String/M theory Compactification and Phenomenology

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Lecture Notes Part II

5b: Low Energy Physics of
M theory on G_2 manifold

In dimension 7, there are only two holonomy groups for a simply connected, compact, manifold, X_7. (orientable).

$$Hol(X_7, g) = \begin{array}{c} SO(7) \\ \text{or} \\ G_2 \end{array}$$

G_2 - holonomy

($\pi_1(X_7) = 0$
X_7 compact)

If (X_7, g_x) have $\mathrm{Hol}(g_x) = G_2$, then

1. • $\exists$ a parallel spinor, $D_{g_x}\eta = 0$
2. • $\mathrm{Ricci}(g_x) = 0$
3. • $\exists$ a parallel 3-form φ, G_2-invariant
4. • $\exists$ a parallel 4-form $*_{g_x}\varphi$, G_2-invariant
5. • $d\varphi = d*\varphi = 0$

Note: $\Lambda^3(\mathbb{R}^7) = 1 + 7 + 27 \cong \Lambda^4(\mathbb{R}^7)$

Fact: (1) $\equiv$ (3) $\equiv$ (4) $\equiv$ (5)

At any pt on X_7, φ can be written (up to $GL(7,\mathbb{R})$ rotations) as

$$\varphi = dx_{125} + dx_{145} + dx_{136} - dx_{246} + dx_{147} + dx_{237} - dx_{567}$$

(where $dx_{ijk} \equiv dx_i \wedge dx_j \wedge dx_k$)

So, $1 = \varphi_{125} = \varphi_{145} = \varphi_{136} = \varphi_{426} = \varphi_{147} = \varphi_{237} = \varphi_{657}$ and zero otherwise.

[φ_{ijk} can be regarded as structure constants of the Octonions, $\mathbb{O}$, so $T_{pt}X_7 \cong \mathrm{Im}\,\mathbb{O}$]

Metric moduli of G_2-hol mfld

Perturb $g_X \to g_X + \delta g$

- $\delta g \in S^2_0(TX)$ has 27 components
- Irreducible under $SO(7) \to G_2$
- $\Lambda^3(\mathbb{R}^7) = \langle \varphi \rangle + \mathbb{R}^7 + 27$

$\therefore \delta g \Longleftrightarrow$ 3-forms $\delta\varphi$.

$$\boxed{\delta g_{ij} = \delta\varphi_i{}^{kl}\,\varphi_{klj} + (j \leftrightarrow i)}$$

To first order in δg $\underline{Ric(g+\delta g) = 0} \Longleftrightarrow \delta\varphi \in \underline{H^3(X,\mathbb{R})}$

So moduli space of G_2 structures (at first order) $\cong H^3(X,\mathbb{R})$ ie harmonic 3-forms.

True to all orders. (see Joyce). OUP 2000

$\left(\dim H^p(X,\mathbb{R}) \equiv b_p\right.$ p-th Betti number of X is a topological invariant)

Particle Physics from G_2-manifolds

Consider M-theory on $X_7 \times \mathbb{R}^{3,1}$ with metric $g_{10,1} = g(X) + \eta(\mathbb{R}^{3,1})$ where $\mathrm{Hol}(g(X)) = G_2$ and η is flat.

If X is smooth (and large), we can use 11d supergravity to describe the low energy physics.

11d Sugra fields $(C_3, g_{11}, \Psi_{3/2})$

C_3: Ω^3 $\quad$ g_{11}: metric $\quad$ $\Psi_{3/2}$: $S(TM)$ gravitino

1. We fix a backround solution of the E-L equations of the form
$$(C_3, g_{11}, \Psi_{3/2}) = (0, g^0(X) + \eta(\mathbb{R}^{3,1}), 0)$$
2. We perturb
3. We extract the ~~zero~~ modes
4. We integrate over $\int_X \sqrt{g^0}\,\mathcal{L}^{11d}$
5. We get left with $\int d^4x\, \mathcal{L}^{4d}$

The zero modes:

- From C_3:
  - $b^2(X)$ $U(1)$ gauge fields $A_I \in \Omega^1(M^{3,1})$
  - $b^3(X)$ S^1-valued scalars $a_i \in \Omega^0(M^{3,1})$ ← axions

- From g_{11}:
  - 1 4d metric $g^{3+1}_{\mu\nu} \in S^2(M^{3,1})$
  - $b^3(X)$ scalars $s_i \in \Omega^0(M^{3,1})$ ← G_2-moduli

- From $\Psi_{3/2}$:
  - 1 4d gravitino $\psi^4_{3/2} \in S^+(TM)$
  - $b^2(X)$ spinors $\lambda_I \in S^+(M)$
  - $b^3(X)$ spinors $\chi_i \in S^+(M)$

(g^4 and $\psi^4_{3/2}$) give the $N=1$ supergravity multiplet

(A_I, λ_I) give $b^2(X)$ vector multiplets

the a_i and s_i's become $\mathbb{R}$ and $\mathbb{I}m$ parts of $b^3(X)$ complex scalars $\boxed{z_j = a_j + \sqrt{-1}\, s_j}$

(z_j, χ_j) give $b^3(X)$ "chiral" multiplets

The z_j's should be local coordinates on the complexified moduli space of G_2 manifolds.

