An effective action for string fields

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This talk is about:

- Conserved correlators in FT
- Effective actions
- Higher spin fields
- Fronsdal equations
- L_∞ symmetry
- Application to SFT

First, some examples, in 3d:

spin 1

Free massive fermion model in 3d

Action

$$S = \int d^3x \left[i\bar{\psi}\gamma^\mu D_\mu \psi - m\bar{\psi}\psi \right], \quad D_\mu = \partial_\mu + A_\mu$$

where $A_\mu = A_\mu^a(x) T^a$ and T^a are the generators of a gauge algebra. The generators are antihermitean, $[T^a, T^b] = f^{abc} T^c$, with normalization $\mathrm{tr}(T^a T^b) = \mathrm{n}\delta^{ab}$. The current

$$J_\mu^a(x) = \bar{\psi}\gamma_\mu T^a \psi$$

is (classically) covariantly conserved on shell

$$(DJ)^a = (\partial^\mu \delta^{ac} + f^{abc} A^{b\mu}) J_\mu^c = 0$$

(see also Dunne,Babu,Das, Panigrahi)

Free massive fermion model in 3d (cont.)

The effective action is given by

$$W[A] = \sum_{n=1}^{\infty} \frac{i^{n+1}}{n!} \int \prod_{i=1}^{n} d^3x_i A^{a_1\mu_1}(x_1)\ldots A^{a_n\mu_n}(x_n)\langle 0|\mathcal{T} J^{a_1}_{\mu_1}(x_1)\ldots J^{a_n}_{\mu_n}(x_n)|0\rangle$$

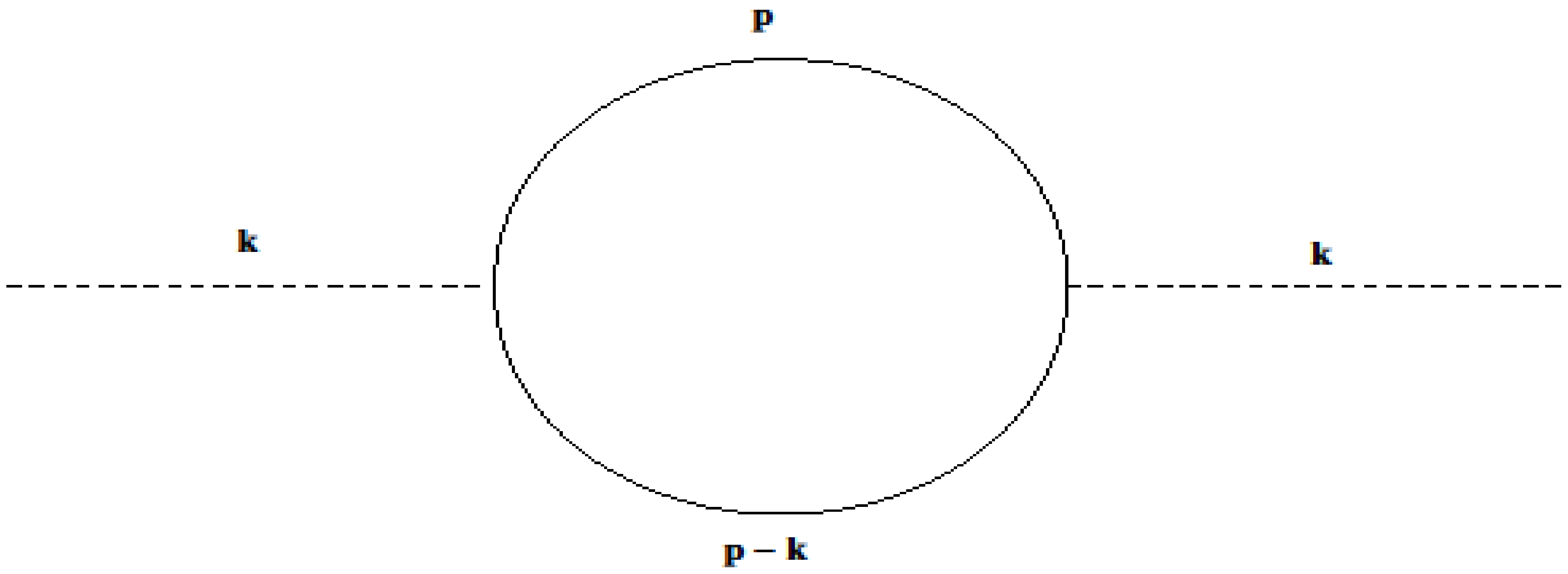
We will consider 2-pt and 3-pt current correlators,

$$\langle 0|\mathcal{T} J^a_\mu(x) J^b_\nu(y)|0\rangle, \quad \text{and} \quad \langle 0|\mathcal{T} J^a_\mu(x) J^b_\nu(y) J^b_\lambda(z)|0\rangle \tag{1}$$

whose Fourier transform are $\tilde{J}^{ab}_{\mu\nu}(k)$ and $\tilde{J}^{abc}_{\mu\nu\lambda}(k_1,k_2)$. The one-loop conservation law in momentum space is

$$k^\mu \tilde{J}^{ab}_{\mu\nu}(k) = 0$$
$$-iq^\mu \tilde{J}^{abc}_{\mu\nu\lambda}(k_1,k_2) + f^{abd}\tilde{J}^{dc}_{\nu\lambda}(k_2) + f^{acd}\tilde{J}^{db}_{\lambda\nu}(k_1) = 0$$

where $q = k_1 + k_2$.


p
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Free massive fermion model:2-pt

The 2-pt function is

$$\tilde{J}^{ab(odd)}_{\mu\nu}(k) \quad = \quad \frac{\mathrm{n}}{2\pi}\delta^{ab}\epsilon_{\mu\nu\sigma}k^{\sigma}\,\frac{m}{k}\,\arctan\frac{k}{2m}$$

where $k = \sqrt{k^2} = E$. The IR and UV limit correspond to $\frac{m}{E} \to \infty$ and 0, respectively. We get

$$\tilde{J}^{ab(odd)}_{\mu\nu}(k) = \frac{\mathrm{n}}{2\pi}\delta^{ab}\epsilon_{\mu\nu\sigma}k^{\sigma}\begin{cases} \frac{1}{2} & \mathrm{IR} \\ \frac{\pi}{2}\frac{m}{k} & \mathrm{UV} \end{cases}$$

Fourier anti-transforming and substituting in $W(A)$ one gets

$$\int d^3x\epsilon^{\mu\nu\lambda}\,A^a_\mu\partial_\nu A^a_\lambda$$

YM in effective action

The IR limit of the 2pt current correlator is given by

$$\tilde{J}^{ab(even)}_{\mu\nu}(k) = \frac{i}{4\pi}\frac{1}{3|m|}\delta^{ab}(k_\mu k_\nu - k^2\eta_{\mu\nu})$$

which is local. Fourier anti-transforming it and inserting it in the formula for the EA

$$S \sim \frac{1}{m}\int d^3x \, \left(A^a_\mu \partial^\mu \partial^\nu A^a_\nu - A^a_\nu \Box A^{a\nu}\right)$$

which is the lowest term in the expansion of the YM action

$$S_{YM} = \frac{1}{g}\int d^3x \, \mathrm{Tr}\,(F_{\mu\nu}F^{\mu\nu})$$

where $g \sim m$.

spin 2

E.m. tensor correlators

Next come the e.m. tensor correlator. It is naturally coupled to the metric. The action in the massive model is

$$S = \int d^3x\, e\, [i\bar\psi E^\mu_a \gamma^a \nabla_\mu \psi - m\bar\psi\psi]\,, \quad \nabla_\mu = \partial_\mu + \frac{1}{2}\omega_{\mu bc}\Sigma^{bc}, \quad \Sigma^{bc} = \frac{1}{4}\left[\gamma^b, \gamma^c\right].$$

The mass term breaks parity!

The energy-momentum tensor

$$T^{\mu\nu} = \frac{i}{4}\bar\psi \left(E^\mu_a \gamma^a \overleftrightarrow{\nabla}^\nu + \mu \leftrightarrow \nu \right) \psi.$$

is covariantly conserved (on shell): $\nabla_\mu T^{\mu\nu} = 0$.

At quantum level (the Fourier trasform of) the 2-pt correlator is

$$\tilde{T}^{(odd)}_{\mu\nu\lambda\rho}(k) \quad = \quad \frac{m}{256\pi}\,\epsilon_{\sigma\nu\rho}\,k^\sigma \left[2m\left(\eta_{\mu\lambda} - \frac{k_\mu k_\lambda}{k^2}\right) + \left(\eta_{\mu\lambda} + \frac{k_\mu k_\lambda}{k^2}\right)\frac{k^2+4m^2}{|k|}\arctan\frac{|k|}{2m}\right].$$

In the effective action the e.m. tensor couples to $h_{\mu\nu}$, where $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} + \ldots$.

