Theta Expansion of Massive Vertex in Pure Spinor

Subhroneel Chakrabarti (HRI)

Discussion Meeting on String Field Theory and String Phenomenology, HRI

11th February, 2018

Introduction

- This is a part of a series of work in collaboration with Ashoke Sen, Sitender P. Kashyap and Mritunjay Verma.
- Our final goal is to compute scattering of first massive states of SO(32) Heterotic strings. We want to use pure spinor formulation to do this computation.
- The required unintegrated vertex was constructed by Berkovits and Chandia (hep-th/0204121).
- This also requires knowing the integrated vertex for first massive states - Sitender's talk.
- However for the amplitude computation one also needs to know the covariant θ expansions of the superfields solely in terms of the physical fields in the spectrum.
- This was done in arXiv:1706.01196 (SC, Sitender P. Kashyap and Mritunjay Verma) .

A quick review of pure spinor

- Throughout we work in $d = 10$, flat background and work with 16x16 gamma matrices - $(\gamma^m)_{\alpha\beta}$ and $(\gamma^m)^{\alpha\beta}$. $m = 0, 1...9$ and $\alpha = 1, 2...16$.
- A pure spinor is defined as a Majorana-Weyl Spinor satisfying $(\lambda\gamma^m\lambda) = 0$ for all m.
- One starts with the World-sheet CFT given by the action

$$S = \frac{2}{\alpha'} \int d^2z \left(\frac{1}{2}\partial X^m \bar{\partial} X_m + p_\alpha \bar{\partial}\theta^\alpha - w_\alpha \bar{\partial}\lambda^\alpha \right) \qquad (0.1)$$

- λ^α is a bosonic right-handed Majorana-Weyl spinor which satisfies the pure spinor constraint.

Basic ingredients

- The ghost Lorentz and ghost number currents

$$N^{mn} = \frac{1}{2} w_\alpha (\gamma^{mn})^\alpha{}_\beta \lambda^\beta \quad , \qquad J = w_\alpha \lambda^\alpha$$

- The supersymmetric invariant combinations

$$d_\alpha = p_\alpha - \frac{1}{2} \gamma^m{}_{\alpha\beta} \theta^\beta \partial X_m - \frac{1}{8} \gamma^m_{\alpha\beta} \gamma_{m\sigma\delta} \theta^\beta \theta^\sigma \partial \theta^\delta$$

$$\Pi^m = \partial X^m + \frac{1}{2} \gamma^m_{\alpha\beta} \theta^\alpha \partial \theta^\beta$$

- The BRST operator $Q = \oint dz \; \lambda^\alpha(z) d_\alpha(z)$
- Our basic ingredients to construct any vertex are $\Pi^m, d_\alpha, \partial\theta^\alpha$ and $N^{mn}, J, \lambda^\alpha$
- We also need D_α , the supercovariant derivative defined as $D_\alpha = \partial_\alpha + (\gamma^m)_{\alpha\beta} \theta^\beta \partial_m$

Conformal weights and Ghost numbers

- Only λ^α has ghost number $= 1$. All of the rest have ghost number$= 0$.
- Conformal weights for various fields are

Π^m	1
d_α	1
$\partial\theta^\alpha$	1
N^{mn}	1
J	1
λ^α	0

OPEs

The various relevant OPEs are

$$d_\alpha(z)d_\beta(w) = -\frac{\alpha'\gamma^m_{\alpha\beta}}{2(z-w)}\Pi_m(w) \quad , d_\alpha(z)\Pi^m(w) = \frac{\alpha'\gamma^m_{\alpha\beta}}{2(z-w)}\partial\theta^\beta(w)$$

$$d_\alpha(z)V(w) = \frac{\alpha'}{2(z-w)}D_\alpha V(w) \quad , \Pi^m(z)V(w) = -\frac{\alpha'}{(z-w)}\partial^m V(w)$$

$$\Pi^m(z)\Pi^n(w) = -\frac{\alpha'\eta^{mn}}{2(z-w)^2}$$

$$N^{mn}(z)\lambda^\alpha(w) = \frac{\alpha'(\gamma^{mn})^\alpha{}_\beta}{4(z-w)}\ \lambda^\beta(w)$$

$$J(z)J(w) = -\frac{(\alpha')^2}{(z-w)^2} \quad , J(z)\lambda^\alpha(w) = \frac{\alpha'}{2(z-w)}\lambda^\alpha(w)$$

$$N^{mn}(z)N^{pq}(w) = -\frac{3(\alpha')^2}{2(z-w)^2}\eta^{m[q}\eta^{p]n} - \frac{\alpha'}{(z-w)}\left(\eta^{p[n}N^{m]q} - \eta^{q[n}N^{m]p}\right)$$

