S-duality in $\mathcal{N} = 1$ orientifold SCFTs

MAX-PLANCK-GESELLSCHAFT

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Montonen-Olive $\mathcal{N}=4$ (S-)duality

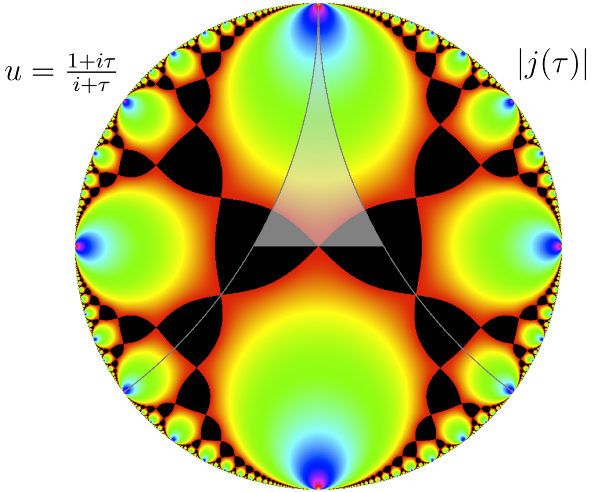
Given a 4d $\mathcal{N}=4$ field theory with gauge group G and gauge coupling $\tau = \theta + i/g^2$ (*), there is a completely equivalent description with gauge group $G^\vee$ and coupling $-1/\tau$ (for $\theta = 0$ this is $g \leftrightarrow 1/g$). Examples:

G	$G^\vee$
$U(1)$	$U(1)$
$U(N)$	$U(N)$
$SU(N)$	$SU(N)/\mathbb{Z}_N$
$SO(2N)$	$SO(2N)$
$SO(2N+1)$	$Sp(2N)$

Very non-perturbative duality, exchanges **electrically charged operators** with **magnetically charged ones**.

(*) I will not describe global structure or line operators here.

S-duality



In this representation $\tau \to -1/\tau$ is $u \to -u$.

S-duality in $\mathcal{N} < 4$

Three main ways of generalizing $\mathcal{N} = 4$ S-duality:

- Trace what relevant susy-breaking deformations do in different duality frames. [Argyres, Intriligator, Leigh, Strassler '99]...
- Make a guess [Seiberg '94].
- Identify some higher principle behind $\mathcal{N} = 4$ S-duality, and find backgrounds with less susy that follow the same principle.
  - $(2, 0)$ 6d theory on $T^2 \to$ Riemann surfaces. [Gaiotto '09], ..., [Gaiotto, Razamat '15], [Hanany, Maruyoshi '15], ...
  - **Field theory S-duality from IIB S-duality.**

Field theories from solitons

One way to construct four dimensional field theories from string theory is to build solitons with a four dimensional core. These can be constructed in type IIB string theory via $D3$ branes.

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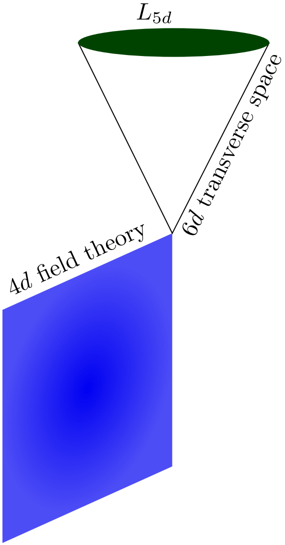


Field theories from solitons

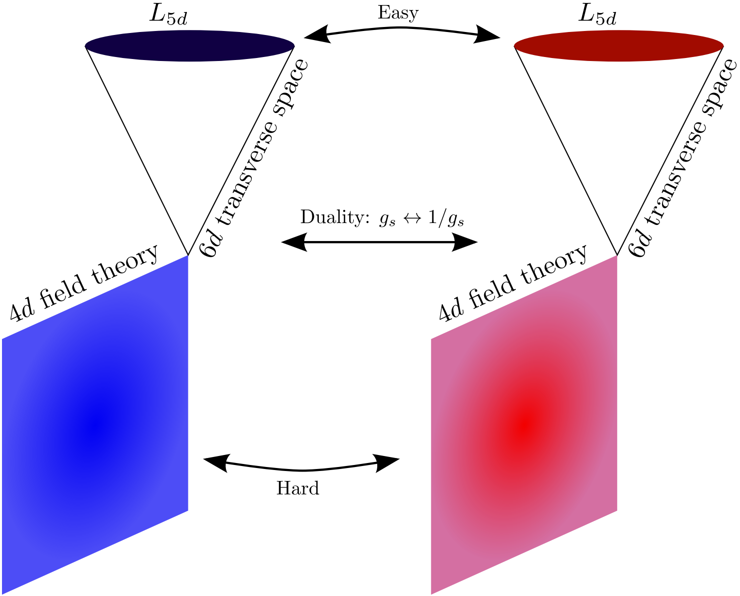
Key idea [Witten '98]

Since the resulting theory is determined by the geometry, one can determine robust results without knowing much of the dynamical details of the duality acting on the core of the soliton.

One just needs to know how the duality acts at infinity.

We then reconstruct the dual theory as that living in the soliton with the right (dual) charge as infinity.

The duality as seen from string theory



Montonen-Olive duality from string theory

The charges of the O3 plane are classified by the cohomology on the $S^5/\mathbb{Z}_2 = \mathbb{RP}^5$ that surrounds the configuration (*). For fields even under the orientifold action (F_5), we have:

$$H^\bullet(\mathbb{RP}^5, \mathbb{Z}) = \{\mathbb{Z}, 0, \mathbb{Z}_2, 0, \mathbb{Z}_2, \mathbb{Z}\},$$

while for fields odd under the orientifold action (H_3, F_3):

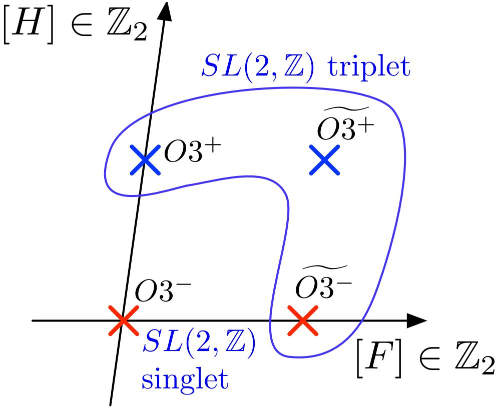
$$H^\bullet(\mathbb{RP}^5, \widetilde{\mathbb{Z}}) = \{0, \mathbb{Z}_2, 0, \mathbb{Z}_2, 0, \mathbb{Z}_2\}.$$

This is *(co)homology with local coefficients*. Working on the S^5 covering space $k \otimes C \simeq \gamma k \otimes \gamma C$. For coefficients in $\mathbb{Z}$ we have $\gamma k = k$ while for coefficients in $\widetilde{\mathbb{Z}}$ we have $\gamma k = -k$. Ordinary (co)homology theory otherwise: $H^\bullet = \ker \partial / \operatorname{im} \partial$.