Gravitational CS

In the IR and UV limit this corresponds to the action term

$$S_{\rm eff}^{\rm P-odd} = \frac{\kappa}{192\pi} \int d^3x \, \epsilon_{\sigma\nu\rho} \, h^{\mu\nu} \, \partial^\sigma (\partial_\mu \partial_\lambda - \eta_{\mu\lambda} \Box) \, h^{\lambda\rho} \qquad (1)$$

This is nothing but the lowest order expansion in $h_{\mu\nu}$ of the gravitational Chern-Simons action in 3d.

$$CS \quad = -\tfrac{\kappa}{96\pi} \int d^3x \, \epsilon^{\mu\nu\lambda} \left(\partial_\mu \omega_\nu^{ab} \omega_{\lambda ba} + \tfrac{2}{3} \omega_{\mu a}{}^b \omega_{\nu b}{}^c \omega_{\lambda c}{}^a \right)$$

- In the IR limit we find $\kappa = 1$ (the action is well defined)
- In the UV limit $\kappa = \frac{3}{2}\pi \frac{m}{|k|}$. So again the limit vanishes unless we consider N flavours, in which case we can take the scaling limit that leaves $\lambda = N \frac{m}{|k|}$ fixed.

EH in the effective action

Now let us go to the IR limit of the even part of the 2pt e.m. tensor correlator.

$$\langle T_{\mu\nu}(k)T_{\lambda\rho}(-k)\rangle^{IR}_{even} = \frac{i|m|}{96\pi}\left[\frac{1}{2}\left((k_\mu k_\lambda \eta_{\nu\rho} + \lambda \leftrightarrow \rho) + \mu \leftrightarrow \nu\right) - \right.$$

$$\left. - (k_\mu k_\nu \eta_{\lambda\rho} + k_\lambda k_\rho \eta_{\mu\nu}) - \frac{k^2}{2}(\eta_{\mu\lambda}\eta_{\nu\rho} + \eta_{\mu\rho}\eta_{\nu\lambda}) + k^2 \eta_{\mu\nu}\eta_{\lambda\rho}\right].$$

This is a local expression multiplied by $|m|$. Fourier anti-transforming it gives rise to the action

$$S \sim |m| \int d^3x \left(-2\partial_\mu h^{\mu\lambda}\partial_\nu h^\nu_\lambda - 2h\,\partial_\mu\partial_\nu h^{\mu\nu} - h^{\mu\nu}\Box h_{\mu\nu} + h\Box h\right)$$

This is the lowest order term in the expansion of the EH action:

$$S_{EH} = \frac{1}{2\kappa}\int d^3x\,\sqrt{g}\,R$$

where $\kappa \sim \frac{1}{|m|}$.

Spin 3 and higher

Higher spin currents

In the massive fermion model in 3d we have other conserved currents. The next after the em tensor is the third order current

$$J_{\mu_1\mu_2\mu_3} = \bar\psi\gamma_{(\mu_1}\partial_{\mu_2}\partial_{\mu_3)}\psi - \frac{5}{3}\partial_{(\mu_1}\bar\psi\gamma_{\mu_2}\partial_{\mu_3)}\psi + \frac{1}{3}\eta_{(\mu_1\mu_2}\partial^\sigma\bar\psi\gamma_{\mu_3)}\partial_\sigma\psi - \frac{m^2}{3}\eta_{(\mu_1\mu_2}\bar\psi\gamma_{\mu_3)}\psi$$

This is conserved (on-shell). We consider the external source $B^{\mu\nu\lambda}$ and couple it to the theory via the action term

$$\int d^3x J_{\mu\nu\lambda}B^{\mu\nu\lambda}$$

Due to current conservation this coupling is invariant under the (infinitesimal) transformations

$$\delta B_{\mu\nu\lambda} = \partial_{(\mu}\Lambda_{\nu\lambda)}$$

In the limit $m \to 0$ we have also invariance under the transformation

$$\delta B_{\mu\nu\lambda} = \Lambda_{(\mu}\eta_{\nu\lambda)}$$

which induces the tracelessness of $J_{\mu\nu\lambda}$ in any couple of indices.

2-pt B correlator

We can construct the effective action for $B_{\mu\nu\lambda}$ with

$$W[B] = \sum_{n=1}^{\infty} \frac{i^{n+1}}{n!} \int \prod_{i=1}^{n} d^3x_i B^{\mu_i\nu_i\lambda_i}(x_1)\langle 0|T J_{\mu_1\nu_1\lambda_1}(x_1)\ldots J_{\mu_n\nu_n\lambda_n}(x_n)|0\rangle.$$

by computing the n-pt functions. For instance, the 2-pt correlator (after subtraction), in the IR is

$$\tilde{J}^{(odd,IR)}_{\mu_1\mu_2\mu_3\nu_1\nu_2\nu_3}(k) = \frac{1}{4\pi}\epsilon_{\mu_1\nu_1\sigma}k^{\sigma}\Big[\frac{1}{60}k^4\eta_{\mu_2\mu_3}\eta_{\nu_2\nu_3} - \frac{8}{135}k^4\eta_{\mu_2\nu_2}\eta_{\mu_3\nu_3}$$
$$-\frac{1}{60}k^2\left(k_{\nu_2}k_{\nu_3}\eta_{\mu_2\mu_3} + k_{\mu_2}k_{\mu_3}\eta_{\nu_2\nu_3}\right) + \frac{16}{135}k^2k_{\mu_2}k_{\nu_2}\eta_{\mu_3\nu_3} - \frac{23}{540}k_{\mu_2}k_{\mu_3}k_{\nu_2}k_{\nu_3}\Big]$$

and in the UV

$$\tilde{J}^{(odd,UV)}_{\mu_1\mu_2\mu_3\nu_1\nu_2\nu_3}(k) = \frac{1}{4}\frac{m}{|k|}\,\epsilon_{\mu_1\nu_1\sigma}k^{\sigma}\Big[\frac{1}{12}k_{\mu_2}k_{\mu_3}k_{\nu_2}k_{\nu_3} - \frac{2}{9}k^2k_{\mu_3}k_{\nu_3}\eta_{\mu_2\nu_2}$$
$$+\frac{k^2}{36}\left(k_{\nu_2}k_{\nu_3}\eta_{\mu_2\mu_3} + k_{\mu_2}k_{\mu_3}\eta_{\nu_2\nu_3}\right) + \frac{1}{9}k^4\eta_{\mu_2\nu_2}\eta_{\mu_3\nu_3} - \frac{1}{36}k^4\eta_{\mu_2\mu_3}\eta_{\nu_2\nu_3}\Big].$$

They are both conserved.

Odd effective action

The UV expression corresponds to the effective action term

$$\sim \int d^3x \quad \epsilon_{\mu_1\nu_1\sigma}\Big[3\partial^\sigma B^{\mu_1\mu_2\mu_3}\partial_{\mu_2}\partial_{\mu_3}\partial_{\nu_2}\partial_{\nu_3}B^{\nu_1\nu_2\nu_3} - 8\partial^\sigma B^{\mu_1\mu_2\mu_3}\Box\partial_{\mu_3}\partial_{\nu_3}B^{\nu_1\nu_3}{}_{\mu_2}$$
$$+2\partial^\sigma B^{\mu_1\lambda}{}_\lambda\Box\partial_{\nu_2}\partial_{\nu_3}B^{\nu_1\nu_2\nu_3} + 4\partial^\sigma B^{\mu_1\mu_2\mu_3}\Box^2 B^{\nu_1}{}_{\mu_2\mu_3}$$
$$-\partial^\sigma B^{\mu_1\lambda}{}_\lambda\Box^2 B^{\nu_1\rho}{}_\rho\Big]$$

where $b_{\mu\nu\lambda} = B_{\mu\nu\lambda} + \dots$.

This is a slight generalization of an action proposed by Pope and Townsend (1989)

$$\sim \int d^3x \qquad \epsilon_{\mu_1\nu_1\sigma}\Big[\frac{3}{2}\partial^\sigma h^{\mu_1\mu_2\mu_3}\partial_{\mu_2}\partial_{\mu_3}\partial_{\nu_2}\partial_{\nu_3}h^{\nu_1\nu_2\nu_3} - 4\partial^\sigma h^{\mu_1\mu_2\mu_3}\Box\partial_{\mu_3}\partial_{\nu_3}h^{\nu_1\nu_3}{}_{\mu_2}$$
$$+2\partial^\sigma h^{\mu_1\mu_2\mu_3}\Box^2 h^{\nu_1}{}_{\mu_2\mu_3}\Big]$$

one can see that they are equal if we set $B^{\mu\lambda}{}_\lambda = 0$

For instance, for the field B one can extract from the even 2-pt function the following action

$$S_B \sim \int \Big((\partial_\mu B_{\mu_1\mu_2\mu_3})^2 - 3(\partial\cdot B_{\mu_1\mu_2})^2 - 3\partial\cdot\partial\cdot B^\mu \frac{1}{\Box}\partial\cdot\partial\cdot B_\mu - \partial\cdot\partial\cdot\partial\cdot B \frac{1}{\Box^2}\partial\cdot\partial\cdot\partial\cdot B \Big)$$

This gives the equation of motion

$$\Box B_{\mu\nu\lambda} - \partial_{\underline{\mu}}\partial\cdot B_{\underline{\nu\lambda}} + \frac{1}{\Box}\partial_{\underline{\mu}}\partial_{\underline{\nu}}\partial\cdot\partial\cdot B_{\underline{\lambda}} - \frac{1}{\Box^2}\partial_\mu\partial_\nu\partial_\lambda\partial\cdot\partial\cdot\partial\cdot B = 0$$

Now take the trace with respect to any two indices of this equation and you will get

$$\partial\cdot\partial\cdot B_\lambda = \Box B'_\lambda - \partial_\lambda\partial\cdot B' + \frac{1}{\Box}\partial_\lambda\partial\cdot\partial\cdot\partial\cdot B$$

Upon replacing this into the third term above one gets precisely the nonlocal Fronsdal equation for massless spin 3.