Unintegrated Vertex

- To find the unintegrated vertex for $m^2 = \frac{1}{\alpha'}$, one starts with the most general ghost no. 1 and conformal weight $= 1$ objects

$$
\begin{aligned}
V \;\; = \;\; & \partial\lambda^\alpha A_\alpha(X,\theta) + :\partial\theta^\beta\lambda^\alpha B_{\alpha\beta}(X,\theta): + :d_\beta\lambda^\alpha C^\beta_\alpha(X,\theta): \\
& + :\Pi^m\lambda^\alpha H_{m\alpha}(X,\theta): + :J\lambda^\alpha E_\alpha(X,\theta): \\
& + :N^{mn}\lambda^\alpha F_{\alpha mn}(X,\theta): \qquad (0.2)
\end{aligned}
$$

- One then solves $QV = 0$ subjected to Gauge condition $V \to V + Q\Omega$

Unintegrated Vertex contd.

- The gauge freedom can be fixed by imposing the following gauge conditions

$$B_{\alpha\beta} \;=\; (\gamma^{mnp})_{\alpha\beta} B_{mnp} \quad , C^{\alpha}_{\;\beta} = (\gamma^{mnpq})^{\alpha}_{\;\beta} C_{mnpq}$$

$$\gamma^{m\alpha\beta} F_{\beta mn} \;=\; 0 \quad , (\gamma^m H_m)_\alpha = 0$$

- The solution given in terms of a single superfield B_{mnp} is

$$H_{s\alpha} = \frac{3}{7} (\gamma^{mn})_\alpha{}^\beta D_\beta B_{mns} \;, \quad C_{mnpq} = \frac{1}{2} \partial_{[m} B_{npq]} \;, E_\alpha = 0 = A_\alpha$$
$$F_{\alpha mn} = \frac{1}{8} \left(7 \partial_{[m} H_{n]\alpha} + \partial^q (\gamma_{q[m})_\alpha{}^\beta H_{n]\beta} \right)$$

- and $\left(\partial^2 - \frac{1}{\alpha'}\right) B_{mnp} = 0$

Field Content of First Massive level

- There are 128 Bosonic (84+44) and 128 Fermionic d.o.f - captured by spin 2 Bosonic fields b_{mnp}, g_{mn} and spin $\frac{3}{2}$ Fermionic field ψ^s_α
- They satisfy

$$\eta^{mn} g_{mn} = 0 \quad \partial^m g_{mn} = 0 \quad \partial^m b_{mnp} = 0$$
$$\partial^m \psi_{m\alpha} = 0 \quad \gamma^{m\alpha\beta} \psi_{m\beta} = 0$$

- So any Superfield describing this multiplet (eg. B_{mnp}) must contain all these physical fields.
- Our goal is 3-fold -
  1. Give a fully covariant θ expansion
  2. Give an expansion solely in terms of the physical fields
  3. Give a systematic procedure to do the θ expansion so it can be done upto all orders without any further input.

Proof that B_{mnp} contains the physical d.o.f

- Berkovits et. al. did the rest frame analysis to exhibit B_{mnp} indeed has the correct d.o.f
- Using SUSY transformation properties they argued

$$D_\alpha B^{abc} = 12(\gamma^{[ab}\Psi^{c]})_\alpha; \quad B^{0ab} = 0; \quad H^c_\alpha = -72\,\Psi^c_\alpha,$$

- Also, Ψ^c_α is an arbitrary tensor-spinor superfield satisfying

$$(\gamma_a)^{\beta\alpha}\Psi^a_\alpha = 0$$

- Further they defined a superfield G_{mn} as

$$G_{ab} \equiv 2\; D_\alpha \gamma^{\alpha\beta}_{(a}\Psi_{b)\beta} \qquad , \qquad \eta_{ab}G^{ab} = 0$$