(*) Well, not really. [Bergman, Gimon, Sugimoto '01]

Montonen-Olive duality from string theory

[Witten '98]



Under S-duality

$$\widetilde{O3}^- \longleftrightarrow O3^+ \quad : \quad SO(2N+1) \longleftrightarrow USp(2N)$$

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Beyond $\mathcal{N} = 4$

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New $\mathcal{N} = 1$ dualities

1. Engineer interesting $\mathcal{N} = 1$ theories in IIB.
2. Figure out the charges characterizing the configuration.
3. Read the effect of S-duality on the charges.
4. Reconstruct the dual $\mathcal{N} = 1$ theories from the dual charges.

Results

We have recently completed this program for a large class of $\mathcal{N} = 1$ SCFTs. They are the ones arising from D3 branes probing orientifolds of singularities. Some simplifying assumptions:

- The geometry is toric, before and after orientifolding. (So I can use the brane tiling formalism of [Franco, Hanany, Martelli, Sparks, Vegh, Wecht '05].)
- The singularity is isolated.
- The orientifold fixed point is isolated.

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Note that despite being $\mathcal{N} = 1$ dualities in 4d, the set of dualities we find here is (essentially) disjoint from known Seiberg dualities.

The $\mathbb{C}^3/\mathbb{Z}_3$ orbifold

The isolated orientifold of $\mathbb{C}^3/\mathbb{Z}_3$ has a horizon manifold

$$X = \mathbb{RP}^5/\mathbb{Z}_3 \sim (S^5/\mathbb{Z}_3)/\widetilde{\mathbb{Z}_2} \equiv Y/\widetilde{\mathbb{Z}_2}\,.$$

It is easier to work in homology and use Poincaré duality

$$H^i(X,\widetilde{\mathbb{Z}}) = H_{\dim(X)-i}(X,\widetilde{\mathbb{Z}})\,.$$

We are thus looking for elements of $H_2(X,\widetilde{\mathbb{Z}})$. This can be conveniently computed using a long exact sequence: [Hatcher]

$$\ldots \longrightarrow H_i(X,\widetilde{\mathbb{Z}}) \longrightarrow H_i(Y,\mathbb{Z}) \xrightarrow{p_*^i} H_i(X,\mathbb{Z}) \longrightarrow H_{i-1}(X,\widetilde{\mathbb{Z}}) \longrightarrow H_{i-1}(Y,\mathbb{Z}) \xrightarrow{p_*^{i-1}} H_{i-1}(X,\mathbb{Z}) \longrightarrow \ldots$$

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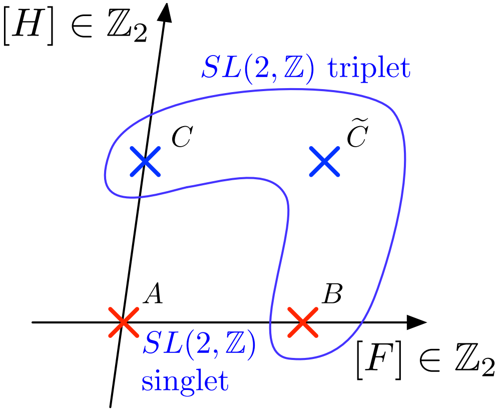
$$H_\bullet(X,\widetilde{\mathbb{Z}}) = \{\mathbb{Z}_2, 0, \mathbb{Z}_2, 0, \mathbb{Z}_2, 0\}$$

$2^2 = 4$ choices of torsion $\implies$ $SL(2,\mathbb{Z})$ singlet plus triplet.

Phases of the $(\mathbb{C}^3/\mathbb{Z}_3)/(\widetilde{\mathbb{Z}_2})$ orientifold



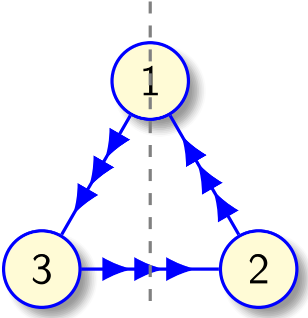
Orientifolding $\mathbb{C}^3/\mathbb{Z}_3$

Orbifolding $\mathcal{N}=4$ duality

Consider the orientifold action with generators $\{\mathcal{R}, \mathcal{I}\,\Omega(-1)^{F_L}\}$:

$$\mathcal{R} : (x, y, z) \longrightarrow (\omega x, \omega y, \omega z)$$
$$\mathcal{I} : (x, y, z) \longrightarrow (-x, -y, -z)$$

with $\omega = \exp(2\pi i/3)$.



A $\mathcal{N}=1$ duality

	$USp(\widetilde{N}+4)$	$SU(\widetilde{N})$	$SU(3)$	$U(1)_R$	$\mathbb{Z}_3$
A^i	$\overline{\square}$	$\square$	$\square$	$\frac{2}{3}-\frac{2}{\widetilde{N}}$	1
B^i	1	$\overline{\square\square}$	$\square$	$\frac{2}{3}+\frac{4}{\widetilde{N}}$	-2

(here $\widetilde{N} \in 2\mathbb{Z}$) is dual to

	$SO(N-4)$	$SU(N)$	$SU(3)$	$U(1)_R$	$\mathbb{Z}_3$
A^i	$\overline{\square}$	$\square$	$\square$	$\frac{2}{3}+\frac{2}{N}$	1
B^i	1	$\overline{\begin{smallmatrix}\square\\\square\end{smallmatrix}}$	$\square$	$\frac{2}{3}-\frac{4}{N}$	-2

in both cases with $W = \frac{1}{2}\lambda\epsilon_{ijk}\mathrm{Tr}\,A^iA^jB^k$.

Global anomalies, the moduli spaces, SCIs and the spectrum of operators match if $\boxed{\widetilde{N} = N-3}$. (D3 charge conservation.)