$\mathcal{M}_{G_2}(X, \varphi, C_3)$

$$z_j \sim \int_{\Sigma_j} C_3 + i\,\Phi_3$$

G_2 form

are periods of a complexified G_2 3-form over a basis $\{\Sigma_j\}$ for $H_3(X)$.

The $\mathcal{L}^4$ that we get from this has a metric on the moduli space, which is a Kähler metric with potential:

$$\boxed{K(z_j, \bar{z}_j) = -3 \ln \int_X \Phi \wedge * \Phi}$$

- We would very much like to know the properties of this Kähler metric!
- Can we compute it approximately for the TCS G_2-manifolds?

- The components of the moduli space metric are homogeneous of degree minus two, so it looks like the metric has "−ve curvature" in some sense that would be good to make precise.

In general, $N=1$ $d=4$ supergravity theories also depend on a superpotential $W(z_i)$, $\partial_j W = 0$, locally holomorphic. Witten/Bagger: W is a section of a line bundle $L \to \mathcal{M}_{G_2}(X_7)$.

Because the $a_i = \mathrm{Re}\, z_i$ is periodic,

$$W = \text{zero}, \text{ up to instantons effects.}$$

– Instantons are associative submanifolds (suitably rigid)

There is also a third function, also holomorphic, called the gauge coupling function $f(z_i)$.

For the $b^2(U(1))$ gauge fields their contribution to $\mathcal{L}^4$ is calculated to be:

$$d^{jIK} \, \mathrm{Im}\, z_j \, F_I \wedge * F_K + d^{jIK} \, \mathrm{Re}\, z_j \, F_I \wedge F_K$$

where $d^{jIK} : H^3(X,\mathbb{Z}) \times H^2(X,\mathbb{Z}) \times H^2(X,\mathbb{Z}) \to \mathbb{Z}$