- The physical fields are now contained as

$$g^{ab} \equiv G^{ab}\Big|_{\theta=0}; \quad b_{abc} \equiv B_{abc}\Big|_{\theta=0}; \quad \psi^a_\alpha = \Psi^a_\alpha\Big|_{\theta=0}$$

Systematic Procedure for θ Expansion

- First of all consider the covariant version of the 3 superfields : B_{mnp}, G_{mn} and Ψ^m_α
- Promote all the algebraic constraints on their zeroth component to the whole superfield, i.e.

$$(\gamma^m)^{\alpha\beta}\Psi_{s\beta} = 0 \quad ; \quad k^m\Psi_{m\beta} = 0 \quad ; \quad k^m B_{mnp} = 0 \quad ; \quad k^m G_{mn} = 0 \ \& \ \eta^{mn} G_{mn} = 0$$

- Find Differential Constraints which will lead to following recursive structure

$$\begin{aligned} D^{(\ell+1)}\Psi_{s\beta} &\sim D^\ell G_{sm} + D^\ell B_{mnp} \\ D^\ell B_{mnp} &\sim D^{(\ell-1)}\Psi_{s\beta} \\ D^\ell G_{mn} &\sim D^{(\ell-1)}\Psi_{s\beta} \end{aligned}$$

- Where $DV \equiv (\theta^\alpha D_\alpha V)\big|_{\theta=0}$,

Our Result

- Let us first quote the final result right away

$$D_\alpha \Psi_{s\beta} = \frac{1}{16} G_{sm} \gamma^m_{\alpha\beta} + \frac{i}{24} k_m B_{nps} (\gamma^{mnp})_{\alpha\beta} - \frac{i}{144} k^m B^{npq} (\gamma_{smnpq})_{\alpha\beta}$$

$$D_\alpha B_{mnp} = 12(\gamma_{[mn} \Psi_{p]})_\alpha + 24\alpha' k^t k_{[m} (\gamma_{|t|n} \Psi_{p]})_\alpha$$

$$D_\alpha G_{sm} = 16 i k^p (\gamma_{p(s} \Psi_{m)})_\alpha$$

- Consistency checks

  1. Must reduce to Berkovits and Chandia's result at rest frame.
  2. Must be consistent with the solutions obtained for $QV = 0$

  .

- Let us consider $D_\alpha \Psi_{s\beta}$ first. We can decompose it to rewrite

$$D_\alpha \Psi_{s\beta} = S_{s;m}(\gamma^m)_{\alpha\beta} + A_{s;mnp}(\gamma^{mnp})_{\alpha\beta} + S_{s;mnpqr}(\gamma^{mnpqr})_{\alpha\beta}$$

note,

$$\gamma^{mnpqr}_{\alpha\beta} = \frac{1}{5!}\epsilon^{mnpqrstuvw}(\gamma_{stuvw})_{\alpha\beta}$$
$$\implies (\gamma_{mnpqr})_{\alpha\beta} = \frac{1}{5!}\epsilon_{mnpqrstuvw}(\gamma^{stuvw})_{\alpha\beta}$$

Thus only the self dual part of $S_{s;mnpqr}$ will be relevant

$$S_{s;mnpqr} = -\frac{1}{5!}\epsilon_{mnpqrtuvwx} S_{s;}{}^{tuvwx}$$
$$\implies S_{s;}{}^{mnpqr} = -\frac{1}{5!}\epsilon^{mnpqrtuvwx} S_{s;tuvwx}$$