Fronsdal eqs for higher spins

The Fronsdal equation of motion for a (completely symmetric) spin 3 field $\varphi_{\mu\nu\lambda}$ is the following:

$$\mathcal{F}_{\mu\nu\lambda} \equiv \Box\varphi_{\mu\nu\lambda} - \partial_{\underline{\mu}}\partial\cdot\varphi_{\underline{\nu\lambda}} + \partial_{\underline{\mu}}\partial_{\underline{\nu}}\varphi'_{\underline{\lambda}} = 0$$

where $\partial_{\underline{\mu}}\partial\cdot\varphi_{\underline{\nu\lambda}} = \partial_\mu\partial\cdot\varphi_{\nu\lambda} + \text{perm.},$. Under $\delta\varphi_{\mu\nu\lambda} = \partial_\mu\Lambda_{\nu\lambda} + \text{perm.}$

$$\delta\mathcal{F}_{\mu\nu\lambda} = 3\partial_\mu\partial_\nu\partial_\lambda\Lambda'$$

So covariance requires tracelessness $\Lambda' = 0$. Unnatural!
D.Francia and A.Sagnotti (2002) proposed a way out via nonlocality.

$$\mathcal{F}_{\mu\nu\lambda} - \frac{1}{\Box^2}\partial_\mu\partial_\nu\partial_\lambda\partial\cdot\mathcal{F}' = 0$$

This is invariant, but nonlocal. However nonlocality is irrelevant (a gauge artifact). This can be seen via a compensator.

The compensator

We can rewrite the non-local Fronsdal equation as

$$\mathcal{F}_{\mu\nu\lambda} \equiv \Box\varphi_{\mu\nu\lambda} - \partial_{\underline{\mu}}\partial\cdot\varphi_{\underline{\nu\lambda}} + \partial_{\underline{\mu}}\partial_{\underline{\nu}}\varphi'_{\underline{\lambda}} = \partial_\mu\partial_\nu\partial_\lambda\alpha$$

where

$$\alpha = \frac{3}{\Box}\partial\cdot\varphi' - \frac{2}{\Box^2}\partial\cdot\partial\cdot\partial\cdot\varphi$$

The field α is called compensator, because its transformation property under $\delta\varphi = 3\partial\Lambda$ is

$$\delta\alpha = 3\Lambda' \qquad \longrightarrow \qquad \delta\mathcal{F} = 3\partial^3\Lambda'$$

It allows to write a local Lagrangian. So, the nonlocality of the initial equation is only a gauge tail which serves to gaurantee covariance.

So what’s going on?

In the effective action of a massive 3d fermion we have found all the local action for spin 1, 2, 3: YM, CS, EH, Fronsdal, Pope-Townsend,….

Is there more?

Yes. Fortunately we can compute the effective action of a 3d fermion exactly for any current.

Spin s currents

The spin s current has the form

$$J^{(s)}_{\mu_1\ldots\mu_s} = \bar{\psi}\gamma_{(\mu_1}\partial_{\mu_2}\ldots\partial_{\mu_s)}\psi + \ldots\ldots$$

We couple it to an external source $a^{\mu_1\ldots\mu_s}$ through the term $\int d^3x\, a^{\mu_1\ldots\mu_s}(x) J^{(s)}_{\mu_1\ldots\mu_s}(x)$, compute the 2-pt function $\langle|\mathcal{T} J^{(s)}_{\mu_1\ldots\mu_s} J^{(s)}_{\nu_1\ldots\nu_s}|0\rangle$, and insert into the generating function

$$W[a,s] = \sum_{n=1}^{\infty} \frac{i^{n+1}}{n!} \int \prod_{i=1}^{n} d^3x_i\, a^{\mu_{11}\ldots\mu_{1p}}(x_1)\ldots a^{\mu_{1n}\ldots\mu_{sn}}(x_n) \\ \times \langle 0|\mathcal{T} J^{(s)}_{\mu_{11}\ldots\mu_{1s}}(x_1)\ldots J^{(s)}_{\mu_{n1}\ldots\mu_{ns}}(x_n)|0\rangle.$$

to obtain the effective action. In particular $a_\mu = A_\mu$, $a_{\mu\nu} = h_{\mu\nu}$ and $a_{\mu\nu\lambda} = b_{\mu\nu\lambda}$.

Exact 2pt correlator for the e.m. tensor

The correlator is

$$\frac{i}{192\pi k}\Bigg(\left(96m^4\coth^{-1}\left(\frac{2m}{k}\right)-48km^3-4k^3m-6k^4\coth^{-1}\left(\frac{2m}{k}\right)\right)\left(n_1\cdot\pi^{(k)}\cdot n_2\right)^2$$

$$+\left(48m^4\coth^{-1}\left(\frac{2m}{k}\right)-24km^3-24k^2m^2\coth^{-1}\left(\frac{2m}{k}\right)+10k^3m\right.$$

$$\left.+3k^4\coth^{-1}\left(\frac{2m}{k}\right)\right)(n_1\cdot\pi^{(k)}\cdot n_1)(n_2\cdot\pi^{(k)}\cdot n_2)\Bigg)$$

where we use the projector $\pi^{(k)}$ and the compact notation:

$$\pi^{(k)}_{\mu\nu}=\eta_{\mu\nu}-\frac{k_\mu k_\nu}{k^2},\qquad (n_1\cdot\pi^{(k)}\cdot n_2)=\pi^{(k)}_{\mu\nu}n_1^\mu n_2^\nu$$

after subtracting

$$\mathcal{O}(m^3):\ \frac{im^3}{3\pi}\left((n_1\cdot n_2)^2+(n_1\cdot n_1)(n_2\cdot n_2)\right)$$

which is not conserved, but local.

Correlator tomography

Expanding in powers of m

$$
\begin{aligned}
\mathcal{O}(m^2): &\quad 0 \\
\mathcal{O}(m): &\quad \frac{im}{12\pi} k^2 \left(\left(n_1 \cdot \pi^{(k)} \cdot n_2 \right)^2 - (n_1 \cdot \pi^{(k)} \cdot n_1)(n_2 \cdot \pi^{(k)} \cdot n_2) \right) \\
\mathcal{O}(m^0): &\quad 0 \\
\mathcal{O}(m^{-1}): &\quad -\frac{i}{80\pi m} k^4 \left(\left(n_1 \cdot \pi^{(k)} \cdot n_2 \right)^2 - \frac{1}{3} (n_1 \cdot \pi^{(k)} \cdot n_1)(n_2 \cdot \pi^{(k)} \cdot n_2) \right) \\
\mathcal{O}(m^{-2}): &\quad 0
\end{aligned}
$$

These terms are all conserved. The $\mathcal{O}(m)$ term is the linearized version of the EH equation of motion. All the other terms differ only by pure gauge parts.

So far we have considered only 3d examples, but the previous analysis holds in every dimension.

We have calculated explicitly all types of 2pt correlators (also mixed ones) for d=3,4,5,6,7,8 for spin s=1,2,3,4,5, both for a scalar and fermion model, and also obtained formulas in any dimension.

$d>3$

Example in 4d

In 4d (in any even dimension) one has an additional problem of regularization. The way out is to do the calculations for $d = 4 + \delta$. For instance in the case of the e.m. tensor the 2pt function in the IR takes the form

$$\begin{aligned}\tilde{T}_{\mu\nu\lambda\rho}(k) \quad = \quad & -\frac{i}{16(2\pi)^2} m^4 (2\eta_{\mu\nu}\eta_{\lambda\rho} + \eta_{\mu\lambda}\eta_{\nu\rho} + \eta_{\mu\rho}\eta_{\nu\lambda}) \left(-\frac{1}{4\delta} + \frac{3}{16} - \frac{1}{8}\left(\gamma + \log\frac{4\pi}{m^2}\right)\right) \\ & -\frac{i}{16(2\pi)^2} m^2 \left[\frac{1}{2}\left((k_\mu k_\lambda \eta_{\nu\rho} + \lambda \leftrightarrow \rho) + \mu \leftrightarrow \nu\right) - \right. \\ & \quad \left. - (k_\mu k_\nu \eta_{\lambda\rho} + k_\lambda k_\rho \eta_{\mu\nu}) - \frac{k^2}{2}(\eta_{\mu\lambda}\eta_{\nu\rho} + \eta_{\mu\rho}\eta_{\nu\lambda}) + k^2 \eta_{\mu\nu}\eta_{\lambda\rho}\right] \\ & \cdot \left(\frac{1}{6\delta} + \frac{1}{12} - \frac{1}{12}(\gamma + \log 4\pi - \log m^2)\right)\end{aligned}$$

The term proportional to m^4 is clearly not conserved, but is local and can be subtracted. The term proportional to m^2 corresponds to the lowest order term of the Einstein-Hilbert action, with a coupling

$$\frac{i}{16(2\pi)^2} m^2 \left(\frac{1}{6\delta} + \frac{1}{12} - \frac{1}{12}(\gamma + \log 4\pi - \log m^2)\right)$$