Superconformal index matching

A very powerful and refined indicator of duality comes from putting the theory on $S^3 \times \mathbb{R}$, and computing the index [Romelsberger '05], [Kinney, Maldacena, Minwalla, Raju '05]:

$$\mathcal{I}(t,x,f) = \int dg \, \mathrm{Tr}\, (-1)^F e^{-\beta\mathcal{H}} t^{\mathcal{R}} x^{2\overline{J}_3} f g \,, \tag{1}$$

with $2\mathcal{H} = \{Q, Q^\dagger\}$. Romelsberger gave a procedure for computing the index from weak coupling quantities. Start with the "letter":

$$i_{\mathcal{T}}(t,x,g,f) = \frac{(2t^2 - t(x+x^{-1}))\chi_{\mathsf{Adj}}(g)}{(1-tx)(1-tx^{-1})} + \frac{\sum_i \left(t^{r_i} \chi_{R_G^i}(g)\, \chi_{R_F^i}(f) - t^{2-r_i} \chi_{\overline{R_G^i}}(g)\, \chi_{\overline{R_F^i}}(f) \right)}{(1-tx)(1-tx^{-1})} \,.$$

and then take the plethystic exponential:

$$\mathcal{I}_{\mathcal{T}}(t,x,f) = \int dg \, \exp\left[\sum_{k=1}^{\infty} \frac{1}{k} i_{\mathcal{T}}(t^k, x^k, g^k, f^k) \right] \,.$$

Superconformal index matching

For $SO(3) \times SU(7) \leftrightarrow USp(8) \times SU(4)$ we get:

$$\begin{aligned}\mathcal{I}_{SO/USp}(t,x,f) = 1 &+ t^{\frac{2}{3}}\left[\chi_{0,2}(f) + \chi_{4,0}(f)\right] \\ &+ t^{\frac{4}{3}}\left[2\chi_{0,4}(f) + 2\chi_{2,0}(f) + \chi_{3,1}(f) + 2\chi_{4,2}(f) + \chi_{8,0}(f)\right] \\ &+ t^{\frac{5}{3}}(x + x^{-1})\left[\chi_{0,2}(f) + \chi_{4,0}(f)\right] \\ &+ t^2\big[3\chi_{0,6}(f) + \chi_{12,0}(f) + \chi_{1,4}(f) + 5\chi_{2,2}(f) + 3\chi_{3,3}(f) \\ &\quad + 2\chi_{4,1}(f) + 3\chi_{4,4}(f) + \chi_{5,2}(f) + 4\chi_{6,0}(f) + \chi_{6,3}(f) \\ &\quad + \chi_{7,1}(f) + 2\chi_{8,2}(f) + 4\big] + \dots\end{aligned}$$

We have checked up to order $t^{11/3}$ for this value of N, higher orders for other values of N, and to all orders in the large N limit:

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A conjecture about elliptic hypergeometric functions

$$\mathcal{I}_{USp} = \mathcal{I}_{SO}$$

Duality in the large N limit

The strict $N \to \infty$ limit simplifies (too much), as the distinction between orientifolds enters as a $1/N$ effect. But some basic checks:

- The SCI of the USp and SO theories can be proven to be equal in this limit.
- The holographic duals transform in a sensible way.

Finite N behavior

For $N > 9$ the dynamics in the IR “stabilizes”: we always get a conformal manifold of complex dimension 1 (from SCI, under the assumption that there are no accidental symmetries).

For $N = 9$ we get a conformal manifold, with extra marginal directions (from the SCI).

For $N = 7$ we can Seiberg dualize the USp theory to an s-confining description (like $N_f = N_c + 1$ SQCD). **(Prediction for SO side.)**

$N = 5$ has a runaway superpotential on both sides.

$N < 5$ is in principle also interesting, but notice that at least one side breaks susy.

$N = 3$ $(\widetilde{N} = 0)$

This case is amusing, we have that $\mathcal{N} = 1$ $USp(4)$ SYM theory is dual to

	"$SO(-1)$"	$SU(3)$	$SU(3)$	$U(1)_R$	$\mathbb{Z}_3$
A^i	$\overline{\square}$	$\overline{\square}$	$\square$	$\frac{2}{3} + \frac{2}{N}$	1
B^i	1	$\square$	$\square$	$\frac{2}{3} - \frac{4}{N}$	-2

where the A field is tachyonic. A realization of the dual Meissner effect? (Related to [Sugimoto '12].)

Generalization to other orbifolds

The proposal generalizes straightforwardly to $\mathbb{C}^3/\mathbb{Z}_n$ singularities, as long as the singularity is isolated (so $n \in 2\mathbb{Z}+1$). Everything works beautifully in these cases too. [Bianchi,Inverso,Morales,Pacifi '13], [I.G.-E., Heidenreich, Wrase '13]

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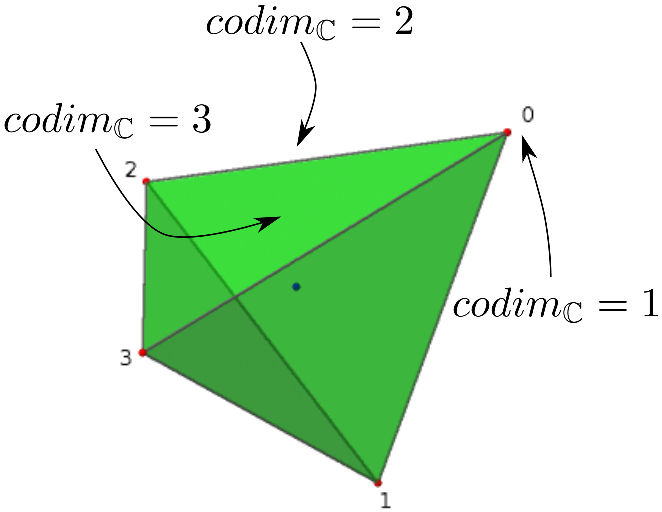
What lies beyond susy orbifolds?

2 minute intro to toric geometry

Basic survival guide



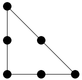
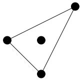
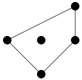
Examples of toric spaces

- Flat space $\mathbb{C}^3 = (|z_1|e^{i\varphi_1}, |z_2|e^{i\varphi_2}, |z_3|e^{i\varphi_3})$.
- Projective spaces $\mathbb{P}^n$ (compact, so not Calabi-Yau).
- Weighted projective spaces (compact, so not Calabi-Yau).
- The conifold: $\sum_{i=1}^{4} z_i^2 = 0$.
- Abelian orbifolds of flat space: $\mathbb{C}^3/(\mathbb{Z}_m \times \mathbb{Z}_n)$.
- Complex cones over the lower del Pezzos $(dP_0, \ldots, dP_3)$.
- $L^{p,q,r}$.

It is a vast class of geometries. Most familiar geometries in string theory are either toric or simply related to toric spaces (hypersurfaces in toric varieties, for example).

Toric diagrams

Toric Calabi-Yau n-manifolds can be described by a $n-1$ convex polytope, the *toric diagram*.