Fields	Number of Components	Irreducible Decomposition
$\boldsymbol{S_{s;m}}$		
$S_{0,0}$	1	$\mathbf{1}$
$S_{0,a}$	9	$\mathbf{9}$
$S_{a,0}$	9	$\mathbf{9}$
$S_{a,b}$	9×9	$\mathbf{1} \oplus \mathbf{36} \oplus \mathbf{44}$
$\boldsymbol{A_{s;mnp}}$		
$A_{0,0ab}$	${}^9C_2 = 36$	$\mathbf{36}$
$A_{0,abc}$	${}^9C_3 = 84$	$\mathbf{84}$
$A_{a,bc0}$	$9 \times {}^9C_2 = 9 \times 36$	$\mathbf{9} \oplus \mathbf{84} \oplus \mathbf{231}$
$A_{a,bcd}$	$9 \times {}^9C_3 = 9 \times 84$	$\mathbf{36} \oplus \mathbf{126} \oplus \mathbf{594}$
$\boldsymbol{S_{s;mnpqr}}$		
$S_{0,0abcd}$	${}^9C_4 = 126$	$\mathbf{126}$
$S_{0,abcde}$	${}^9C_5 = 126$	$\mathbf{126}$
$S_{a,bcde0}$	$9 \times {}^9C_4 = 9 \times 126$	$\mathbf{84} \oplus \mathbf{126} \oplus \mathbf{924}$
$S_{a,bcdef}$	$9 \times {}^9C_5 = 9 \times 126$	$\mathbf{84} \oplus \mathbf{126} \oplus \mathbf{924}$

Table 2.1: Irreducible decomposition of the fields $S_{s;m}$, $A_{s;mnp}$ and $S_{s;mnpqr}$

Group Theory contd.

- The idea is to systematically impose covariant conditions on $S_{s;m}, A_{s;mnp}$ etc. so as to keep only the physical representations **44** and **84**.
- $k^s\Psi_{s\alpha} = 0$ becomes

$$k^s S_{s;m} = 0 \qquad k^s A_{s;mnp} = 0 \qquad k^s S_{s;mnpqr} = 0$$

- **36** , **84** of $A_{s;mnp}$ and the first two **126** in $S_{s;mnpqr}$ are therefore removed.
- $S_{0;m} = 0$ allows us to get rid of **1** and one of **9**
- Self-duality of $S_{s;mnpqr}$ implies we only need to consider one among $S_{a;bcdef}$ or $S_{a;bcde0}$.

Removing non-physical irreps

- $(\gamma^m \Psi_m)_\beta = 0$ constraint gives

$$S^s{}_s = 0$$
$$S_{[m,n]} + 3A^s{}_{,smn} = 0$$
$$A_{[m,npq]} - 10S^s{}_{;smnpq} = 0$$

- This encourages us to guess $S_{[sm]} = 0$ and $A_{(sm)np} = 0$
- This gets rid of all the remaining non-physical irreps in S_{sm}
- For $A_{s;mnp}$ we write

$$A_{a,bc0} = A_{(a,b)c0} \oplus A_{[a,bc]0}$$
$$A_{a,bcd} = A_{(a,b)cd} \oplus A_{[a,bcd]}$$

The $A_{(a,b)c0}$ contains **9** and **231**. $A_{(a,b)cd}$ contains **36** and **594**. All of these are gotten rid by our guess. The component $A_{[a,bc]0}$ represents the desired **84**.

Removing non-physical irreps

- To remove the remaining unphysical irrep, we set $A_{[abcd]} = 0$ and covariantize the constraint as

$$P^{mm'} P^{nn'} P^{pp'} P^{qq'} A_{[m';n'p'q']} = 0$$

where, $P^{mn} = \eta^{mn} + \alpha' k^m k^n$ is a projector that projects onto completely spatial part.

- This simplifies to

$$A_{[s;mnp]} + 3\alpha' A_{[s;|t|mn} k_{p]} k^t = 0$$

- In rest frame this is precisely just $A_{[abcd]} = 0$

Removing non-physical irreps

- The remaining irreps of $S_{s;mnpqr}$ are

$$S_{a,bcde0} \quad = \quad S_{(a,b)cde0} \oplus S_{[a,bcde]0}$$

  The 2nd term represents **126** whereas 1st term represents **84** and **924**.

- Note that ${S^s}_{;sabc0}$ is precisely the **84**. Further $A_{[a,bc]0} = -10 {S^s}_{;sabc0}$. So we have only one **84**.
- In addition ${S^s}_{,sabcd} = 0$. This kills the remaining **126** of $S_{s,mnpqr}$.
- One can get rid of the remaining **924** as well.
- But the bottomline is the physical d.o.f are contained in the components $A_{[s,mnp]}$ and $A_{[m,np]s}$.