Example in 6d

In 6d for spin 3 the full 2pt correlator is too big.
Here is a part of the expansion in $\frac{m}{k}$ (tomography)

$$
\begin{aligned}
\mathcal{O}_{UV}(m^2) \quad - \quad \mathcal{O}_{IR}(m^2) &= \frac{im^2k^6}{3175200\pi^3}\Bigg(32\left(-210\log\left(-\frac{k^2}{m^2}\right)+389\right)\left(n_1\cdot\pi^{(k)}\cdot n_2\right)^3 \\
&+\left(3885\log\left(-\frac{k^2}{m^2}\right)-9454\right)\left(n_1\cdot\pi^{(k)}\cdot n_2\right)(n_1\cdot\pi^{(k)}\cdot n_1)(n_2\cdot\pi^{(k)}\cdot n_2)\Bigg) \\
\mathcal{O}_{UV}(m^4) \quad - \quad \mathcal{O}_{IR}(m^4) &= \frac{im^4k^4}{64800\pi^3}\Bigg(16\left(-17+30\log\left(-\frac{k^2}{m^2}\right)\right)\left(n_1\cdot\pi^{(k)}\cdot n_2\right)^3 \\
&+\left(-480\log\left(-\frac{k^2}{m^2}\right)+617\right)\left(n_1\cdot\pi^{(k)}\cdot n_2\right)(n_1\cdot\pi^{(k)}\cdot n_1)(n_2\cdot\pi^{(k)}\cdot n_2)\Bigg) \\
\mathcal{O}_{UV}(m^6) \quad - \quad \mathcal{O}_{IR}(m^6) &= -\frac{im^6k^2}{864\pi^3}\Bigg(\frac{64}{3}\left(n_1\cdot\pi^{(k)}\cdot n_2\right)^3 \\
&-\left(18\log\left(-\frac{k^2}{m^2}\right)-17\right)\left(n_1\cdot\pi^{(k)}\cdot n_2\right)(n_1\cdot\pi^{(k)}\cdot n_1)(n_2\cdot\pi^{(k)}\cdot n_2)\Bigg)
\end{aligned}
$$

The last two lines are a version of nonlocal Fronsdal equation for spin 3.

Important remark!

We have seen eom’s for various spins appearing in the IR of 2pt correlators (or, equivalently, of the quadratic EA). But in fact the EA is entirely based on the corresponding differential operator.

You can find plenty of complete formulas in ArXiv:1609.020088, 1709:01738

(miracles of Mathematica)

These results can be cast in a more ‘geometrical’ form…

Correlator tomography for spin s

For spin s the 2-pt correlators can be calculated exactly. After subtracting some local terms, their structure is a generalization of the one for the e.m. tensor. It is a superposition with k, m-dependent coefficient of

$$\tilde{E}^{(s)}(k, n_1, n_2) = \sum_{l=0}^{[s/2]} a_l \tilde{A}_l^{(s)}(k, n_1, n_2)$$

where a_l are numbers, and

$$\begin{aligned}
\tilde{A}_0^{(s)}(k, n_1, n_2) &= \frac{1}{(s!)^2}(n_1 \cdot \pi^{(k)} \cdot n_2)^s \\
\tilde{A}_1^{(s)}(k, n_1, n_2) &= \frac{1}{(s!)^2}(n_1 \cdot \pi^{(k)} \cdot n_2)^{s-2}(n_1 \cdot \pi^{(k)} \cdot n_1)(n_2 \cdot \pi^{(k)} \cdot n_2) \\
\ldots &\quad \ldots\ldots \\
\tilde{A}_l^{(s)}(k, n_1, n_2) &= \frac{1}{(s!)^2}(n_1 \cdot \pi^{(k)} \cdot n_2)^{s-2l}(n_1 \cdot \pi^{(k)} \cdot n_1)^l(n_2 \cdot \pi^{(k)} \cdot n_2)^l \\
\ldots &\quad \ldots\ldots
\end{aligned}$$

General eom's

We can represent the eom symbolically as $k^2 \sum_{l=0}^{[s/2]} a_l \tilde{A}_l^{(s)}(k, n_1, n_2) = 0$. We can write this equation in the form

$$k^2 (n_1 \cdot \pi^{(k)} \cdot n_2)^{s-2l} (n_1 \cdot \pi^{(k)} \cdot n_1)^l (n_2 \cdot \pi^{(k)} \cdot n_2)^l$$
$$= \sum_{p=l}^{\lfloor s/2 \rfloor} \left(-\frac{1}{2}\right)^p \binom{\lfloor s/2 \rfloor}{p} \binom{\lfloor s/2 - l \rfloor}{p-l} \frac{p!(D+s-2p-4)!!}{(D+s-4)!!} (n_1 \cdot \pi^{(k)} \cdot n_1)^p \, \tilde{\mathcal{G}}^{(n)[p]}(k, n_1, n_2$$

where $\mathcal{G}^{(n)}$ are generalizations of the linearized Einstein tensor. On the other hand

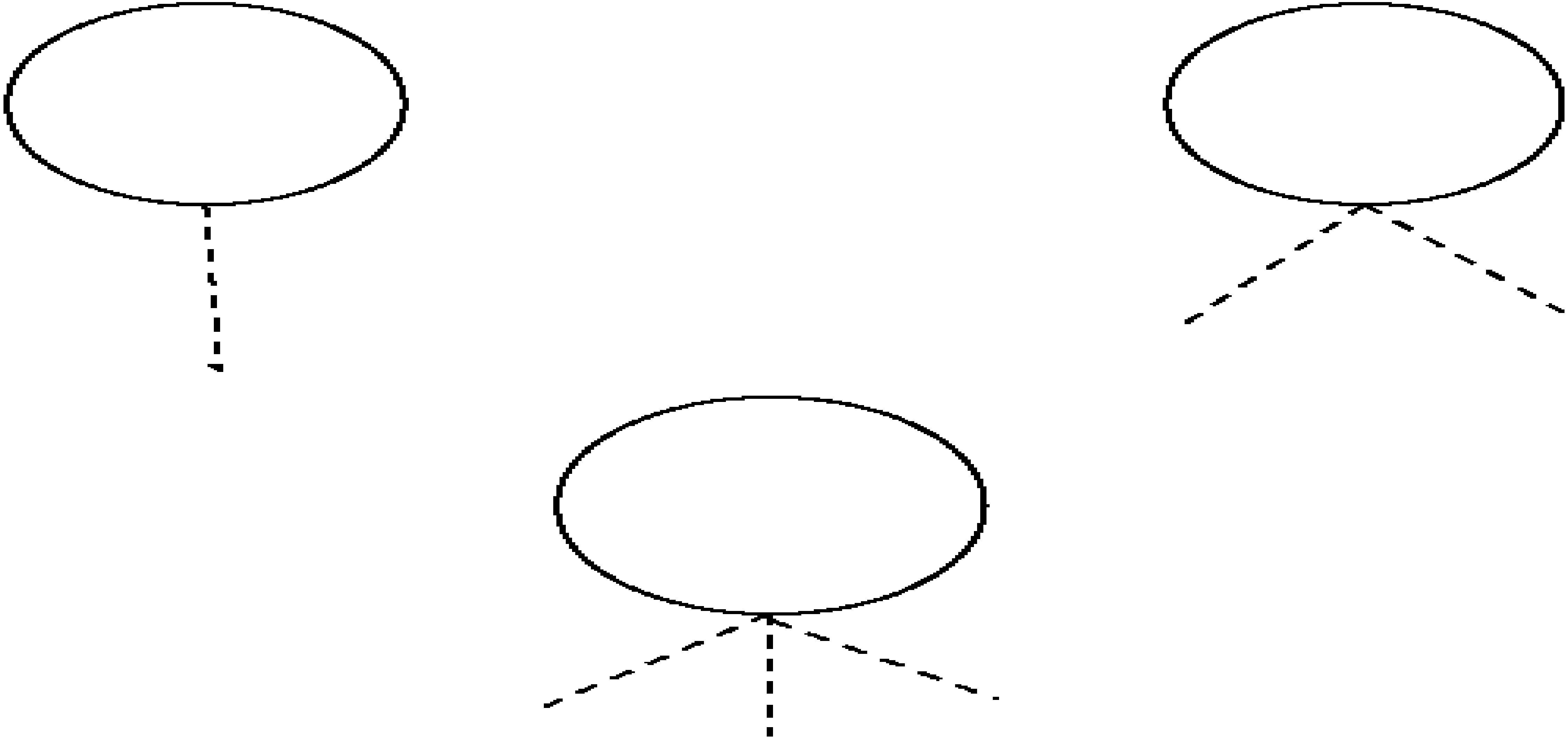
$$\mathcal{G}^{(n)} = \sum_{p=0}^{n} (-1)^p \frac{(n-p)!}{2^p n!} \eta^p \, \mathcal{F}^{(n)[p]}, \qquad \mathcal{F}^{(n+1)} = \begin{cases} \frac{1}{\Box^n} \mathcal{R}^{(s)[n+1]} & s = 2(n+1) \\ \frac{1}{\Box^n} \partial \cdot \mathcal{R}^{(s)[n]} & s = 2n+1 \end{cases}$$

where $\mathcal{F}^{(n)}$ are the Fronsdal operators and $\mathcal{R}$ the generalized Jacobi tensors, defined by

$$\frac{1}{(s!)^2} (m^s \cdot \mathcal{R}^{(s)} \cdot n^s) = \sum_{l=0}^{s} \frac{(-1)^l}{s!(s-l)!l!} (m \cdot \partial)^{s-l} (n \cdot \partial)^l (m^l \cdot \varphi \cdot n^{s-l})$$

Tadpoles and seagull terms

Tadpole and seagull terms



An example: spin 2 in any dim.