Calabi-Yau	Toric diagram
$\mathbb{C}^3$	
Conifold	
$\mathbb{C}^3/(\mathbb{Z}_2 \times \mathbb{Z}_2)$	
dP_0	
dP_1	

General case

By a computation in algebraic topology one can see that for a toric $O3/O7$ orientifold of a toric CY_3 cone, with

- k sides
- isolated conical singularity of the cone
- fixed points of the orientifold only at the conical singularity

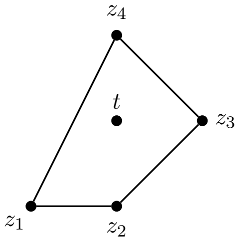
$$H^3(X, \widetilde{\mathbb{Z}}) = \mathbb{Z}_2^{\oplus(k-2)}$$

For example, for $\mathcal{C}_{\mathbb{C}}(dP_1) = \mathcal{C}_{\mathbb{R}}(Y^{2,1})$

$$H^3\left(Y^{2,1}/\mathbb{Z}_2, \widetilde{\mathbb{Z}}\right) = \mathbb{Z}_2 \oplus \mathbb{Z}_2$$

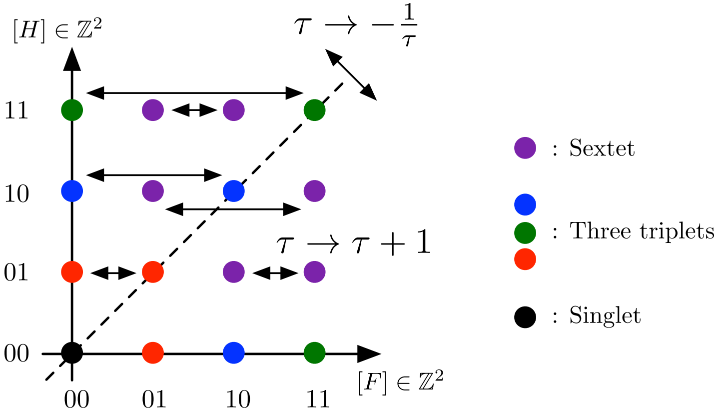
so there are $2^{2\cdot 2} = 16$ orientifold types:

- 1 $SL(2,\mathbb{Z})$ singlet, 3 triplets, 1 sextet.
- $\rightsquigarrow$ 10 different weakly coupled limits.



Orientifold phases for dP_1

More graphically:



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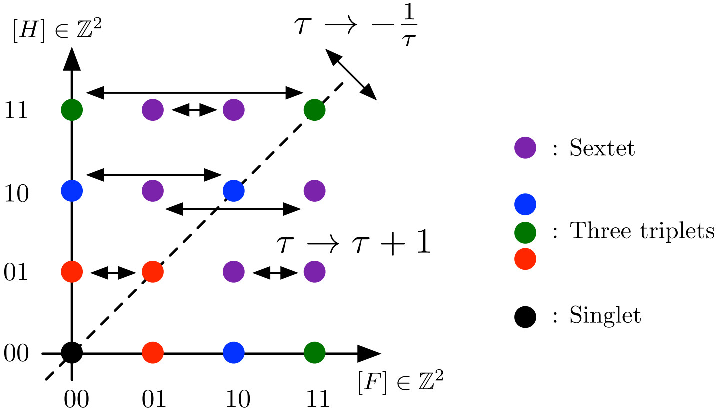
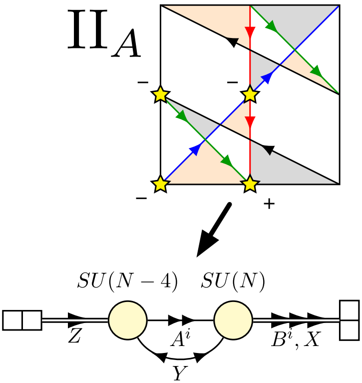


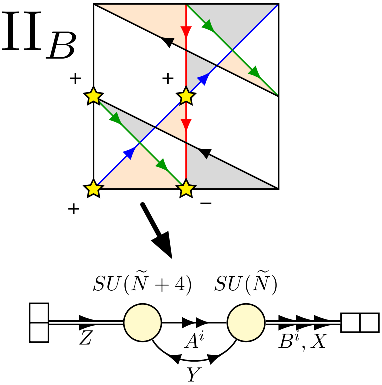
Our task is to map the dots to theories.

Known orientifolds of dP_1

The previously known orientifolds for branes at the dP_1 singularity can be obtained via brane tiling methods
[Franco,Hanany,Krefl,Park,Uranga,Vegh '07]



$$W = \epsilon_{ij} B^i A^j Y + \tfrac{1}{2}\epsilon_{ij} X A^i Z A^j$$



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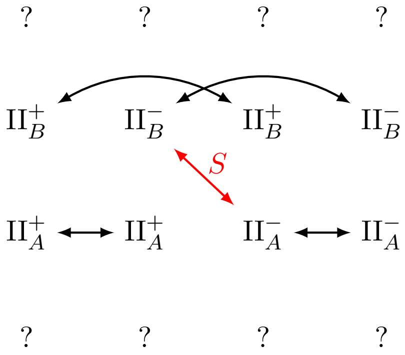
Known orientifolds of dP_1

Anomaly and SCI matching tell us that theories II_A and II_B are dual to each other **iff** N is odd (and $N = \widetilde{N} + 2$). Furthermore, partially resolving $dP_1 \to \mathbb{C}^3/\mathbb{Z}_3 + \mathbb{C}^3$ allows us to read where these orientifolds are located in the torsion diagram:



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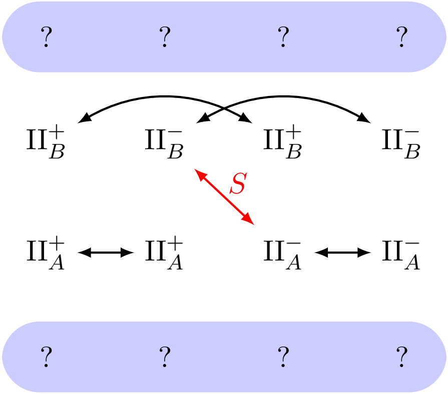

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We can construct the theory for a brane at a singularity by T-dualizing and giving a **brane tiling** instead: a configuration of NS5, D5 and O5. [. . . , Imamura, Kimura, Yamazaki]. What is the most general brane tiling for branes at the (orientifolded) dP_1 singularity?

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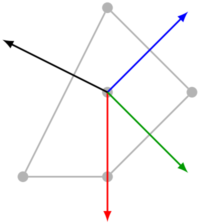

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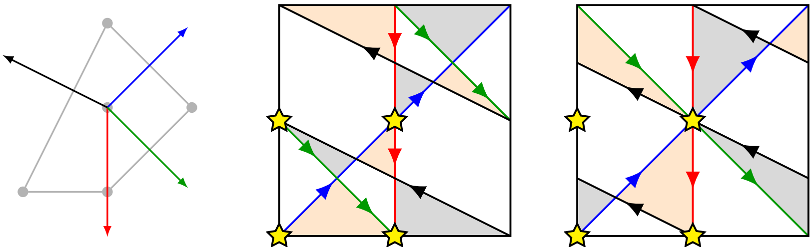

New orientifold phases: heuristic meaning

The new phases can be interpreted as the theory "stuck" (due to the orientifold) at the infinite coupling point connecting Seiberg dual brane configuration.