Obtaining the answer

- Consistency with $QV = 0$ implies

$$A_{[s,mnp]} = -\frac{1}{12} C_{smnp} = -\frac{i}{24} k_{[s} B_{mnp]}$$

and

$$S_{s,mnpqr} = \frac{1}{12} \eta_{s[m} A_{n,pqr]} - \frac{1}{1440} \epsilon_{smnpqr}{}^{tuvw} A_{t,uvw}$$

- writing an ansatz for $A_{s;mnp}$ as

$$A_{s,mnp} = -\frac{ia}{96} k_s B_{mnp} - \frac{ib}{96} k_{[m} B_{np]s}$$

- one can determine a and b by imposing various constraints

$$A_{s,mnp} = \frac{i}{24} k_{[m} B_{np]s}$$

The final answer

- Therefore

$$S_{s,mnpqr}(\gamma^{mnpqr})_{\alpha\beta} = \frac{1}{6} A_{n,pqr}(\gamma_s{}^{npqr})_{\alpha\beta}$$

- Using this, we obtain

$$\begin{aligned} D_\alpha \Psi_{s\beta} \quad = \quad & \frac{1}{16} G_{s,m}\gamma^m_{\alpha\beta} + \frac{i}{24} k_{[m} B_{np]s}\gamma^{mnp}_{\alpha\beta} \\ & - \frac{i}{144} k_{[n} B_{pqr]}(\gamma_s{}^{npqr})_{\alpha\beta} \end{aligned}$$

- One can do similar group theoretic analysis for other 2 relations.
- Alternatively with this relation and proper covariantization of rest frame result one can also arrive at the same results.

The θ expansions

$$
\begin{aligned}
\Psi_{s\beta} \quad = \quad & \psi_{s\beta} + \frac{1}{16}(\gamma^m\theta)_\beta\; g_{sm} - \frac{i}{24}(\gamma^{mnp}\theta)_\beta k_m b_{nps} - \frac{i}{144}(\gamma_s{}^{npqr}\theta)_\beta k_n b_{pqr} \\
& - \frac{i}{2}k^p(\gamma^m\theta)_\beta(\psi_{(m}\gamma_{s)p}\theta) - \frac{i}{4}k_m(\gamma^{mnp}\theta)_\beta(\psi_{[s}\gamma_{np]}\theta) - \frac{i}{24}(\gamma_s{}^{mnpq}\theta)_\beta k_m(\psi_q\gamma_{np}\theta) \\
& - \frac{i}{6}\alpha' k_m k^r k_s(\gamma^{mnp}\theta)_\beta(\psi_p\gamma_{rn}\theta) + \frac{i}{288}\alpha'(\gamma^{mnp}\theta)_\beta k_m k^r k_s(\theta\gamma^q{}_{nr}\theta)\; g_{pq} \\
& - \frac{i}{192}(\gamma^{mnp}\theta)_\beta k_m(\theta\gamma^q{}_{[np}\theta)g_{s]q} - \frac{i}{1152}(\gamma_{smnpq}\theta)_\beta k^m(\theta\gamma_{npt}\theta)\; g^{qt} \\
& - \frac{i}{96}k^p(\gamma^m\theta)_\beta(\theta\gamma_{pq(s}\theta)\; g_{m)q} - \frac{1}{1728}(\gamma^{mnp}\theta)_\beta k_m(\theta\gamma^{tuvw}{}_{nps}\theta)k_t b_{uvw} \\
& - \frac{1}{864\alpha'}(\gamma_s\theta)_\beta(\theta\gamma^{npq}\theta)b_{npq} - \frac{1}{10368}(\gamma_s{}^{mnpq}\theta)_\beta k_m(\theta\gamma_{tuvwnpq}\theta)k^t b^{uvw} \\
& - \frac{1}{864}(\gamma^m\theta)_\beta(\theta\gamma^{npq}\theta)b_{npq}k_m k_s - \frac{1}{576}(\gamma_{smnpq}\theta)_\beta k^m(\theta\gamma^{tun}\theta)b_u{}^{pq}k_t \\
& - \frac{1}{96\alpha'}(\gamma^m\theta)_\beta(\theta\gamma^{qr}{}_{(s}\theta)b_{m)rq} + \frac{1}{96}(\gamma^m\theta)_\beta(\theta\gamma^{nqr}\theta)k_n k_{(s}b_{m)qr} \\
& + \frac{1}{96}(\gamma^{mnp}\theta)_\beta k_m(\theta\gamma^r{}_{q[n}\theta)b_{ps]r}k^q \quad + \quad O(\theta^4)
\end{aligned}
$$