The transverse and non-transverse parts of the bubble diagram are

$$\tilde{T}_t^{\mu\mu\nu\nu}(k) = -2^{-3-d+\lfloor\frac{d}{2}\rfloor} i\, m^d \pi^{-\frac{d}{2}} \sum_{n=1}^{\infty} \frac{m^{-2n}\Gamma\left(n-\frac{d}{2}\right)}{2^n(2n+1)!!} k^{2n}\left((2n-1)\pi^{\mu\nu}\pi^{\mu\nu} - \pi^{\mu\mu}\pi^{\nu\nu}\right)$$

and

$$\tilde{T}_{nt}^{\mu\mu\nu\nu}(k) \quad = \quad -2^{-3-d+\lfloor\frac{d}{2}\rfloor}\, i\, m^d \pi^{\frac{d}{2}} \Gamma\left(-\frac{d}{2}\right)\left(\eta^{\mu\nu}\eta^{\mu\nu} - \eta^{\mu\mu}\eta^{\nu\nu}\right)$$

The tadpole and seagull contributions are, respectively,

$$\tilde{\Theta}^{\mu\mu}(k) = -2^{-2-d+\lfloor\frac{d}{2}\rfloor}\, i\, m^d \pi^{\frac{d}{2}} \Gamma\left(-\frac{d}{2}\right)\eta^{\mu\mu} = \tilde{\Theta}\,\eta^{\mu\mu}$$

and

$$\tilde{T}_{(s)}^{\mu\mu\nu\nu}(k) = 2^{-3-d+\lfloor\frac{d}{2}\rfloor}\, i\, m^d \pi^{\frac{d}{2}} \Gamma\left(-\frac{d}{2}\right)\left(3\eta^{\mu\nu}\eta^{\mu\nu} - 2\eta^{\mu\mu}\eta^{\nu\nu}\right)$$

The Ward identity in momentum space is

$$k_\mu \tilde{T}^{\mu\mu\nu\nu}(k) = \left[-k^\nu \eta^{\mu\nu} + \frac{1}{2}k^\mu \eta^{\nu\nu}\right]\tilde{\Theta}$$

Inserting in $\tilde{T}^{\mu\mu\nu\nu}(k)$ all the contributions, this identity is satisfied.
For instance, in the IR the effective action is

$$W \overset{\text{IR}}{\sim} -2^{-1-d+\lfloor\frac{d}{2}\rfloor} m^d \pi^{-\frac{d}{2}} \int \mathrm{d}^d x \sqrt{g} \times \left[\Gamma\left(-\frac{d}{2}\right) - \frac{\Gamma\left(1-\frac{d}{2}\right)}{24m^2} R \right.$$
$$\left. - \frac{\Gamma\left(2-\frac{d}{2}\right)}{80m^4}\left(R_{\mu\nu\lambda\rho}R^{\mu\nu\lambda\rho} - 2R_{\mu\nu}R^{\mu\nu} + \frac{1}{3}R^2\right) + \ldots\right] + O(h^3)$$

$$W \overset{UV}{\sim} (-1)^{\frac{d}{2}} \frac{2^{-4-2d+\lfloor\frac{d}{2}\rfloor}\pi^{\frac{3}{2}-\frac{d}{2}}}{(-1+e^{i\pi d})\Gamma\left(\frac{d+3}{2}\right)} \int \mathrm{d}^d x \sqrt{g}\left[(d-4)R_{\mu\nu\lambda\rho}\Box^{\frac{d}{2}-2}R^{\mu\nu\lambda\rho}\right.$$
$$\left. +6\left(R_{\mu\nu\lambda\rho}\Box^{\frac{d}{2}-2}R^{\mu\nu\lambda\rho} - 2R_{\mu\nu}\Box^{\frac{d}{2}-2}R^{\mu\nu} + \frac{1}{3}R\Box^{\frac{d}{2}-2}R\right) + \ldots\right] + O(h^3)$$

In view of these results the first consideration is: HS's theories are among us, they are natural developments of ordinary theories. They are nothing exotic.

For HS theories, see:

Vasiliev, Prokushkin, Metsaev,… Bekaert, Young, Mourad, Francia, Iazeolla, Sagnotti, Campoleoni, Fredenhagen, Fotopoulos, Tsulaia, Taronna,…

Their ambition is to construct sensible HS theories and, to a certain extent, they have succeeded (3d, 4d AdS…)

Provisional conclusion:

Free field theories generate one-loop effective actions which contain information (action, eom,…) about a very large spectrum of (if not all) local field theories physicists have been able to invent.

But the higher spin theories so far are in linearized form. Is the correspondence only valid for free higher spin theories?

The answer is: no! The correspondence may extend also to interactions.

Let us consider some examples

interaction

Free massive fermion model:3-pt

The 3-pt function is more complicated

$$\tilde{J}^{1,abc}_{\mu\nu\lambda}(k_1,k_2) = i\int \frac{d^3p}{(2\pi)^3}\mathrm{Tr}\left(\gamma_\mu T^a \frac{1}{\not p - m}\gamma_\nu T^b \frac{1}{\not p - \not k_1 - m}\gamma_\lambda T^c \frac{1}{\not p - \not q - m}\right)$$

The result is a generalized Lauricella function (Boos,Davydychev). In the IR we find

$$\tilde{J}^{1,abc(odd)}_{\mu\nu\lambda}(k_1,k_2) \approx i\frac{\mathrm{n}}{32\pi}\sum_{n=0}^{\infty}\left(\frac{\sqrt{E}}{m}\right)^{2n} f^{abc}\tilde{I}^{(2n)}_{\mu\nu\lambda}(k_1,k_2)$$

and, in particular,

$$I^{(0)}_{\mu\nu\lambda}(k_1,k_2) = -6\epsilon_{\mu\nu\lambda}$$

which corresponds to the action term

$$\sim \int d^3x\, \epsilon^{\mu\nu\lambda} f^{abc} A^a_\mu A^b_\nu A^c_\lambda$$

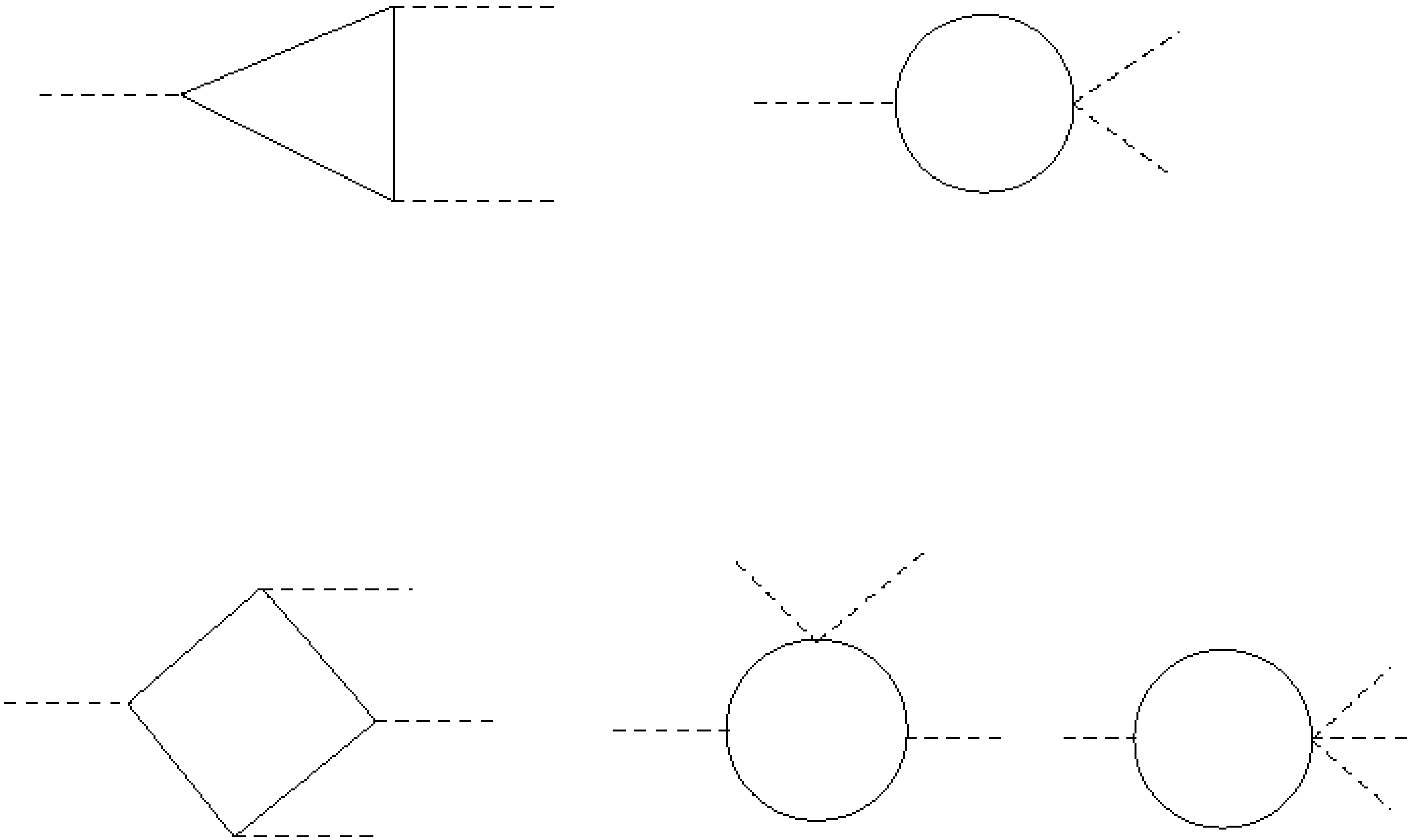
Free massive fermion model:3-pt (cnt.)