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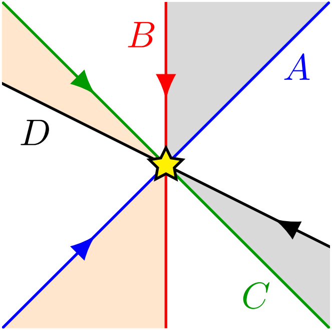

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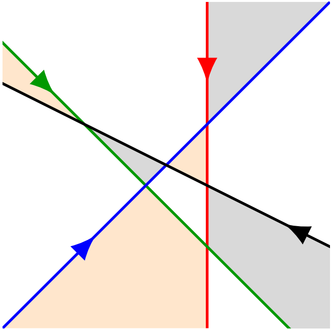

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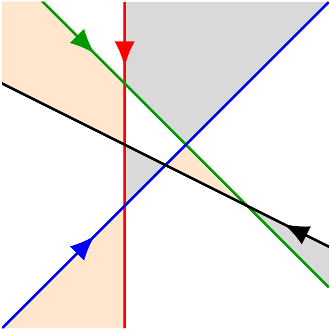
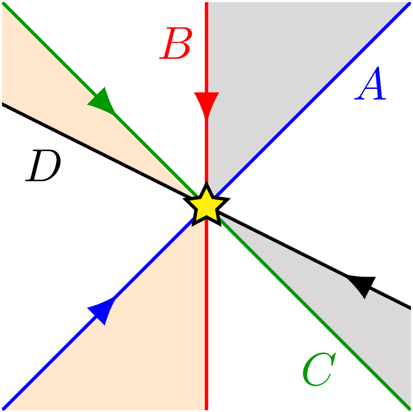


Some properties of the conformal matter

(Without attempting any careful derivation)

The basic building block in this class of $\mathcal{N} = 1$ dualities is the theory living on the intersection of $2k$ NS5 branes on top of an O5 plane. For instance, for $k = 2$



Deconfinement

We can deform (a generalization of **anti-symmetric deconfinement** [Berkooz '95], [Pouliot '95]) the multiple intersection into something which is (conjecturally) in the same universality class.

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The original observation: consider $USp(N-4)$ with N chiral superfields Z^i in the vector representation. This theory is believed to confine, and be described at low energies in terms of the meson $Z^{ij} = \Omega^{ab} Z^i_a Z^j_b$ ($\begin{smallmatrix}\square\\\square\end{smallmatrix}$ of $SU(N)_f$), with an effective superpotential [Intriligator, Pouliot '95]:

$$W = \mathrm{Pf}(Z)\,. \tag{2}$$

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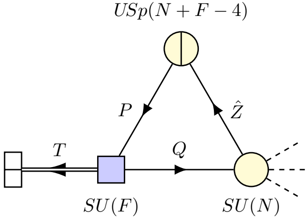
We can obtain a theory of an unconstrained antisymmetric by complicating the theory slightly:

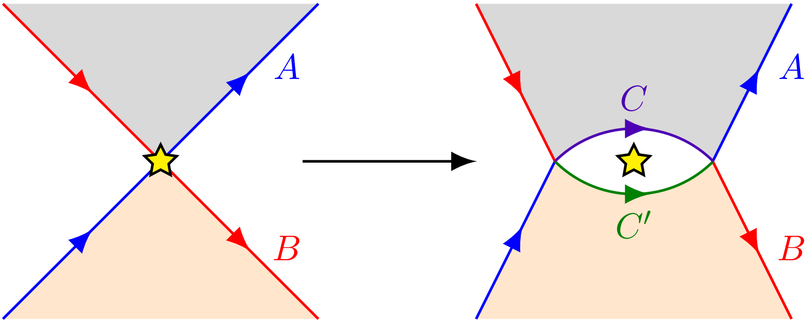


with superpotential

$$W = PQ\hat{Z} + TP^2 \tag{3}$$

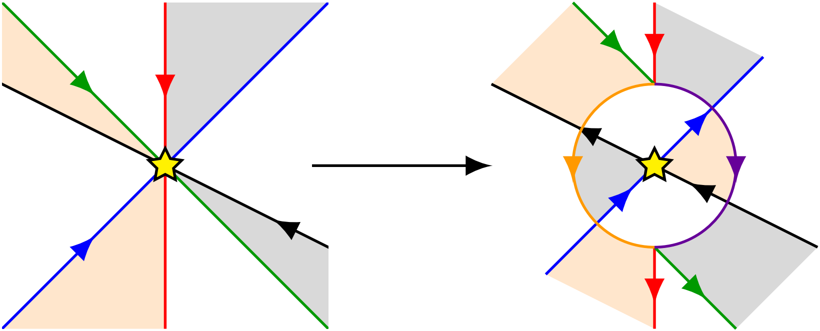
Brane tiling description of deconfinement

This somewhat involved structure is very naturally generated by deconfinement in the brane tiling

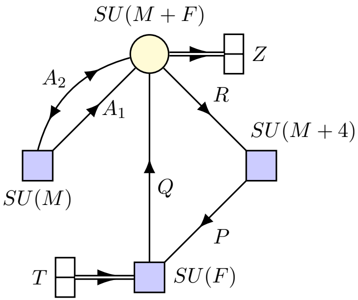


Generalizing to multiple intersections

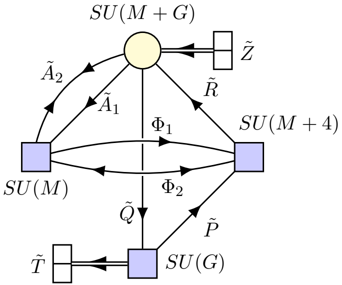
This picture is straightforward to generalize to intersections of multiple NS5 branes:



$\mathfrak{q}_{USp}$

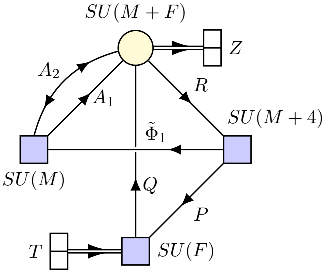


$W = A_1A_2Z + PQR + TQ^2Z$

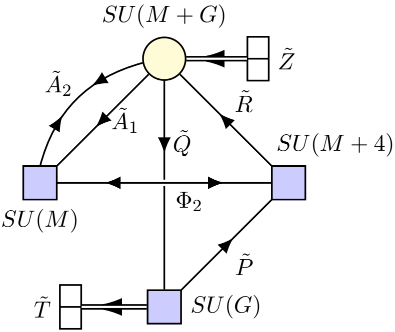


$W = \tilde{A}_1\tilde{A}_2\tilde{Z} + \Phi_1\tilde{A}_1\tilde{R} + \Phi_2\tilde{A}_2\tilde{R} + \tilde{P}\tilde{Q}\tilde{R} + \tilde{T}\tilde{Q}^2\tilde{Z}$

