Connecting the ambitwistor and the sectorized heterotic strings

Renann Lipinski Jusinskas

February 11th - 2018

Discussion Meeting on String Field Theory and String Phenomenology - HRI, Allahabad, India

Outline

- CHY amplitudes;
- ambitwistor strings;
- pure spinor formalism;
- the sectorized model;
- infinite length limit;
- conclusion;

CHY amplitudes

CHY formulae

In JHEP07(2014)033 (arXiv:1309.0885 [hep-th]), Cachazo, He and Yuan, based on previous results, proposed the following formula for the tree level scattering amplitudes of n massless particles:

$$\mathcal{M}_n^s = \int \frac{d^n\sigma}{\mathrm{vol}\, SL(2,\mathbb{C})} \prod_a{}' \delta(\sum_{b\neq a} \frac{s_{ab}}{\sigma_a - \sigma_b}) \left(\frac{\mathrm{Tr}(T^{a_1} T^{a_2} \dots T^{a_n})}{(\sigma_1 - \sigma_2)\cdots(\sigma_n - \sigma_1)} + \dots\right)^{2-s} \mathrm{Pf}'(\Psi)^s. \quad (1)$$

It is valid for gravitons ($s = 2$), gluons ($s = 1$) and scalars ($s = 0$).

Scattering equations

Motivation: "connection between the scattering data of n massless particles and maps from the n-punctured sphere into the null cone in D dimensional momentum space"

$$k_a^m = \oint \frac{p^m(z)}{\prod_{b=1}^{n}(z-\sigma_b)}. \tag{2}$$

By requiring $p^m(z)p_m(z) = 0$, the so-called scattering equations are obtained:

$$\sum_{b\neq a} \frac{k_a \cdot k_b}{(\sigma_a - \sigma_b)} = 0. \tag{3}$$

Is it possible to obtain such structures from a string theory?

Ambitwistor strings

Quick review

- Mason and Skinner proposed them soon after CHY, JHEP07(2014)048 (arXiv:1311.2564);
- First order chiral string (only holomorphic fields), e.g.

$$\int d^2z \, \{\partial X^m \bar{\partial} X_m\} \to \int d^2z \, \{P_m \partial X^m\} \, . \qquad (4)$$

- Massless condition $P^m P_m = 0$ is imposed as a constraint in the action;
- It can be seen as an $\alpha' \to 0$ limit of the (super)string (infinite tension).

Bosonic ambitwistor string

The bosonic action, after gauge fixing, is

$$S = \int d^2 z \, \{P_m \bar{\partial} X^m + b\bar{\partial} c + \bar{b}\bar{\partial}\bar{c}\}, \tag{5}$$

with BRST charge

$$Q = \oint \{cT - bc\partial c + \bar{c} \overbrace{P^m P_m}^{\equiv H}\}. \tag{6}$$

The energy-momentum tensor T is written as

$$T = -P_m \partial X^m - b\partial c - \partial(bc) - \bar{b}\partial\bar{c} - \partial(\bar{b}\bar{c}), \tag{7}$$

and nilpotency of Q implies $D = 26$.

Vertex operators

The spectrum of the bosonic ambitwistor string consists of a symmetric traceless tensor, g^{mn}, identified with the graviton.

The unintegrated vertex operator is

$$U = g^{mn} c\bar{c} P_m P_n \, e^{ik\cdot X}. \tag{8}$$

The integrated vertex operator is more subtle and can be written as

$$V = \int d^2 z \, \bar{\delta}(k \cdot P) \, g^{mn} P_m P_n \, e^{ik\cdot X}. \tag{9}$$

The operator $\bar{\delta}(k \cdot P)$ is responsible for the localization of the scattering amplitudes as it appears in the CHY formulae!

Heterotic ambitwistor strings

If we add worldsheet supersymmetry, the heterotic model has the simple gauge-fixed action

$$S = \int d^2z \{P_m \bar{\partial} X^m + \frac{1}{2}\psi_m \bar{\partial}\psi^m + b\bar{\partial}c + \beta\bar{\partial}\gamma + \bar{b}\bar{\partial}\bar{c} + \mathcal{L}_C\}. \quad (10)$$

with BRST charge

$$Q = \oint \{cT + \gamma G_{matter} - bc\partial c - \frac{1}{2}b\gamma^2 + \bar{c}P^m P_m\} \quad (11)$$

Cohomology: massless sector of the usual heterotic string plus an unexpected 3-form. The gluon amplitudes agree with the CHY formulae but the gravity sector resembles conformal gravity.

Pure spinors

Quick review of the pure spinor formalism

- "Super-Poincaré Covariant Quantization of the Superstring", N. Berkovits; JHEP04(2000) (arXiv:hep-th/0001035).
- Based on a bosonic spinor variable satisfying $(\lambda\gamma^m\lambda) = 0$.
- Field content: $X^m, \theta^\alpha, p_\alpha, \lambda^\alpha, w_\alpha$.