$$
\begin{aligned}
B_{\alpha\beta} \quad = \quad \gamma^{mnp}_{\alpha\beta}\Big[& b_{mnp} + 12(\psi_p\gamma_{mn}\theta) + 24\alpha' k^r k_m(\psi_p\gamma_{rn}\theta) + \frac{3}{8}(\theta\gamma_{mn}{}^{q}\theta)\, g_{pq} - \frac{3i}{4}(\theta\gamma^{tu}{}_{m}\theta)k_t b_{unp} \\
& + \frac{3}{4}\alpha' k^r k_m(\theta\gamma_{rn}{}^{q}\theta)\, g_{pq} - \frac{i}{24}(\theta\gamma_{tuvwmnp}\theta)k^t b^{uvw} - \frac{1}{6} ik_s\, (\psi_v\gamma_{tu}\theta)\, (\theta\gamma_{stuvmnp}\theta) \\
& -4i\alpha k_s k_t k_m\, (\theta\gamma_{tun}\theta)\, (\psi_p\gamma_{su}\theta) + ik_s\, (\theta\gamma_{tmn}\theta)\, (\psi_p\gamma_{st}\theta) + ik_s\, (\theta\gamma_{tmn}\theta)\, (\psi_t\gamma_{sp}\theta) \\
& +2ik_s\, (\theta\gamma_{stm}\theta)\, (\psi_n\gamma_{tp}\theta) - ik_s\, (\theta\gamma_{stm}\theta)\, (\psi_t\gamma_{np}\theta) \; + \; O(\theta^4)\Big]
\end{aligned}
$$

- Straightforward calculation in principle but tedious to do by hand.
- A Mathematica code has been developed for this purpose and it has been benchmarked with our known results up to θ^3.

Calculating Amplitudes

- Let us consider a toy example of tree-level 3 point function - 2 Gluons and 1 b_{mnp} field.
- The tree-level amplitude prescription in pure spinor is

$$\mathcal{A}_N = \langle V^1 V^2 V^3 \int U^4 \cdots \int U^N \rangle$$

- For our example it is simply

$$\mathcal{A}_3 = \langle V^1 V^2 V^3 \rangle$$

- the pure spinor measure is normalized as

$$\langle (\lambda\gamma^m\theta)(\lambda\gamma^n\theta)(\lambda\gamma^p\theta)(\theta\gamma_{mnp}\theta) \rangle = 1$$

- The evaluation of amplitudes requires several Pure Spinor superspace identities, given by Berkovits, Mafra.

Steps to compute the amplitudes

- Write down the θ expansion of each vertex operator V^i to the desired order.
- From the product, we keep only those terms which have exactly five factors of θ.
- Therefore each term in the product $V^1 V^2 V^3$ always has exactly three factors of λ^α.
- Express every physical field in terms of its polarization and plane wave basis. For example,

$$A^{m_1 \ldots m_n}_{\alpha_1 \ldots \alpha_k}(X) = a^{m_1 \ldots m_n}_{\alpha_1 \ldots \alpha_k} \, e^{ik \cdot X}$$

  where $a^{m_1 \cdots m_n}_{\alpha_1 \ldots \alpha_k}$ is the constant tensor-spinor polarization corresponding to the physical field $A^{m_1 \cdots m_n}_{\alpha_1 \ldots \alpha_k}(X)$.
- Compute the correlation function $\langle : e^{ik_1 \cdot X} :: e^{ik_2 \cdot X} :: e^{ik_3 \cdot X} f(X^m) : \rangle_{Disk}$ separately, where typically $f(X^m)$ is just a product of various ∂X^m.
- The only thing left to compute at this stage is the correlation function in Pure Spinor Superspace.