$\mathfrak{q}_{SO}$



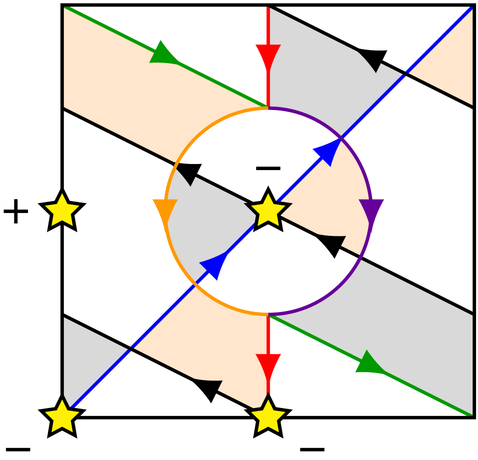
$$W = A_1 A_2 Z + \tilde{\Phi}_1 A_1 R + PQR + TQ^2 Z$$



$$W = \tilde{A}_1 \tilde{A}_2 \tilde{Z} + \Phi_2 \tilde{A}_2 \tilde{R} + \tilde{P}\tilde{Q}\tilde{R} + \tilde{T}\tilde{Q}^2 \tilde{Z}$$

Two new orientifold phases for dP_1

Phase I_A



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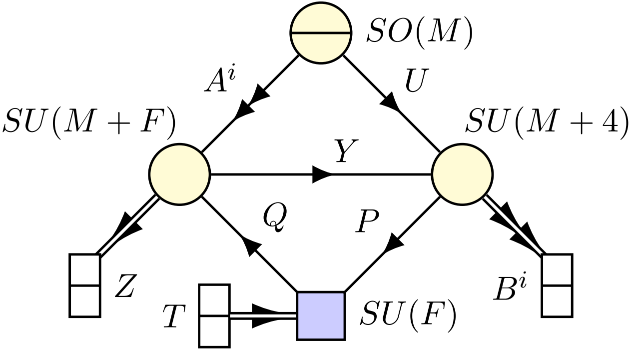

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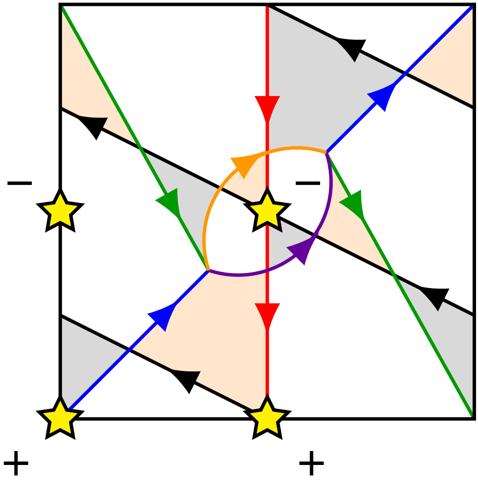
Phase I_B



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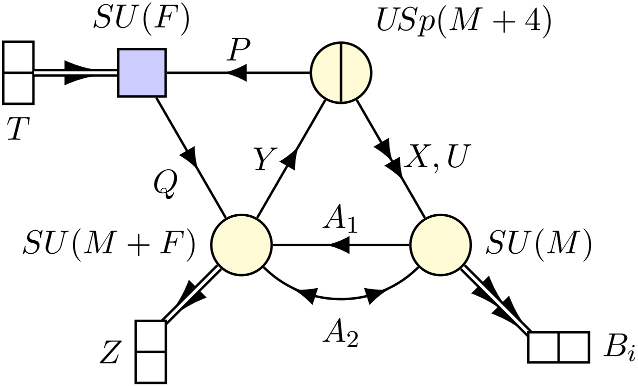
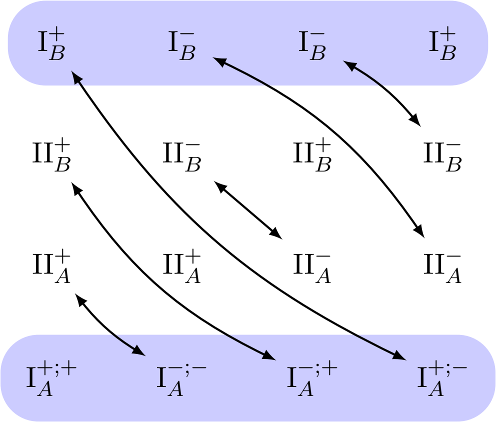


The full duality diagram



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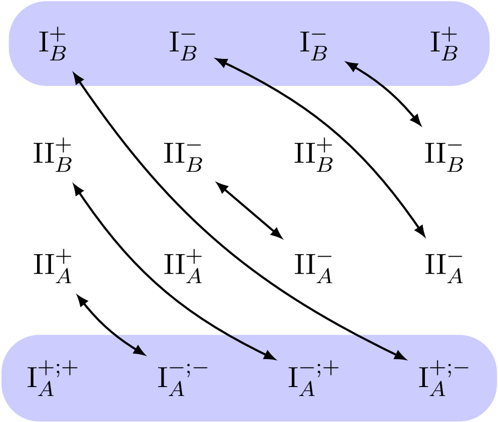

Perfect agreement between SCIs, agrees with partial resolution, etc.

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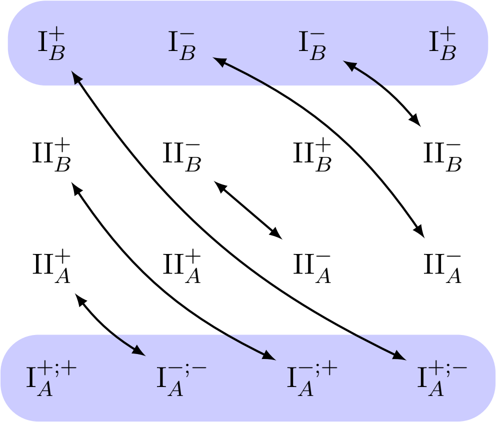
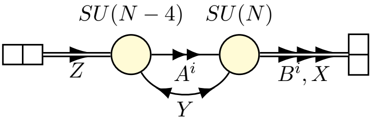
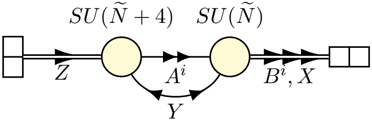


Perfect agreement between SCIs, agrees with partial resolution, etc.
SCI disagrees for non-dual theories.