Putting things together we find the effective CS action

$$CS = \frac{\kappa}{4\pi} \int d^3x \mathrm{Tr} \left(A \wedge dA + \frac{2}{3} A \wedge A \wedge A \right)$$

- In the IR $\kappa = 1$, so the CS action is invariant also under large gauge transformation.
- - In the UV things are more complicated. Eventually we get the same action with $\kappa = \pi \frac{m}{k}$. So, the UV limit is 0.
  - If ψ carries a flavour index $i = 1, \ldots, N$, the previous result is multiplied by N, and $\kappa = \pi N \frac{m}{k}$. So we can consider the scaling limit $N \to \infty$, $\frac{m}{k} \to 0$ and κ fixed and finite.

Important! Both 2-pt and 3-pt correlator satisfy the WI's of CFT (and they are pure contact term)!



However it is clear that we cannot proceed by trial and error… it's too complicated!

We have to find a systematic way, a method to construct HS theories from EA's.

Another approach: worldline quantization

In simple words: quantize a particle worldline X^m with the Weyl quantization method and interpret a field as a function of the quantum X^m

Strassler, Segal, Bastianelli, Bonezzi, Boulanger, Corradini, Latini, Bekaert, Joung, Mourad ...

Worldline quantization of a fermion system

Let us consider a free fermion theory

$$S_0 = \int d^d x\, \overline{\psi}(i\gamma\cdot\partial - m)\psi,$$

coupled to external sources and use the Weyl quantization method of a worldline, which becomes a second quantization when applied to fields. The full action is expressed as

$$S = \langle\overline{\psi}| - \gamma\cdot(\widehat{P} - \widehat{H}) - m|\psi\rangle$$

The symbol of $\widehat{P}^\mu$ is the momentum p^μ, and the symbol of $\widehat{H}$ is $h(x,p)$, where

$$h^\mu(x,p) = \sum_{n=0}^{\infty} \frac{1}{n!} h_{(s)}^{\mu\mu_1\cdots\mu_n}(x) p_{\mu_1}\cdots p_{\mu_n}$$

Now interpret $\psi(x) = \langle x|\psi\rangle$, then

$$S = S_0 + \sum_{n=0}^{\infty} \int d^d z\, i^n \frac{\partial}{\partial z^{\mu_1}} \cdots \frac{\partial}{\partial z^{\mu_n}} \overline{\psi}\left(x + \frac{z}{2}\right) \gamma_\mu h^{\mu\mu_1\cdots\mu_n}(x)\psi\left(x - \frac{z}{2}\right)\Big|_{z=0}$$

The tensor $h^{\mu\mu_1\cdots\mu_n}$'s is linearly coupled to the conserved currents

$$J^{(s)}_{\mu\mu_1\ldots\mu_n} = \frac{1}{n!}\frac{\partial}{\partial z^{\mu_1}}\cdots\frac{\partial}{\partial z^{\mu_n}}\overline{\psi}\left(x+\frac{z}{2}\right)\gamma_\mu\psi\left(x-\frac{z}{2}\right)\Big|_{z=0}$$

For instance

$$\begin{aligned} J^{(1)}_{\mu} &= \overline{\psi}\gamma_\mu\psi \\ J^{(2)}_{\mu\mu_1} &= \frac{i}{2}\left(\partial_{(\mu_1}\overline{\psi}\gamma_{\mu)}\psi - \overline{\psi}\gamma_{(\mu}\partial_{\mu_1)}\psi\right) \\ &\cdots \end{aligned} \qquad (1)$$

and S is off-shell symetric under

$$\delta\gamma\cdot h(x,p) = \gamma\cdot\partial_x\epsilon(x,p) - i[\gamma\cdot h(x,p) \overset{*}{,} \epsilon(x,p)]$$

and

$$\delta\tilde{\psi}(x,p) = i\epsilon(x,p) * \tilde{\psi}(x,p), \qquad \tilde{\psi}(x,p) = \int d^dy\,\psi\left(x-\frac{y}{2}\right)e^{iy\cdot p}.$$

The quantum effective action is formally $W[h] = N\,\mathrm{Tr}[\ln \widehat{\mathrm{G}}]$ which is regularized via

$$W_{reg}[h,\epsilon] = -N\int_{\epsilon}^{\infty}\frac{dt}{t}\mathrm{Tr}\left[\mathrm{e}^{-\mathrm{t}\widehat{\mathrm{G}}}\right] =$$

The crucial factor is the heat kernel

$$K[g|t] \equiv \mathrm{Tr}\left[\mathrm{e}^{-\mathrm{t}\widehat{\mathrm{G}}}\right] = \mathrm{Tr}\left[\mathrm{e}^{\mathrm{t}(\gamma(\widehat{\mathrm{P}}-\widehat{\mathrm{H}})+\mathrm{m})}\right]$$

This kernel can be perturbatively expanded, $K[g,t] = \sum_{n=0}^{\infty}\langle\langle K^{(n)\mu\ldots\mu}(t)|h_{\mu}^{\otimes n}\rangle\rangle$, where

$$K^{\mu_1\cdots\mu_n}(x_1,u_1,\ldots,x_n,u_n|t) = \prod_{j=1}^{n} e^{ip_j\cdot\left(x_{j+1}-x_j+i\frac{u_{j+1}+u_j}{2}\right)}\widetilde{K}^{\mu_1\cdots\mu_n}(p_1,\ldots,p_n|m,t)$$

and, setting $\omega' = \omega - i\epsilon$,

$$\widetilde{K}^{\mu_1\cdots\mu_n}(p_1,\ldots,p_n|m,t) = \frac{t}{n}\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}e^{i\omega t}$$
$$\times\,\mathrm{tr}\left[\gamma^{\mu_1}\frac{1}{\not{p}_1+\mathrm{m}-\mathrm{i}\omega'}\gamma^{\mu_2}\ldots\gamma^{\mu_{n-1}}\frac{1}{\not{p}_{n-1}+\mathrm{m}-\mathrm{i}\omega'}\gamma^{\mu_n}\frac{1}{\not{p}_n+\mathrm{m}-\mathrm{i}\omega'}\right]$$

Advantages and disadvantages of the two methods

The WLQ method gives directly conserved currents and symmetry transformation properties. On the other hand the latter are rigidly defined. The technology of perturbative evaluation is not yet well explored.

The Feynman diagram technique does not determine the conserved currents and symmetry properties, but leaves it to us to construct them. On the other hand it has more freedom and its technology is well developed.

At this point there is an important new entry:

the L_∞ symmetry

Toward L_∞ ...

The effective action is

$$W[h] = \sum_{n=1}^{\infty} \frac{1}{n!} \int \prod_{i=1}^{n} d^d x_i \, \frac{d^d p_i}{(2\pi)^d} \, \mathcal{W}^{(n)}_{\mu_1,\ldots,\mu_n}(x_1,p_1,\ldots,x_n,p_n)\, h^{\mu_1}(x_1,p_1)\ldots h^{\mu_n}(x_n,p_n)$$

Varying w.r.t. $h^\mu(x,p)$ one obtains the generalized EoM

$$\begin{aligned}\mathcal{F}_\mu(x,p) \quad &\equiv \quad \sum_{n=0}^{\infty} \frac{1}{n!} \int \prod_{i=1}^{n} d^d x_i \, \frac{d^d p_i}{(2\pi)^d} \, \mathcal{W}^{(n+1)}_{\mu,\mu_1\ldots,\mu_n}(x,p,x_1,p_1,\ldots,x_n,p_n,\epsilon) \\ &\qquad \times h^{\mu_1}(x_1,p_1)\ldots h^{\mu_n}(x_n,p_n) = 0\end{aligned}$$

and from $\delta_\varepsilon \mathcal{W}[h] = 0$ one can prove that

$$\delta_\varepsilon \mathcal{F}_\mu(x,p) = i[\varepsilon(x,p) \stackrel{*}{,} \mathcal{F}_\mu(x,p)]$$

L_∞ symmetry

A *strongly homotopic algebra* L_∞ is defined by vector spaces X_i and (multi)linear maps L_i of degree $d_i = i - 2$ among them, that have to satisfy certain quadratic relations.
In our case we will need only three spaces X_0, X_{-1}, X_{-2} and the complex

$$X_0 \xrightarrow{L_1} X_{-1} \xrightarrow{L_1} X_{-2} \xrightarrow{L_1} 0$$

The degree assignment is as follows: $\varepsilon(x,p)$ has degree 0, $h^\mu(x,p)$ degree -1 and $\mathcal{F}_\mu(x,p)$ degree -2.
The relations are

$$\sum_{i+j=n+1} (-1)^{i(j-1)} \sum_\sigma (-1)^\sigma \epsilon(\sigma;x)\, L_j(L_i(x_{\sigma(1)},\ldots,x_{\sigma(i)}),x_{\sigma(i+1)},\ldots,x_{\sigma(n)}) = 0$$

In detail

$$L_1^2 = 0, \qquad L_1L_2 - L_2L_1 = 0, \qquad L_3L_1 + L_2L_2 + L_1L_3 = 0, \quad \ldots$$

Stasheff, Lada, Hohm, Zwiebach, Gaberdiel,Barnich, Zeitlin, Blumenhagen, Fuchs, Traube,. . .