$$\int \alpha^j \wedge \beta^I \wedge \beta^K$$

So $f(Z) = \int_X (C_3 + i\varphi) \wedge \beta^I \wedge \beta^K$

$\{\beta^I\}$ basis for $H^2(X, \mathbb{Z})$

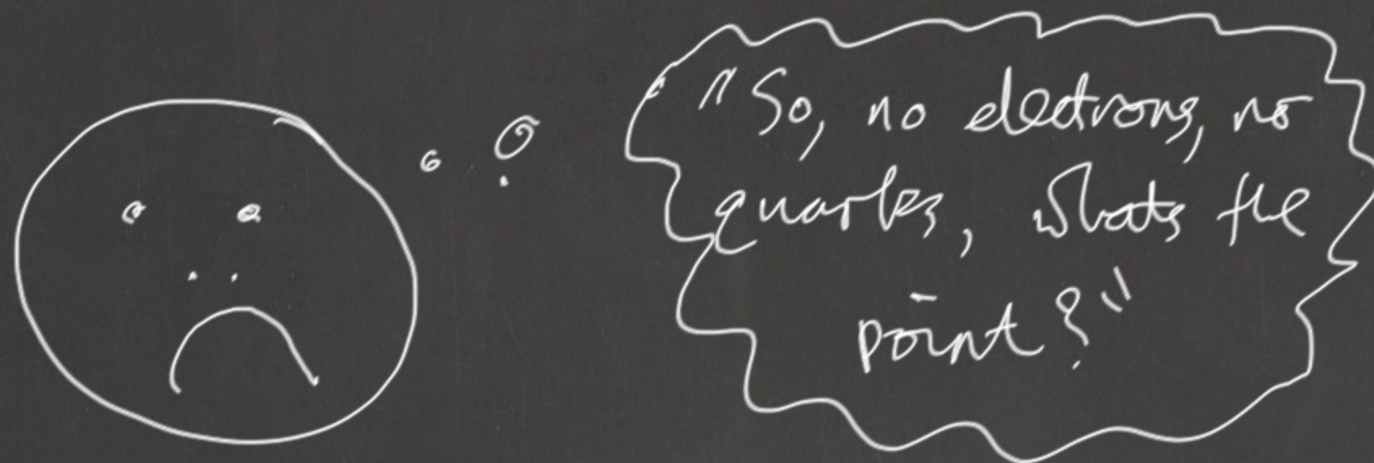
Can ~~wr~~ K

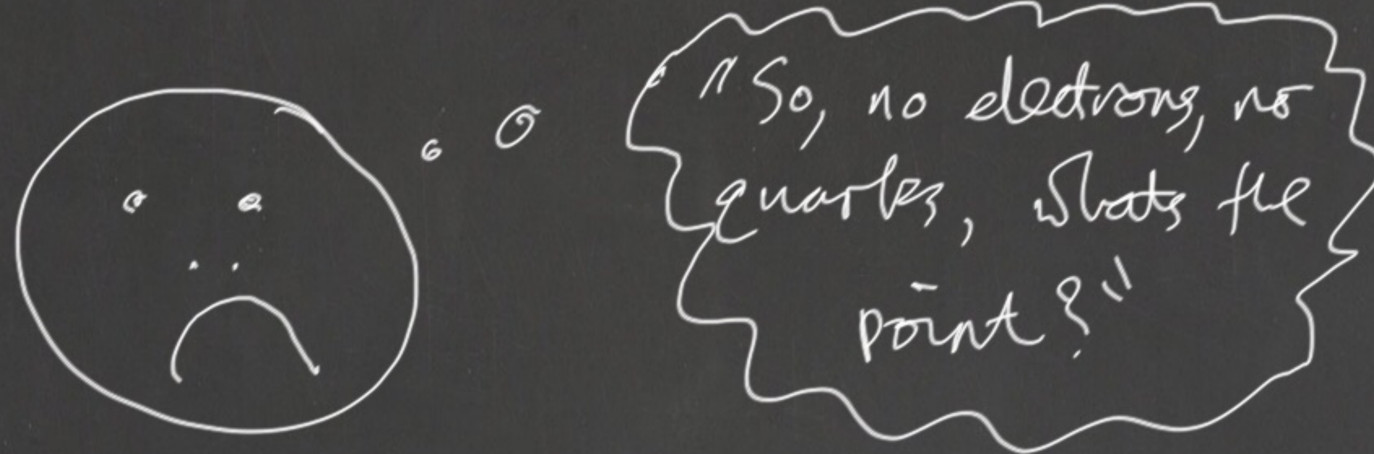
The low energy physics is Gravity + $b^2(X)$ $U(1)$'s and

- $b^3(X)$ neutral moduli multiplets

- There are no light charged particles!

- No non-Abelian gauge symmetries


"So, no electrons, no quarks, whats the point?"



We will try and show that particles like quarks and leptons and non-Abelian gauge bosons emerge from special kinds of <u>singularities</u> of G_2-manifolds.

Yang-Mills Fields from codim 4 singularities

Consider a special case when
$X = K3 \times \mathbb{R}^3$ w/ product metric
Then $M^{10,1} = X \times \mathbb{R}^{3,1} = K3 \times \mathbb{R}^{6,1}$, leading
to a 6+1 d Lagrangian.
In this case the moduli space = space
of Einstein metrics on $K3 = \mathbb{R}^+ \times \dfrac{SO(3,19)}{SO(3) \times SO(19)}$

There are also 22 $U(1)$ gauge fields

C_3 } $b^2(K3)$ $U(1)$ gauge fields

g_n } 58 scalars $= \left(\mathrm{Vol}(K3), \int_{\Sigma_\alpha} \omega_I \equiv \phi_{I\alpha}\right)$

"SD cycles"

$\psi_{3/2}$ } fermions which make everything supersymmetric

This looks v. similar to the theory obtained by considering $E_8 \times E_8$ Heterotic superstring theory on

$$M^{9,1} = T^3 \times \mathbb{R}^{6,1}.$$

Massless Bose fields in Het string on $T^3 \times \mathbb{R}^{6,1}$

10d	$\int_{T^3}$	6+1d massless fields
dilaton $\phi \in \Omega^0(M^{10})$	INTEGRATE	1 scalar $\in \mathbb{R}^+$
metric $g^{10} \in S^2 T M^{10}$	OVER	6 scalars in $SL(3,\mathbb{R})/SO(3)$
B-field $B \in \Omega^2(M)$		3 scalars in $H^2(T^3, U(1))$
$E_8\times E_8$ gauge field $A \in \Omega^1(\mathrm{Lie}(E_8\times E_8))$ $E \to M$ $E_8\times E_8$ v bundle	T^3	Flat $E_8\times E_8$ connections on T^3. The Identity connected component is 48-dim$^{\underline{\text{l}}}$ and is $\dfrac{(\hat{T}^3)^{16}}{W(E_8\times E_8)}$ $W(E_8\times E_8)$ is Weyl group

So the moduli space is 58-dim$^{\text{l}}$

$$58 = \underset{\underset{\phi}{\uparrow}}{1} + \underset{\underset{\frac{SL(3)}{SO(3)}}{\uparrow}}{6} + \underset{\underset{H^2(T^3)}{\uparrow}}{3} + \underset{\underset{\left(H_1(T^3)\right)^{rk(E_8\times E_8)}}{\uparrow}}{48}$$

What about gauge bosons?

$g_{\mu\nu} \longrightarrow U(1)^3$ from the 3 Killing vectors on T^3.

$B \longrightarrow U(1)^3$ from the 3 harmonic 1 forms on T^3.

$A \longrightarrow U(1)^{16}$ at generic points in space of flat $E_8 \times E_8$ connections (Id comp)

So $U(1)^{22}$, as in M-theory on K3.

- In fact, the Heterotic moduli space on T^3 is also, locally $\mathbb{R}^+ \times \dfrac{SO(3,19)}{SO(3) \times SO(19)}$

- String dualities assert that the heterotic string on T^3 is equivalent to M theory on K3.