All duality phases for dP1



$$W = \epsilon_{ij} B^i A^j Y + \tfrac{1}{2}\epsilon_{ij} X A^i Z A^j$$



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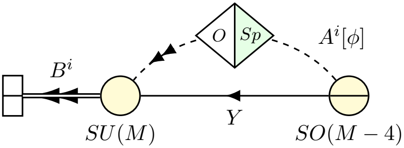

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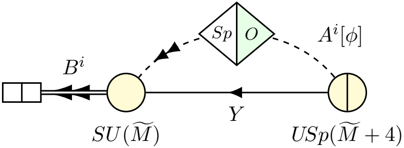

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A constructive approach

The discussion so far was not entirely satisfactory: I constructed a set of theories using orientifolded brane tilings, and then used consistency conditions (SCI matching, etc) to guess which discrete RR and NSNS torsion is associated to each theory. It works for simple examples, but it is a fairly labour intensive process in practice, so minimally complicated examples like $\mathcal{C}(dP_2)$ are already very painful.

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We can do much better: in [I.G.-E., Heidenreich '16] we provide a (operationally) simple and systematic way of reading the torsion associated to a given brane tiling.

Appearance of conformal matter is very general

An easy theorem

For any toric polytope with more than four sides **all** phases are non-classical.

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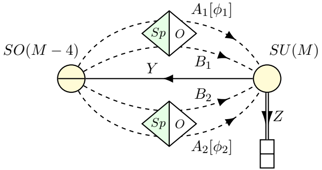
A corollary

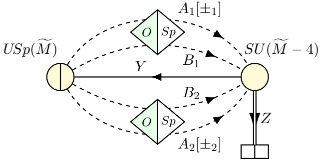
We were very lucky that we decided to study $\mathbb{C}^3/\mathbb{Z}_3$ first ...

A more involved example

For $\mathcal{C}(dP_2)$, **every** duality phase includes TO_2 matter:



$W = Y(B_1 - B_2) + A_1 A_2 Z$



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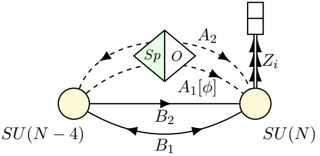
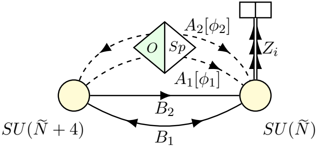


$W = A_1 B_1 Z_1 + A_2 B_2 Z_2 + B_1 B_2 Z_3$



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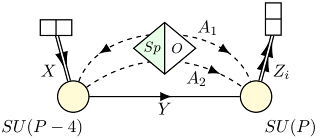
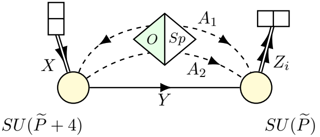


$W = A_1 Y Z_1 + X A_2 Y Z_2$



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Recapitulation

Our philosophy for finding duals:

1. Build a configuration of branes at singularities.
2. Measure its conserved charges, including torsion.
3. Apply IIB S-duality to the charges.
4. Construct the brane configuration in the same geometry with the dual charges.

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Branes at singularities are somewhat special, in that step **4** can be done using perturbative ingredients + some universal strongly coupled blocks.

Conclusions

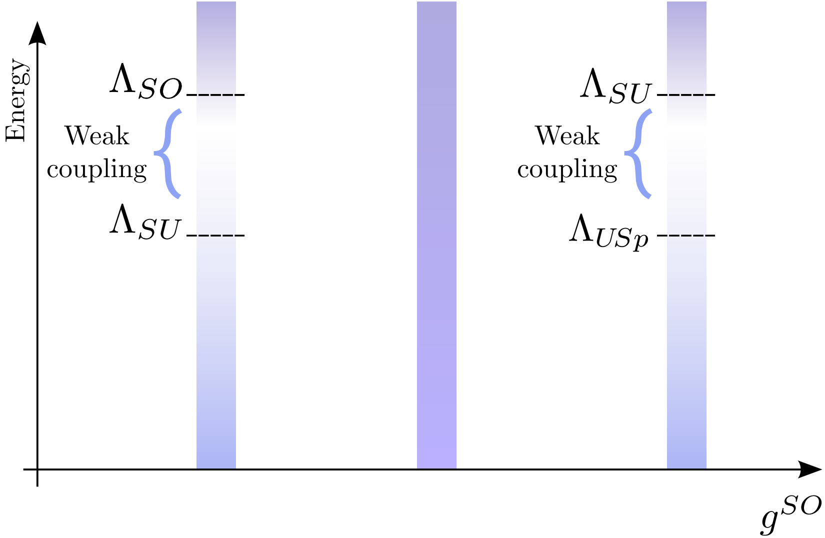
- We find very strong evidence for the existence of a new (in $\mathcal{N}=1$, but closely related to $\mathcal{N}=4$ Montonen-Olive in spirit) class of S-dual descriptions for certain interesting $\mathcal{N}=1$ theories: non-conformal, chiral, ...
- The whole idea works thanks to the existence of a class of hitherto unknown $\mathcal{N}=1$ theories for orientifolded singularities, coming from gauging flavor symmetries of a class of isolated strongly coupled $\mathcal{N}=1$ SCFTs.
- Our approach is **constructive**: given a toric singularity we can read off the physics at all cusps in the associated conformal manifold.

Some open questions

- M5 brane description? The emerging picture is very reminiscent of what happens for dualities in class $\mathcal{S}$ theories:
  - The dual descriptions are built by gauging global symmetries of ($\mathcal{N}=1$) isolated SCFTs.
  - The physics is determined by a toric diagram plus discrete data $\sim$ decorated Riemann surface (via mirror symmetry).
  - **Ongoing work:** Generalization to $\mathcal{N}=2$ theories, and M-theory uplift. Can we generalize deconfinement to this case?
  - Alternatively, any intersection with class $\mathcal{S}_k$?
- Implications for strong coupling dynamics? [Sugimoto '12]
- 3d dualities from circle reduction.
- Dynamics on duality walls.
- Can we study $\mathcal{N}=0$ dualities? [Hook, Torroba '13]
- Global structure of the duality? Extended operators? [Aharony, Seiberg, Tachikawa '13]

Supplementary material

Physical interpretation



Inherited duality

In fact, this situation of having a UV cutoff is familiar from $\mathcal{N}=1$ inherited S-duality [Argyres, Intriligator, Leigh, Strassler '99]. Start with $\mathcal{N}=4$ SYM, and give a mass to an adjoint. One ends up with the non-renormalizable superpotential

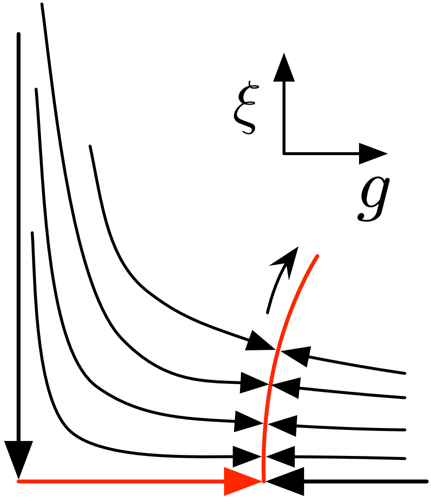
$$W = h\mathrm{Tr}\left([\phi_1,\phi_2][\phi_1,\phi_2]\right). \tag{4}$$

Note that away from $h=0$ we require a cut-off. (In this case there is always a natural UV completion, of course.)