Computing the amplitudes

- The unintegrated massless vertex is given by $\lambda^\alpha A_\alpha$. It's θ expansion is

$$\begin{aligned} A_\alpha(X,\theta) &= \frac{1}{2}a_m(\gamma^m\theta)_\alpha - \frac{1}{3}(\xi\gamma_m\theta)(\gamma^m\theta)_\alpha - \frac{1}{32}F_{mn}(\gamma_p\theta)_\alpha(\theta\gamma^{mnp}\theta) \\ &+\frac{1}{60}(\gamma^m\theta)_\alpha(\theta\gamma^{mnp}\theta)(\partial_n\xi\gamma_p\theta) + \frac{1}{1152}(\gamma^m\theta)_\alpha(\theta\gamma^{mrs}\theta)(\theta\gamma^{spq}\theta)\partial_r F_{pq} + \cdots \end{aligned}$$

$V_a^{(1)}$	$V_a^{(2)}$	V_b
1	1	3
1	3	1
3	1	1

Table: The possible ways of getting five θs from product of 3 Vertex operators

- Let us see how it works for one such term
- we choose our polarizations as

$$a_m^{(1)}(X) = e_m^{(1)} e^{ip_1 \cdot X}, \quad a_m^{(2)}(X) = e_m^{(2)} e^{ip_2 \cdot X}, \quad b_{mnp} = e_{mnp} e^{ik \cdot X}$$

- the polarization tensors satisfy the transversality conditions

$$e_m^{(1)} p_1^m = 0 \qquad , \qquad e_m^{(2)} p_2^m = 0 \qquad , \qquad e_{mnp} k^m = 0$$

- consider 1st term of $\Pi^m \lambda^\alpha H_{m\alpha}$
- Only the term containing ∂X^m in Π^m will be relevant

computing the amplitude

$$I = \frac{1}{1920\alpha'} \epsilon^{mnp} e_p^{(1)} e_n^{(2)} \Gamma_m$$

using

$$\langle(\lambda\gamma^{pqr}\theta)(\lambda\gamma_m\theta)(\lambda\gamma_n\theta)(\theta\gamma_{stu}\theta)\rangle = \frac{1}{70}\delta^{[p}_{[m}\eta_{n][s}\delta^q_t\delta^{r]}_{u]}$$

with the world-sheet correlator Γ_m involving the X fields

$$\left\langle : e^{ip_1\cdot X(z_1)} :: e^{ip_2\cdot X(z_2)} :: e^{ik\cdot X(z_3)}\partial X^m(z_3) : \right\rangle = i\alpha' \left(\frac{p_1^m z_{23} + p_2^m z_{13}}{z_{12}}\right)$$

where, $z_{ij} = z_i - z_j$

$$
\begin{aligned}
I &= \frac{i}{1920} e^{mnp} e_p^{(1)} e_n^{(2)} \left(\frac{(p_1)_m z_{23} + (p_2)_m z_{13}}{z_{12}} \right) \\
&= \frac{i}{1920} e^{mnp} e_p^{(1)} e_n^{(2)} (p_2)_m \left(\frac{-z_{23} + z_{13}}{z_{12}} \right) \\
&= \frac{i}{1920} e^{mnp} e_p^{(1)} e_n^{(2)} (p_2)_m
\end{aligned}
$$

- This way one needs to compute all the terms
- Terms involving d_α has to be handled carefully, since $d_\alpha(z)\theta^\beta(w) \sim \frac{\alpha' \delta_\alpha^\beta}{2(z-w)}$ and one θ can get absorbed by one d_α
- Once again, straightforward computation, but tedious.
- A Cadabra code is under development for computing any amplitude given external states.

Conclusion

- We have given a systematic way to perform fully covariant θ expansion of massive vertex solely in terms of the physical fields.
- The basic strategy seems independent of the mass level in question, so one should be able to replicate this for higher mass levels.
- With the θ expansion completely fixed, this will ensure there are no new inputs are required for performing the θ expansion of the integrated vertex.
- With the integrated vertex now constructed as well we are ready to compute scattering of massive states and hope to report the results in near future.

Thank you for listening!