- Non-Abelian gauge symmetry is present in Het string from the start

Non-Abelian symmetries in Het on T^3

The $U(1)^{16}$ gauge group is identified as the commutant of a generic flat connection on T^3 inside $E_8 \times E_8$.

But: at special codim 3 and 3n subspaces, the commutant enhances to non-Abelian groups

E.g. consider flat $SU(2)$ connections on T^3. These are globally

$$\mathcal{M}_{flat}(SU(2), T^3) = \mathrm{Hom}(\pi_1(T^3), SU(2)) = \frac{\hat{T}^3}{\mathbb{Z}_2}$$

At a general pt $\in \mathcal{M}$, the <u>commutant</u> of the flat connection is $T(SU(2)) = U(1)$

At the origin $(0,0,0)$, the commutant is the full $SU(2)$.

In general for $\mathcal{M}_{flat}(E_8 \times E_8; T^3)$, one requires choosing at least one $U(1) \subset T(E_8 \times E_8)$ and the 3 holonomies of this connection must vanish in order to ENHANCE the GAUGE SYMMETRY.

ie <u>codim 3</u> singularities

McKay Correspondence for M-theory on K3

$$H_2(K3,\mathbb{Z}) \cong \Gamma(E_8)\oplus\Gamma(E_8)\oplus \underbrace{3\sigma\begin{pmatrix}0&1\\1&0\end{pmatrix}}_{\substack{\text{related to}\\ (g,B,\varphi \text{ on } T^3)}}$$

sig $(19,3)$ lattice.

* According to Het/M duality, there is a codim^{n} 10 subspace of $\mathcal{M}_{\text{Einstein}}(K3)$ which one is tempted to identify with $\mathcal{M}_{\text{flat}}(E_8\times E_8, T^3)$. Coords $\varphi_{I\alpha}=\int_{\Sigma_\alpha}\omega_I \quad \Sigma_\alpha\in\Gamma(E_8)\oplus\Gamma(E_8)$

M - branes

$dC_3 = G_4$ In the absence of branes and $[G_4] = 0$ coh

$d G_4 = 0$

$d *_{11} G_4 = 0$

M-branes are sources of currents

M5: $d G_4 = q_5 \delta_5 (M^{5,1} \subset M^{10,1})$

M2: $d * G_4 = q_2 \delta_8 (M^{2,1} \subset M^{10,1})$

Near singularities of $\mathcal{M}^{(K3)}_{\text{EINSTEIN}}$,

The K3 becomes singular and has codim 4 orbifold singularities. We can model these on orbifold singularities of the form $\mathbb{R}^4/\Gamma_{ADE}$ where Γ_{ADE} is a finite subgroup of $SU(2) \subset SO(4)$ acting on $\mathbb{C}^2 \cong \mathbb{R}^4$ in the fundamental rep$^{\underline{n}}$.

The flat metric on $\mathbb{R}^4/\Gamma_{ADE}$ admits a desingularisation with a smooth hyperkähler metric

π : Smooth $\widetilde{\mathbb{R}^4/\Gamma_{ADE}} \longrightarrow$ Singular, flat $\mathbb{R}^4/\Gamma_{ADE}$

$\pi^{-1}(0)$ is a set of rk(A-D-E) S^2's which intersect according to the Dynkin diagram of A-D-E.

$H_2(\widetilde{\mathbb{R}^4/\Gamma_{ADE}}, \mathbb{Z})$ = Root lattice of ADE

Intersection form = $-$ Cartan matrix

$$A_\alpha = \int_{\Sigma_\alpha \subset H_2\left(\widetilde{\mathbb{R}^4/\Gamma_{ADE}}\right)} C_3$$

are a set of $rk(ADE)$ $U(1)$ gauge fields. The gauge group, $U(1)^{rk(ADE)}$ is naturally identified with $T(ADE \text{ group})$.

max torus

Moreover, since $d * G_4 = q_2 \delta_8(M^{2+1})$,

M2-branes wrapped on the $\Sigma_\alpha \subset H_2$ are like charged particles in $\mathbb{R}^{6,1}$:

$$d * G_4 = \tilde{\Sigma}_\alpha \wedge d *_{6+1} dA_\alpha = q_2 \delta_6(P+) \wedge [\Sigma_\alpha]$$

$\tilde{\Sigma}_\alpha$ = Poincaré dual of Σ_α.

Because $H_2\left(\mathbb{R}^4 / \Gamma_{ADE}\right)$ = root lattice ADE,

These M2-branes, plus the rk(ADE) "photons" have charges of the adjoint rep^n of ADE.

We also get $3 \times \mathrm{rk}\,(ADE)$ moduli fields from the moduli space of Einstein metrics.

$$\int_{\Sigma_\alpha} \omega_I \sim \phi_{I\alpha} \qquad I = 1,2,3$$

The wrapped M2-branes are "BPS" states whose masses are exactly given by the volumes of the exceptional S^2's

ie $\quad \mathrm{Mass}_\alpha = |\phi_{I\alpha}|$

$\therefore$ At the origin of moduli space, we have a copy of the adjoint rep^n of ADE which is MASSLESS.