L_∞: defining L_i

From $\delta_\epsilon h$ one extracts

$$
\begin{aligned}
L_1(\varepsilon)^\mu &= \partial_x^\mu \varepsilon(x,p) \\
L_2(\varepsilon,h)^\mu &= -i[h^\mu(x,p) \overset{*}{,} \varepsilon(x,p)] = -L_2(h,\varepsilon)^\mu \\
L_j(\varepsilon,h,h)^\mu &= 0 \qquad , \qquad j \geq 3
\end{aligned}
$$

From the generalized EoM operator $\mathcal{F}_\mu$ one extracts

$$
\begin{aligned}
\ell_n(h,\ldots,h) &= (-1)^{\frac{n(n-1)}{2}} \int \prod_{i=1}^{n} d^d x_i \, \frac{d^d p_i}{(2\pi)^d} \, \mathcal{W}^{(n+1)}_{\mu,\mu_1\ldots,\mu_n}(x,p,x_1,p_1,\ldots,x_n,p_n,\epsilon) \\
&\quad \times h^{\mu_1}(x_1,p_1)\ldots h^{\mu_n}(x_n,p_n)
\end{aligned}
$$

and then

$$
L_n(h_1,h_2,\ldots,h_n) = \mathrm{Symm}\ \ell_n(h_1,h_2,\ldots,h_n)
$$

L_∞: verifying the relations

One can prove the L_∞ relations using two crucial properties

$$\delta_\varepsilon \gamma\cdot h(x,p) = \gamma\cdot\partial_x \varepsilon(x,p) - i[\gamma\cdot h(x,p) \overset{*}{,} \varepsilon(x,p)] \equiv \gamma\cdot\mathcal{D}_x^* \varepsilon(x,p)$$

and

$$\delta_\varepsilon \mathfrak{F}_\mu(x,p) = i[\varepsilon(x,p) \overset{*}{,} \mathfrak{F}_\mu(x,p)]$$

which can be decomposed as follows

$$\begin{aligned}
& i[\varepsilon \overset{*}{,} \langle\!\langle \mathcal{W}^{(n)}_{\mu\nu_1\ldots\nu_{n-1}} , h^{\mu_1} \ldots h^{\mu_{n-1}} \rangle\!\rangle] \\
& -i \sum_{i=1}^{n-1} \langle\!\langle \mathcal{W}^{(n)}_{\mu\mu_1\ldots\mu_i\ldots\mu_{n-1}} , h^{\mu_1} \ldots [\varepsilon \overset{*}{,} h^{\mu_i}] \ldots h^{\mu_{n-1}} \rangle\!\rangle \\
& -\frac{1}{n} \sum_{i=1}^{n} \langle\!\langle \mathcal{W}^{(n+1)}_{\mu\mu_1\ldots\mu_i\ldots\mu_n} , h^{\mu_1} \ldots \partial_x^{\mu_i} \varepsilon \ldots h^{\mu_n} \rangle\!\rangle = 0
\end{aligned}$$

HS symmetry anomalies

The effect of a HS symmetry transformation

$$\delta_\varepsilon h^\mu(x,p) = \partial_x^\mu \varepsilon(x,p) - i[h^\mu(x,p) \stackrel{*}{,} \varepsilon(x,p)] \equiv \mathcal{D}_x^{*\mu}\varepsilon(x,p)$$

on the effective action $\mathcal{W}$ is

$$\begin{aligned} 0 &= \delta_\varepsilon W[h] \\ &= \sum_{n=1}^{\infty} \frac{1}{(n-1)!} \int \prod_{i=1}^{n} d^d x_i \frac{d^d p_i}{(2\pi)^d} \\ &\quad \times \mathcal{W}^{(n)}_{\mu_1,\ldots,\mu_n}(x_1,p_1,\ldots,x_n,p_n,\epsilon)\, \mathcal{D}_x^{*\mu_1}\varepsilon(x_1,p_1)\, h^{\mu_2}(x_2,p_2)\ldots h^{\mu_n}(x_n,p_n) \end{aligned}$$

But since

$$(\delta_{\varepsilon_2}\delta_{\varepsilon_1} - \delta_{\varepsilon_1}\delta_{\varepsilon_2})\, h^\mu(x,p) \quad = \quad i\,\mathcal{D}_x^{*\mu}[\varepsilon_1 \stackrel{*}{,} \varepsilon_2](x,p)$$

A would-be anomaly $\delta_\varepsilon W[h] = \mathcal{A}[\varepsilon, h]$ must satisfy a consistency condition

$$\delta_{\varepsilon_2}\mathcal{A}[\varepsilon_1, h] - \delta_{\varepsilon_1}\mathcal{A}[\varepsilon_2, h] = \mathcal{A}[[\varepsilon_1 \stackrel{*}{,} \varepsilon_2], h]$$

The L_∞ symmetry is characteristic of many theories:

- Gauge FT's
- CS theories
- Double FT
- Closed SFT… and now….

- Worldline QFT's

We have indications that in order to describe quantum gravitational effect we need a theory with infinite many fields. (Camanho-Edelstein-Maldacena-Zhiboedov, 2014)

What this theory is, string theory, HS theories, … at present we don't know…

A guiding tool could be the L_∞ symmetry

From the previous results it seems that it is sort of incomplete to consider an ordinary field theory (with one single field) in isolation, because its quantization calls immediately for an infinite set of other fields.

It would seem that for a theory to be complete it must contain all the fields it is able to excite upon quantization (involutive). Is this possible or is there an unsurmountable duality between matter and gauge fields?

String field theory may be the favorite playground to answer the previous questions.

So let us try to apply these ideas to SFT.

Effective SFT

The previous results for HS theories prompt us to apply the same approach to open SFT, i.e. to treat free SFT as the previous free theories and try to compute the corresponding effective action. The appropriate action is

$$S = \frac{1}{2g_o} \int \left(\Phi * Q\Phi + \frac{2}{3} \Phi * \Psi * \Phi \right)$$

where Φ is the original string field of SFT and Ψ represents the string field of external sources. This action is on-shell invariant under $\delta\Psi = Q\Lambda$.

Our program is to compute the one-loop effective action following the worldline quantization method.

Worldline quantization for SFT

OSFT relies on the string expansion

$$x(\sigma) = x_0 + \sqrt{2}\sum_{n=1}^{\infty} x_n \cos(n\,\sigma)$$

The dynamical modes are

$$x_n = \frac{i}{\sqrt{2n}}\left(a_n - a_n^\dagger\right), \qquad p_n = -i\frac{\partial}{\partial x_n} = \sqrt{\frac{n}{2}}\left(a_n + a_n^\dagger\right)$$

beside x_0 and its conjugate $p_0 = -i\frac{\partial}{\partial x_0}$.
The string field Φ is a functional of the modes x_n and can be represented as $\Phi\,[x(\sigma)]$ or $\Phi\,[x_n]$.

The worldline (second) quantization consists in Weyl-quantizing the x_n's, treating them as separate worldline particles.

We look for an analogy with the fermion model where the kinetic term $\int \overline{\psi}\gamma \cdot \partial\psi$ can be represented by means of the Moyal product

$$\int d^d x \, \overline{\widetilde{\psi}}(x,p) * \gamma \cdot \partial_x \widetilde{\psi}(x,p), \quad \text{where} \quad \widetilde{\psi}(x,p) = \int d^d y \, \psi \left(x - \frac{y}{2} \right) e^{iy \cdot p}.$$

To this end one normally uses the split string formalism

$$l_{2n-1} = x_{2n-1} + \sum_{k=1}^{\infty} \mathcal{X}_{2n-1.2k} \, x_{2n}, \qquad r_{2n-1} = -x_{2n-1} + \sum_{k=1}^{\infty} \mathcal{X}_{2n-1.2k} \, x_{2n}.$$

where $\mathcal{X}$ is an invertible numerical matrix, with inverse $\mathcal{Y}$. Then we define (symbolically) the partial Fourier transform

$$\widetilde{\Phi}[\mathcal{X}x_e, \mathcal{Y}p_e] = \int [dx_o] e^{-2ip_e \mathcal{Y} \cdot x_o} \Phi[\mathcal{X}x_e + x_o, \mathcal{X}x_e - x_o]$$

and represent the Witten $*$ product by means of the Moyal product

$$\Phi *_W \Psi \longrightarrow \widetilde{\Phi} *_M \widetilde{\Psi}$$

(Bars, Matsuo, Kishimoto, Erler, Douglas, Liu, Moore, Zwiebach, Belov, ...)

