} + h^{3,1} + 1)$ complex scalars.

$E_8 \times E_8$ symmetry in $d = 9+1$ and A^a_μ and λ^a are in adjoint of $E_8 \times E_8$ $(248, 1) + (1, 248)$

- In vacuum $\langle A^a_i \rangle \neq 0$

From 3+1 perspective this is an adjoint scalar vev ⇒ Higgs gauge group.

- Unbroken gauge symmetry is the subgroup of $\mathcal{L}(E_8 \times E_8)$, commuting with $\langle A_i^a \rangle$.

- $\langle A_i^a \rangle$ = spin connection of g_Z

W_Z^{spin} is an SU(3) gauge field

requires choosing an SU(3) subgroup of $E_8 \times E_8$

Fact E_8 has a maximal compact subgroup which is $E_6 \times SU(3)$

We choose this $SU(3)$.

Then commutant of $SU(3)$ is

$$E_6 \times E_8 . = 3+1 \text{ d Gauge Symmetry}$$

$$\mathrm{Adj}(E_8) \longrightarrow E_6 \times SU(3) \checkmark$$

$$248 \longrightarrow (78, 1) + (1, 8)$$

$$+ \underset{\nwarrow}{(27, 3)} + (\overline{27}, \underset{\nearrow}{\overline{3}})$$

Decomposing λ^a under $E_6 \times SU(3)$ gives modes in the $(27, 3)$ rep^{n} of $E_6 \times SU(3)$.

$$E_6 \supset SO(10) \supset SU(5) \supset \underline{G_{SM}}$$

$$27 = 1 + 10 + 16 = 1 + \underline{5 + \bar{5}} + 1 + \underbrace{5 + \overline{10}}_{\text{exactly one } R_{\text{fermion}} \text{ in } G_{SM}.}$$

We want to calculate zero mods from λ^a field.

$\lambda^a \in S^+ \otimes \mathcal{L}(E_8 \times E_8)$

S^+ is +ve chirality spinor of $Spin(9,1)$

16 dimensional repⁿ.

$S^+ = Spin(6) \times Spin(3,1) = SU(3) \times Spi(3,$

(note $Spin(3,1) \cong SL(2,\mathbb{C})$)

$16 = (4,2) + (\bar{4},\bar{2}) = (1,2) + (1,\bar{2})$

$\underline{(3,2)} + \underline{(\bar{3},\bar{2})}$

λ^a contains modes in the

$3\otimes 3$, $\bar{3}\otimes 3$, $\bar{3}\otimes\bar{3}$, $3\otimes\bar{3}$.

$$4 \longrightarrow 1+3 \cong \Lambda^{3,0}+\Lambda^{1,0}$$

$$\bar{4} \longrightarrow 1+\bar{3} \cong \Lambda^{0,0}+\Lambda^{2,0}$$

All of these are (p, q) forms.

Eqn $\not{D}_{10}\lambda^a = 0$

$$\Rightarrow \not{D}_6\lambda^a = 0 \qquad D_6 = \not{\partial}_6 + i(\not{A})$$

$$3 = \Lambda^{1,0} = \Lambda^{0,2} \qquad \bar{3} = \Lambda^{0,1} = \Lambda^{2,0}$$

$$\therefore\ 3 \otimes \bar{3} = \Lambda^{1,1} = \bar{3} \otimes 3$$

$$3 \otimes 3 = \Lambda^{1,2} \qquad \bar{3} \otimes \bar{3} = \Lambda^{2,1}$$

$$\begin{tikzcd}
\text{So} & \underset{h^{1,2}}{\overset{L}{(27,3)\otimes 3}} \arrow[drr, leftrightarrow, "C"] & + & \underset{h^{1,1}}{\overset{R}{(27,3)\otimes \bar{3}}} \arrow[dll, leftrightarrow] \\
& \underset{h^{1,1}}{(\overline{27},\bar{3})\otimes 3} & + & \underset{h^{1,2}}{(\overline{27},\bar{3})\otimes \bar{3}}
\end{tikzcd}$$

$$\text{Since} \quad 27_L \underset{\text{by } C}{=} \overline{27_R} \left.\vphantom{\int}\right\} \begin{array}{l} (\overline{27},\bar{3})\otimes 3 \text{ zero} \\ \text{modes} = (27,3)\otimes\bar{3} \\ \text{zero modes} \end{array}$$

Dirac operator $\equiv$ Dolbeaut Operator ∂ or $\bar{\partial}$

Zero modes are <u>harmonic</u> (p, q) forms. (again), but now these are charged. [under 27 or $\overline{27}$ of unbroken E_6].