- The whole system is also supersymmetric
- How do we describe this?

★ At the two derivative level, $\exists$ a unique supersymmetric $\mathcal{L}$agrangian theory in $\mathbb{R}^{6,1}$ with non-Abelian gauge symmetry: 6+1 d Super Yang-Mills. ★

∴ The physics of M-theory on

$\left(\dfrac{\mathbb{R}^4}{\Gamma_{ADE}} \times \mathbb{R}^{6,1}, \omega_I \right)$ is described

ω_I — H-K

by $(6+1)d$ Super-Yang-Mills theory with $G =$ A-D-E gauge symmetry:

$$\mathcal{L}^{6+1d} = -\frac{1}{4} \operatorname{tr} F_A * F_A + i\bar{\lambda} \not{D}_A \lambda + \sum_I D_A \Phi_I * D_A \Phi_I - \operatorname{tr} [\phi_I, \phi_J]^2$$

$$A \in \Omega^1(\mathcal{L}ie(ADE))$$

$$\Phi_I \in \Omega^0(\mathcal{L}ie(ADE) \otimes \mathbb{R}^3)$$

$$\lambda \in \underset{\substack{\uparrow \\ \text{Spin}}}{S}(\mathcal{L}ie(ADE) \otimes \mathbb{C}^2)$$

$\exists$ $SU(2)$ global symmetry ($SU(2)_R$)

The $SO(3) \subset SU(2)$ rotates the HK ω_I and the 3 ϕ_I's

The λ's take values in the fundamental of $SU(2)$

The vacua are described by the set of

$$\Phi_I \in \Omega^0(\mathcal{L}ie(ADE) \otimes \mathbb{R}^3)$$

with $[\phi_I, \Phi_J] = 0$

modulo gauge transforms.

ie a <u>local</u> description of a flat <u>ADE</u> connection on T^3.

[commuting triple]

This is exactly the moduli space of hyperkähler metric on $\widetilde{\mathbb{R}^4 / \Gamma_{ADE}}$. (Kronheimer)

To obtain an identity connected flat A-D-E connection on T^3, one simply exponentiates the 3 ϕ_I's

$$g_I = e^{i \Phi_I} \quad \left(e^{i \int_\Sigma \omega_I} \right)$$

Non-Abelian symmetry in M-theory on G_2-manifolds, X.

One can consider codim 4 singularities in X.

These are supported on 3d subspaces $Q \subset X$.

Nice local model: Bryant-Salamon metric on $S(S^3) \cong \mathbb{R}^4 \times S^3$

This $Hol(G_2)$ metric has $SU(2)^3$ symmetry.

There is an <u>associative</u>

$\{0\} \times S^3 \subset \mathbb{R}^4 \times S^3$.

One can take a quotient by Γ_{ADE}, $S(S^3)/\Gamma_{ADE}$, which fixes the associative S^3 ie

X modelled by $\frac{\mathbb{R}^4}{\Gamma_{ADE}} \times S^3$.

The volume of the associative S^3 is a free parameter.

In the large volume limit, the manifold gets flatter and flatter and looks more and more like $\mathbb{R}^4/\Gamma_{ADE} \times \mathbb{R}^3$ locally.

$$\mathrm{Vol}(S^3) \sim \frac{1}{g^2}$$

i.e. <u>weak</u> coupling limit in gauge theory!

charge or gauge coupling $\longrightarrow g$

So we can describe the physics of the singularity by taking the 6+1 d Super Yang-Mills on $\{0\} \times S^3$, and any suitably large associative Q supported at a codim 4 singularity.

Expanding in "harmonics",
keeping only zero modes and
integrating over Q, we find that:
low energy physics of a
codim 4 ADE singularity on Q
is given by
3+1 d Super Yang-Mills theory
plus $b_1(Q)$ "chiral" multiplets
$(\Omega^0(\mathrm{Lie}(ADE)), S^+(\mathrm{Lie}(ADE)))$

Actually, precise statement is chiral multiplets are the moduli space of complex flat $ADE_{\mathbb{C}}$ connections on Q.

Essentially, when we consider the 6+1 d Super Yang-Mills theory on $Q \times \mathbb{R}^{3,1} \subset X_7 \times \mathbb{R}^{3,1}$, the three scalars of the 6+1 d theory in flat spacetime, Φ_I, become components of a 1-form, Φ on Q.

$\Phi \in \Omega^1(Q, \mathrm{Lie}(ADE))$

The components of the gauge field A on Q then combine with Φ to give a complex ADE connection

$$\mathcal{A}_\delta = A + i\,\Phi$$

The "BPS" or supersymmetry conditions then assert that $F_{\mathcal{A}_\delta}$ is flat and $d_A{}^*\Phi = 0$

i.e., stable, complex, flat connections

One expects to find situations in which there is a coassociative fibration over Q, where the fibers are diffeomorphic to the $\widetilde{\mathbb{R}^4/\Gamma_{ADE}}$.
(cf S. Donaldson)
If we assume the generic fiber is smooth, then we have a $U(1)^{rk(ADE)}$ gauge theory on Q

There are rk(ADE) complex gauge fields A^α_μ which are essentially the integrals of the complexified 3-form $C_3 + i\,\varphi$ over the 2-cycles in $\mathbb{R}^4/\Gamma_{ADE}$

$$A^\alpha_\mu \sim \int_{\Sigma_\alpha} (C_3 + i\,\varphi)$$

Note that for the Cartan directions in Lie (ADE), the one form Φ_α is both closed and co-closed and is presumably related to the 1-form discussed by Donaldson recently in "Adiabatic Co-associative Fibrations".