The point $h=0$ is believed to be interacting. [Intriligator, Seiberg '94], [Intriligator, Wecht '03].

By a-maximization, we read that the operator $\mathcal{O}=\mathrm{Tr}\left([\phi_1,\phi_2][\phi_1,\phi_2]\right)$ is exactly marginal, so it moves us in the conformal manifold. A RG-invariant parameterization of this motion is by $\xi = h^N\Lambda^N$. So $\xi \ll 1$ implies $\Lambda \ll h^{-1}$, i.e. we have a weakly coupled description for a large range of scales.

Inherited duality



Banks-Zaks fixed point

The IR fixed point is typically interacting in our class of constructions, but for large N we have a weakly interacting description of the theory close to $W = 0$, $g_{SU} = 0$:

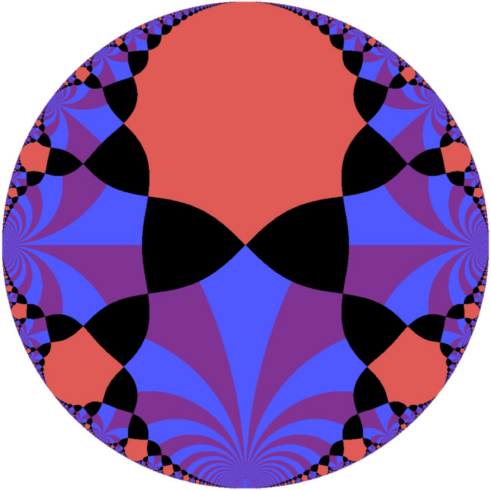
	$USp(\widetilde{N}+4)$	$SU(\widetilde{N})$	$SU(3)$	$U(1)_R$	$\mathbb{Z}_3$
A^i	$\overline{\square}$	$\square$	$\square$	$\frac{2}{3} - \frac{2}{\widetilde{N}}$	1
B^i	1	$\overline{\square\square}$	$\square$	$\frac{2}{3} + \frac{4}{\widetilde{N}}$	-2

with $W = \frac{\widetilde{\lambda}}{2}\,\Omega_{ab}\,\epsilon_{ijk}\,A_m^{i;a}A_n^{j;b}B^{k;mn}$, and

$$\frac{(\widetilde{N}+4)(\widetilde{N}+5)}{2}\cdot\frac{g_{1*}^2}{8\pi^2} = \frac{18(\widetilde{N}-1)}{\widetilde{N}} + (\widetilde{N}^2-1)\frac{g_{2*}^2}{8\pi^2}\,,$$

$$\widetilde{N}(\widetilde{N}+4)\frac{|\widetilde{\lambda}_*|^2}{8\pi^2} = \frac{6(\widetilde{N}-6)}{\widetilde{N}} + (\widetilde{N}+2)(\widetilde{N}-1)\frac{g_{2*}^2}{8\pi^2}\,.$$

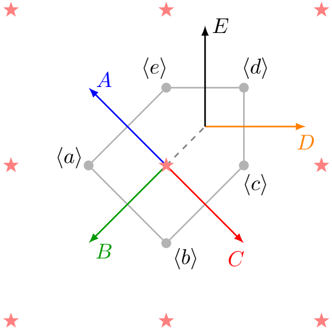
Duality for the $SL(2,\mathbb{Z})$ triplet



Red is $SO(N-4) \times SU(N)$, blue/purple is $USp(N+1) \times SU(N-3)$.

Reading the NSNS torsion from the tiling

A basic fact: a torsion class in $H^3(X, \widetilde{\mathbb{Z}}) = \mathbb{Z}_2^{\oplus(k-2)}$ is generated by a choice of signs associated to each corner of the toric diagram

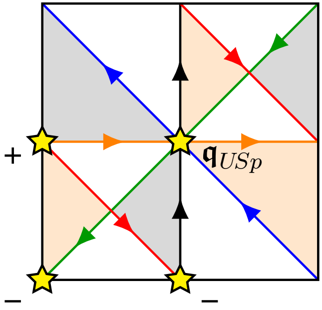


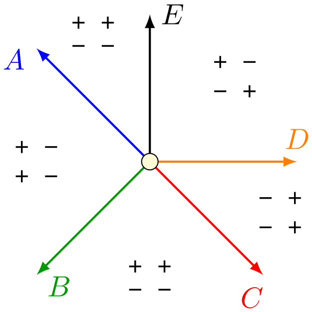
up to the equivalences

$$\langle a \rangle + \langle c \rangle = \langle b \rangle + \langle e \rangle = \langle d \rangle \tag{5}$$

Reading the NSNS torsion from the tiling

On the other hand, at each fixed point of the tiling we have an O5, with local charges jumping as we cross the NS5:





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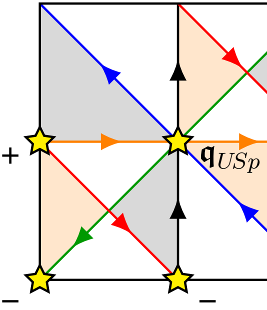

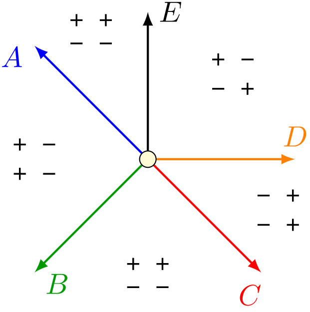

A theorem

Both objects are the same.

Reading the RR torsion from the tiling

Requires more formalism to state, but the conjecture is also very clean in the right language:

$$F_\alpha \equiv [F] \cdot \sum_{i \in V_\alpha} \langle i \rangle \pmod 2 \tag{6}$$

where

- F_α are the parities associated with the TO_k theories.
- $[F]$ is the RR torsion.
- V_α encodes a specific subset of O5 local charges.
- "$\cdot$" is a natural geometric product between torsion classes.

Not proven, but lots of supporting evidence.

Intuitively, it encodes the existence of D5 domain walls across which the RR torsion jumps. [Witten '98] These should not change the geometry of the dual tiling.