The worldline quantization is different. We define the partial Fourier transform

$$\widetilde{\Phi}[\{x_n, p_n\}] = \int [d\{y_n\}] \prod_n e^{ip_n \cdot y_n} \widetilde{\Phi}[\{x_n - \frac{y_n}{2}\}]$$

and use the ordinary Moyal product

$$\widetilde{\Phi}[\{x_n, p_n\}] * \widetilde{\Psi}[\{x_n, p_n\}] = \widetilde{\Phi}[\{x_n, p_n\}] e^{\frac{i}{2}\left(\overleftarrow{\partial}_x \cdot \overrightarrow{\partial}_p - \overleftarrow{\partial}_p \cdot \overrightarrow{\partial}_x\right)} \widetilde{\Psi}[\{x_n, p_n\}]$$

The OSFT model linearly coupled to external sources is

$$S_e = \int [d\{x\}][d\{p\}] \left(\widetilde{\Phi}(L_0 - 1)\widetilde{\Psi} - \frac{2}{3}\widetilde{\Phi} * \Psi * \widetilde{\Phi}\right)$$

What remains to be done is the integration over the 'matter' string field $\widetilde{\Phi}$. We can use the Feynman rules for SFT with propagator

$$\frac{1}{L_0 - 1}$$

Symmetry and perturbative expansion

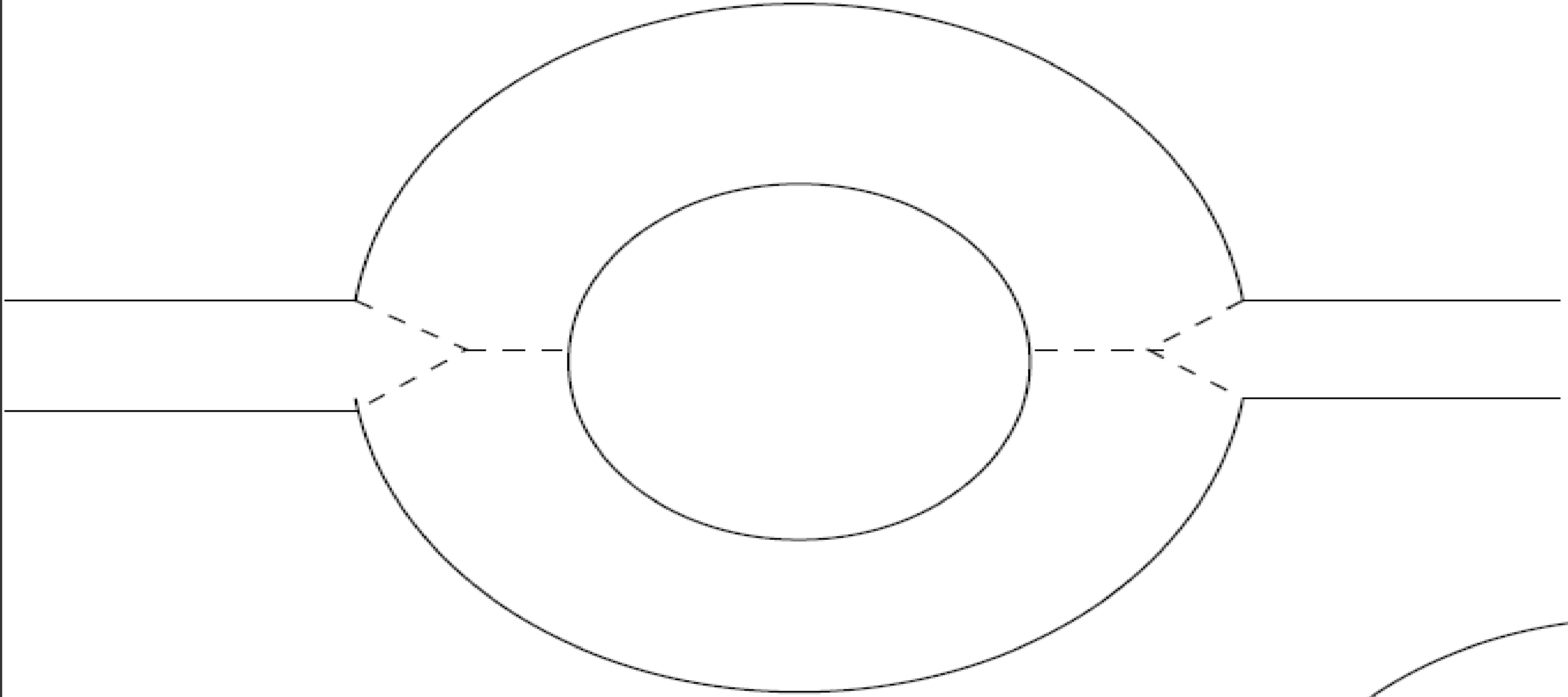
The symmetry induced on the external string field Ψ is

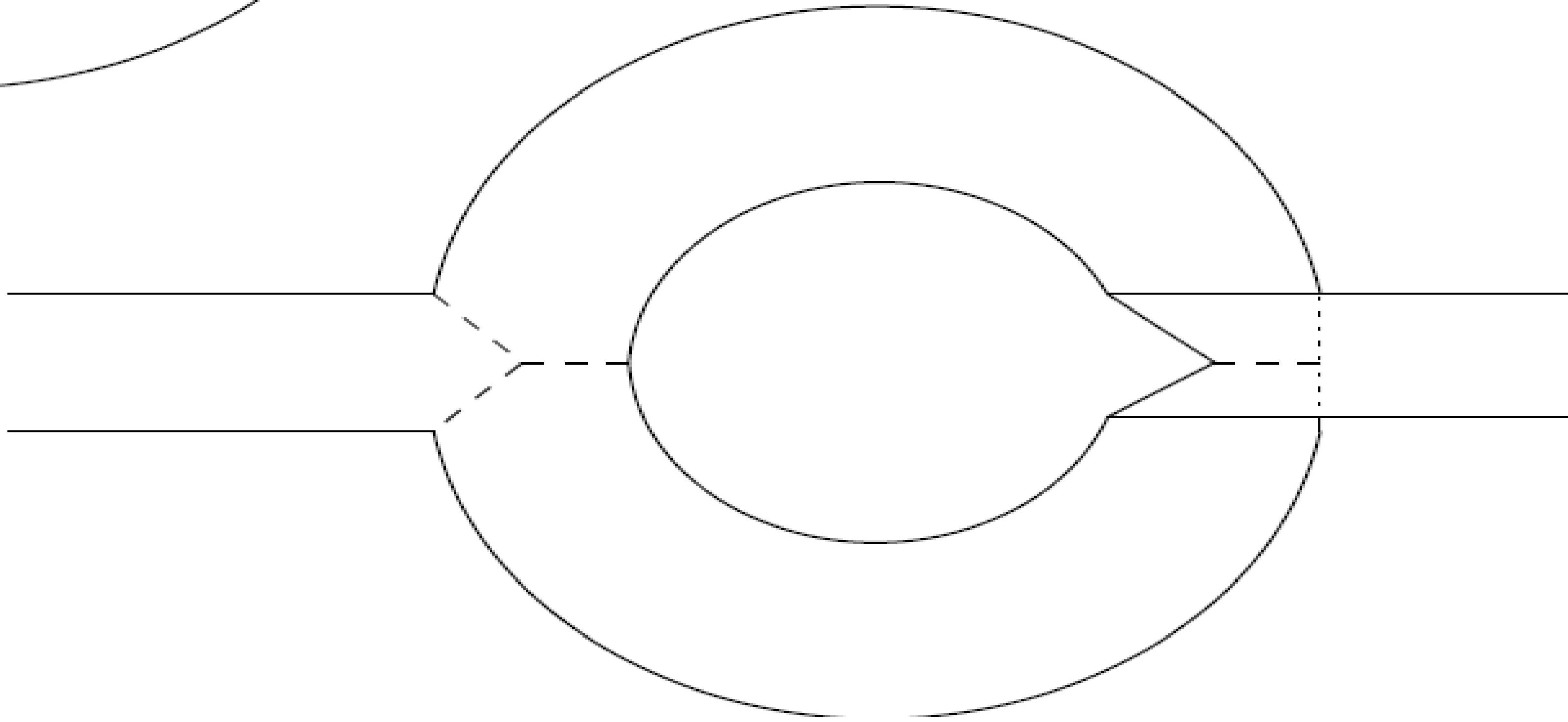
$$\delta_E \Psi[\{x\},\{p\}] = i(L_0 - 1)E[\{x\},\{p\}] - i\Big[\Psi[\{x\},\{p\}], E[\{x\},\{p\}]\Big]$$

The string gauge parameter $E[\{x\},\{p\}]$ contains many more modes than the usual string gauge parameter of SFT.

This leads to another L_∞ symmetry for the effective action.

On the other hand the calculation of the one-loop effective action requires the same Feynman diagrams as the OSFT.





The one-loop spectrum

The analysis of these one-loop amplitudes was carried out long ago. The on-shell case by Lovelace, Cremmer, Sherk, Shapiro, Thorn in the framework of dual models; the off-shell case, using OSFT, by Freedman, Giddings, Shapiro, Thorn, Bluhm, Samuel.

- The one-loop planar diagram has singularities corresponding to emissions of closed tachyons and dilatons.
- The non-planar diagram has on-shell poles corresponding to the closed string modes ($\alpha' = 1$):

$$M^2 = 4n - 4, \qquad n = 0, 1, 2, \ldots$$

- But there are also off-shell poles at

$$M^2 = \frac{1}{2}n - 4, \qquad n = 0, 1, 2, \ldots$$

Concluding:

• there are indications that the so obtained effective action contains to open-closed SFT.

• do the additional poles suggest a larger theory?

Thanks