Chiral Fermions from Codim 7 Singularities

- We have seen that a key ingredient of the SM of particles, ie non-Abelian gauge fields, reside at codim 4 orbifold singularities.
- The other key ingredient are chiral fermions: fermions whose left and right handed components transform in different complex representations of G_{SM}.
- These have to arise from higher codimension singularities.

- One expects this to be codim 7, because, lower codimension singularities will effect-ively give L and R spinors in the same rep$^{\underline{n}}$ of G_{SM}.

- We will motivate this study via duality with the heterotic string on T^3

M theory $\approx$ Het String $E_8 \times E_8$

$K3 \to X \to S^3$, $\mathrm{Hol}(G_2)$

Adiabatic fibration to give a duality between M theory on X and het string on Z.

$T^3 \to Z \to S^3$ (Calabi Yau)

T

Hol vector bundle required here

2012 Corti Haskins Nordstrom Pacini

T^3 fibred CY 3-folds arise in SYZ mirror symmetry

Chiral Fermions in Het String on Z

- Data for the background:

(Z, g) – Calabi-Yau ; $(E \to Z)$ $E_8 \times E_8$ hol vector bundle

A – a Hermitian Yang-Mills connection on E with $c_2(E) = c_2(TZ)$.

- Chiral fermions arise from zero modes of the Dirac operator on Z, coupled to the connection A.

$$\not{D}_A \psi = 0.$$

- We suppose Z is T^3-fibered over a large S^3, with small fibers, of length L.
- The Dirac equation on Z will have a contribution from $\not{D}_A \Psi\big|_{T^3}$ ie the Dirac operator on the fibers, coupled to the connection restricted to the fibers.
- We do not have to worry about singular fibers in what follows!

- The restriction of the HYM eq^ts to the fibers, require the connection A to be flat on each fiber.
- This is expected from our discussion of M on K3 vs flat on T^3.
- If $\tilde{A}$ is a generic flat connection on T^3, $\not{D}_{\tilde{A}}\psi$ has no zero modes, hence will have eigenvalues of order $1/L$

- But $1/L \gg$ <u>any</u> of the other terms in $\not{D}_A \psi$ on Z, hence we require zero modes from the $\not{D}_A \psi|_{T^3}$ contribution.
- Zero modes of $\not{D}_A$ on T^3 arise precisely when A has <u>non-Abelian</u> commutant in $E_8 \times E_8$.
- These naturally occur at <u>isolated</u> points on the base S^3.

So chiral fermions are localised at pts on S^3 above which the T^3 has a connection with "enhanced symmetry".

- In M theory, these are precisely the K3 fibers with ADE singularities.

- Lets describe the local models for these codim 7 singularities.

Ex 1: Bryant-Salamon Cone on $\mathbb{R}^+ \times \mathbb{CP}^3$

$$\widetilde{\mathbb{R}^4/\mathbb{Z}_2} \hookrightarrow \underset{\overset{\parallel}{\mathbb{R}^8/U(1)}}{\mathbb{R}^+ \times \mathbb{CP}^3} \longrightarrow \mathbb{R}^3$$

$\mathbb{R}^+ \times \mathbb{CP}^3$ is a 3d family of "Eguchi-Hanson" 4-mflds. At the origin of $\mathbb{R}^3$, the S^2 in $\widetilde{\mathbb{R}^4/\mathbb{Z}_2}$ collapses so the fiber over this point is singular, $\frac{\mathbb{R}^4}{\mathbb{Z}_2}$, and the total space has a conical singularity.

$U(1)$ + 1 fundamental

Conical, $dr^2 + r^2 g(\mathbb{CP}^3)$

$\mathbb{C}^4 \cong \underline{\mathbb{H}} + \underline{\mathbb{H}}$

2 - Hypers

$\mathbb{CP}^3$

$\widetilde{\mathbb{R}^4/\mathbb{Z}_2} \cong$ moduli space of $N=2$ susy gauge theory, $U(1)$ and 2 Hypers.