In general even dimension (compact) # of +ve chirality zero modes of $\not{D}$ $\neq$ # −ve chirality zero modes

This follows from Atiyah-Singer Index theorem, which has a corollary that net # of zero modes i.e. $\underline{+ve} - \underline{-ve}$ is a topological invariant. True also for Dirac operator twisted by a gauge field.

In our case, $+ve = h^{1,1}$

$-ve = h^{2,1}$

Hodge numbers =
of CY in 6d

$$\begin{array}{ccccccc}
 & & & h^{0,0} & & & \\
 & & h^{1,0} & & h^{0,1} & & \\
 & h^{2,0} & & h^{1,1} & & h^{0,2} & \\
h^{3,0} & & h^{2,1} & & h^{1,2} & & h^{0,3} \\
 & h^{3,1} & & h^{22} & & h^{1,3} & \\
 & & h^{3,2} & & h^{2,3} & & \\
 & & & h^{3,3} & & &
\end{array}
=
\begin{array}{ccccccc}
 & & & 1 & & & \\
 & & 0 & & 0 & & \\
 & 0 & & h^{1,1} & & 0 & \\
1 & & h^{2,1} & & h^{1,2} & & 1 \\
 & 0 & & h^{1,1} & & 0 & \\
 & & 0 & & 0 & & \\
 & & & 1 & & &
\end{array}$$

So only $h^{1,1}$, $h^{2,1} = h^{1,2}$ non-trivial.

Thm If $E = TZ$,

$$\mathrm{Ind}\,\not{D}_E = h^{1,1}(Z) - h^{2,1}(Z)$$

$$= \frac{1}{2}\chi(Z).$$

$=$ net # of 27's $-$ $\overline{27}$'s

for Standard Model want this to be $3 =$ # families.

Interactions

$$W = \int H \wedge \Omega$$

Superpotential
of Heterotic
string.

Contains interactions eg Yukawas.

Consider 27's ↔ $\Lambda^{1,1}$ forms on Z.

in 3+1d, can have a unique
$27 \cdot 27 \cdot 27$ E_6 invariant.

27 correspond to some $\alpha^{1,1}_{\alpha=1,\dots h^{1,1}}$

One Can write $\int_Z \alpha_\alpha \wedge \alpha_\beta \wedge \alpha_\gamma \equiv d_{\alpha\beta\gamma}$

is topol-gical invariant, $d_{\alpha\beta\gamma}$.

$\alpha^{1,1} \xleftrightarrow[\text{Dual}]{\text{Poincare}}$ 4 d surface $[\alpha] \subset Z$

$d_{\alpha\beta\gamma}$ counts the # of oriented intersections of $[\alpha_\alpha] \cap [\alpha_\beta] \cap [\alpha_\gamma]$.

$d_{\alpha\beta\gamma}$ are integers

$d_{\alpha\beta\gamma}$ is coeffient of Yukawa interactn of $27_\alpha 27_\beta 27_\gamma$.

$d_{\alpha\beta\gamma}$ is an integer i.e. gives an $O(1)$ Yukawa.

SM has $y_t \simeq \frac{1}{\sqrt{2}}$, as $\langle\phi\rangle = 246\,\text{GeV}$
$m_t = 172\,\text{GeV}$

For some choices of α, β, γ,
$d_{\alpha\beta\gamma} = 0$; for topological reasons.

Since the values of $d_{\gamma\beta\gamma}$ are either integers or zero we end up, at tree level with some order one Yukawas and some = zero.

Not bad, since SM has $y_{top} \sim 1$ and all other Yukawas $<< 1/40$.

- Perturbatively exact result, (as superpotential not renormalised) broken non-perturbatively by world-sheet instantons.

Holomorphic $f:\ \text{world sheet} \longrightarrow S^2 \cong \mathbb{P}^1 \subset Z.$

Gromov Witten Invariants

$$y :\simeq e^{-\int(\omega+iB)} = e^{-\mathrm{Vol}(\mathbb{P}^1)}\, e^{i\theta}$$

$\Longrightarrow$ Natural exponential hierarchy of Yukawas.

$$F = d_{\alpha\beta\gamma}(z^\alpha - \bar{z}^\alpha)(\quad)(\quad)$$

$$K = -\ln F$$

$$G_{\alpha\bar{\beta}} = \partial_\alpha \bar{\partial}_{\bar{\beta}} K$$

Thus, we have shown, that the generic 4d supersymmetric compactification of $E_8 \times E_8$ heterotic string theory has:

- Non-Abelian gauge symmetry
- Chiral Fermions
- Hierarchical Yukawa Couplings
- Many will also have spontaneous symmetry breaking

These are the four key properties of the Standard Model.

Part 5b and beyond will continue in second set of lecture notes.