$U(1)$ $\quad F \sim$ quadratic hol polynom

$D \sim$ real quadmit

are moment maps of the $U(1)$ action on $\mathbb{H}+\mathbb{H}$

$\mathbb{C}^4$
$$\begin{cases} F \sim z_1 z_2 - z_3 z_4 \\ D_\xi \sim |z_1|^2 + |z_2|^2 - |z_3|^2 - |z_4|^2 - \xi \end{cases}$$

$$\widetilde{\mathbb{R}^4 / \mathbb{Z}_2} = \frac{\{F = D_\xi = 0\}}{U(1)}$$

Ex2 Conjectural $Hol(G_2)$ metric on

$$\mathbb{R}^+ \times \mathbb{W}\mathbb{CP}^3_{NN11} :$$

$$\mathbb{R}^4/\mathbb{Z}_{N+1} \longrightarrow \mathbb{R}^+ \times \mathbb{W}\mathbb{CP}^3_{NN,11} \longrightarrow \mathbb{R}^3$$

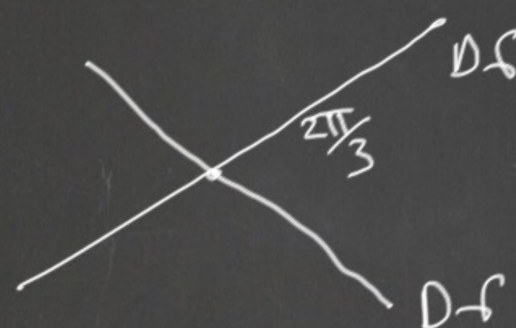
↑ fibers are a <u>partial</u> resolution of $\mathbb{R}^4/\mathbb{Z}_{N+1}$ along the "N-th node"

ie each fiber has an $\mathbb{R}^4/\mathbb{Z}_N$ orbifold singularity and a non-zero 2-sphere. At the origin in $\mathbb{R}^3$ the fiber is $\frac{\mathbb{R}^4}{\mathbb{Z}_{N+1}}$.

$$\frac{\mathbb{R}^7 \times \mathbb{CP}^3}{U(1)} \cong \mathbb{R}^6$$

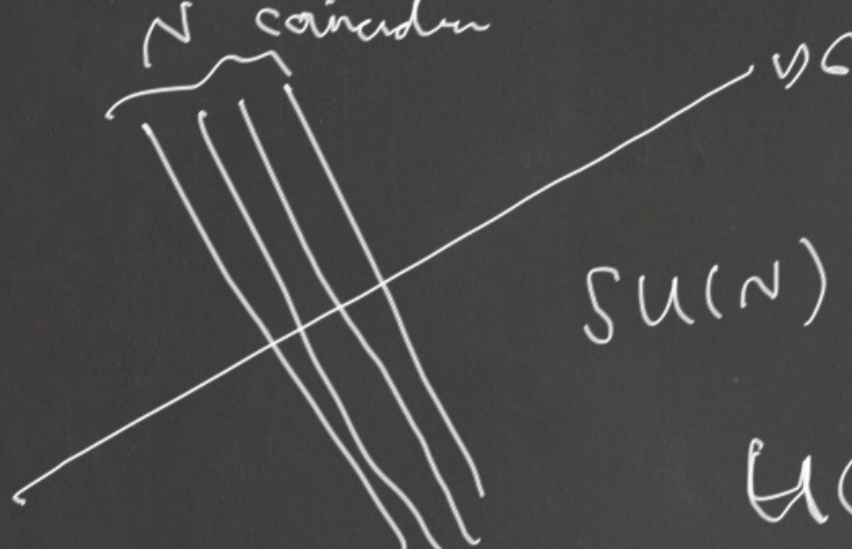
- Fixed pts of $U(1)$ are 2 $\mathbb{R}^3$'s which intersect at $\{0\} \in \mathbb{R}^6$
- $U(1) \cong S^1 =$ M theory circle

IIA picture is



$U(1)$ + 1 fundamental

$$\frac{\mathbb{R}^+ \times W\mathbb{CP}^2_{NN11}}{U(1)} = \mathbb{R}^6$$



$SU(N) + 1$ fundamental

$U(N)$?

In general, expect Hol(G_2) metrics on:

1) Start with any flat orbifold

$$\mathbb{R}^4/\Gamma_{ADE} \quad \text{with } rk(ADE) = k+1$$

2) The nodes on the "boundary" of the ADE Dynkin diagram correspond to 2-spheres in partial resolutions of $\mathbb{R}^4/\Gamma_{ADE}$. Each of these nodes breaks ADE to a $rk(k)$ subgroup

- Each S^2 has three associated moment maps $(\underline{\mu})$ in Kronheimer's construction
- So we get a 3d family of partially resolved ALE spaces

$$\widetilde{\mathbb{R}^4/\Gamma_{ADE}} \longrightarrow \left(\widetilde{\mathbb{R}^4/\Gamma_{ADE}}, \underline{\mu}\right) \longrightarrow \mathbb{R}^3$$

where the fibers each have an $\mathbb{R}^4/\Gamma_G$ orbifold singularity.

" At the origin, the fiber becomes

$$\frac{\mathbb{R}^4}{\Gamma_{ADE}} \quad \text{of } rK(K+1).$$

These 7d spaces are conjectured to have holonomy G_2 metrics.

(BSA, Witten)

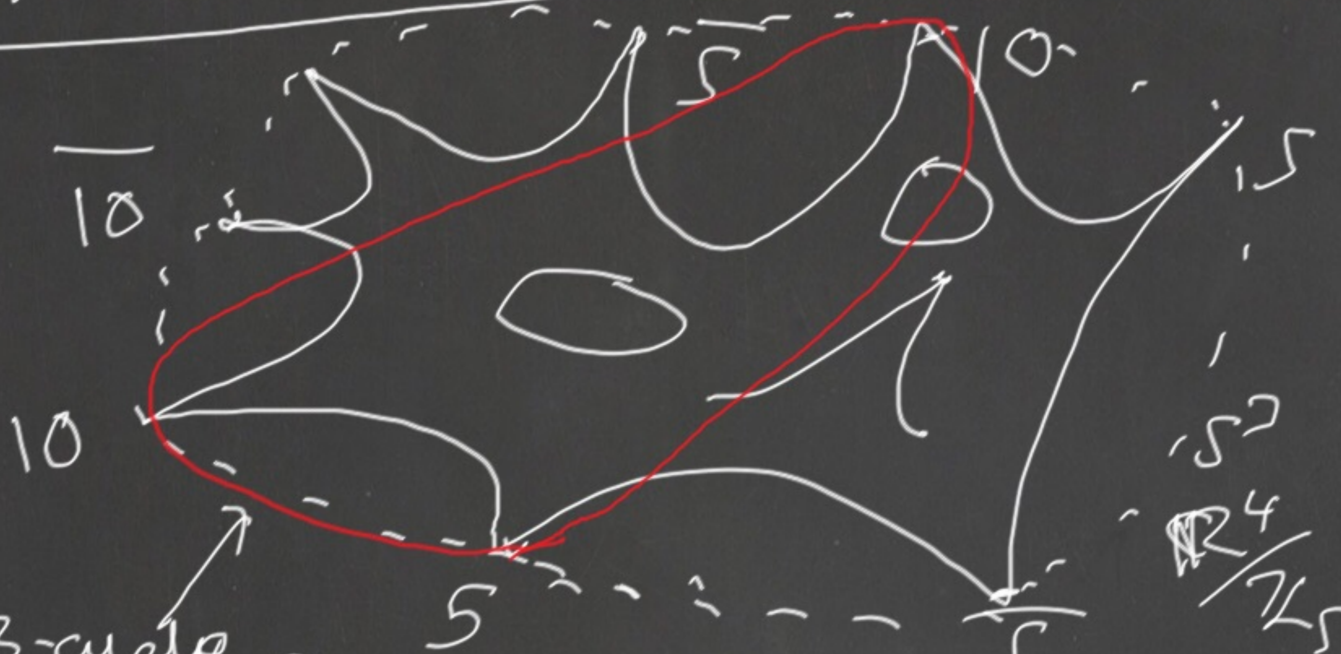
This establishes the picture of

- non-Abelian gauge symmetry
- chiral fermions

in M theory on G_2.

Next: Yukawa couplings

Yukawa Couplings in M theory



Susy 3-cycle Σ_3 $\rightsquigarrow$ gives a M2-brane instanton

generates a $5\cdot10\cdot10$ Yukawa

$$y_{5\cdot10\cdot10} \sim e^{-\mathrm{Vol}(\Sigma_3)}$$

So all Yukawas have

$$|y| \sim e^{-\mathrm{Vol}(\Sigma_3)}$$

for some Σ_3.

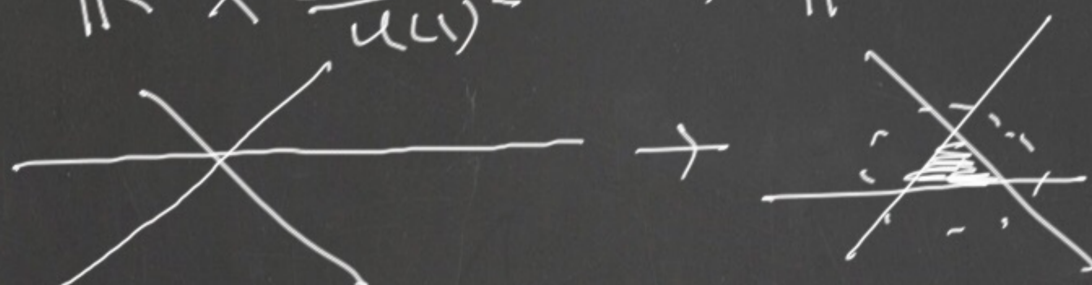
So, in M theory, Yukawas are naturally, exponentially hierarchical.

· What about y_{top}?

Need $\mathrm{Vol}(\Sigma_3) \sim O(1)$

ie $5, 10, 10$ must be close together, or $\mathrm{Vol}(\Sigma_3) \to 0$.

<u>Ex</u> $\mathbb{R}^+ \times \frac{SU(3)}{U(1)^2} \longrightarrow \mathbb{R}^6$



So, we have seen in <u>both</u> heterotic and M theory that

- non-Abelian gauge symmetry
- chiral fermions
- hierarchical Yukawas

arise <u>naturally</u>,

Seems to be a <u>generic</u> prediction of string/M compactified to 3+1d !

In third part of notes (next file) we will try and extract some further generic predictions for Dark Matter and cosmology.