- The postulated BRST charge $Q \equiv \lambda^\alpha d_\alpha$ is nilpotent only for a pure spinor,

$$Q^2 \propto (\lambda\gamma^m\lambda)\Pi_m. \tag{12}$$

- The cohomology is automatically described in terms of superfields;
- Unknown origin!

Heterotic ambitwistor string in Berkovits' proposal

The action of the model is given by

$$S = \int d^2z\{P_m\bar{\partial}X^m + p_\alpha\bar{\partial}\theta^\alpha + w_\alpha\bar{\partial}\lambda^\alpha + \bar{b}\bar{\partial}\bar{c} + \mathcal{L}_C\}, \tag{13}$$

while the BRST charge is defined as

$$Q \equiv \oint\{\lambda^\alpha[\overbrace{p_\alpha + (\gamma^m\theta)_\alpha P_m}^{d_\alpha}] + \bar{c}T - \bar{b}\bar{c}\partial\bar{c}\}. \tag{14}$$

- Fails to reproduce the expected gauge transformations for the gravity sector;
- Does not include the particle-like Hamiltonian $H = P^m P_m$;

Cohomology I

The cohomology includes the non-abelian super Yang-Mills fields and the "would-be" $\mathcal{N} = 1$ supergravity. The former can be encoded by the vertex operator

$$U_{SYM} = \lambda^\alpha \bar{c} A^I_\alpha(\theta) J^I e^{ik\cdot X}, \tag{15}$$

where $A^I_\alpha(\theta)$ satisfies

$$D_\alpha \gamma^{\alpha\beta}_{mnpqr} A^I_\beta = 0. \tag{16}$$

The gauge transformations of U_{SYM} are described by the BRST-exact operator

$$\begin{aligned} \delta U_{SYM} &\equiv \{Q, \bar{c}\Lambda^I(\theta) J^I e^{ik\cdot X}\}, \\ &= \lambda^\alpha \bar{c}(D_\alpha \Lambda^I) J^I e^{ik\cdot X}. \end{aligned} \tag{17}$$

Cohomology II

As for the supergravity states, the vertex operator is

$$U_{SG} = \lambda^\alpha \bar{c} A^m_\alpha(\theta) P_m e^{ik\cdot X}. \tag{18}$$

BRST-closedness implies the usual e.o.m.

$$\begin{aligned} D_\alpha \gamma^{\alpha\beta}_{mnpqr} A^s_\beta &= 0, && (19a) \\ k_m A^m_\alpha &= 0, && (19b) \end{aligned}$$

However, the expected gauge transformation $\delta A^m_\alpha = D_\alpha \Lambda^m + k^m \Lambda_\alpha$ does not come from a BRST-exact state:

$$\begin{aligned} \delta U_{SG} &\equiv \lambda^\alpha \bar{c}(D_\alpha \Lambda^m + k^m \Lambda_\alpha) P_m e^{ik\cdot X}, \\ &\neq \{Q, \mathrm{something}\}. && (20) \end{aligned}$$

The sectorized model

Quick review of the sectorized model

- Background coupling $\rightarrow$ modification of the ambitwistor model (Chandia and Vallilo);
- Reinterpretation of the variables in terms of two sectors, "+"and -", resembling the left and right movers of the string;
- Correct description of the massless heterotic spectrum;
- Background constraints obtained solely from nilpotency of the BRST charge;
- Quantum corrections are easily obtained;
- Problem: the construction of the integrated vertex remains unknown;

The two sectors are represented by

$$\mathcal{O}_+ = \{P_m^+, \bar{b}, \bar{c}, J^I\}, \tag{21}$$
$$\mathcal{O}_- = \{P_m^-, d_\alpha, \theta^\alpha, w_\alpha, \lambda^\alpha\}, \tag{22}$$

with

$$P_m^+ = P_m + \partial X_m, \tag{23a}$$
$$P_m^- = P_m - \partial X_m - (\theta\gamma_m\partial\theta), \tag{23b}$$
$$d_\alpha = p_\alpha - \frac{1}{2}(\gamma^m\theta)_\alpha P_m^- - \frac{1}{4}(\theta\gamma^m\partial\theta)(\gamma_m\theta)_\alpha. \tag{23c}$$

The $\mathcal{N} = 1$ supersymmetry of the heterotic string is explicit.

The two sectors have characteristic (pseudo) energy-momentum tensors defined by

$$T_- \equiv \frac{1}{4}\eta^{mn}P^-_m P^-_n - d_\alpha \partial\theta^\alpha - w_\alpha \partial\lambda^\alpha, \tag{24a}$$

$$T_+ \equiv -\frac{1}{4}\eta^{mn}P^+_m P^+_n + T_C - \bar{b}\partial\bar{c} - \partial(\bar{b}\bar{c}), \tag{24b}$$

which combine to form the full energy-momentum tensor

$$\begin{aligned} T &= T_+ + T_-, \qquad (25) \\ &= -P_m\partial X^m - p_\alpha\partial\theta^\alpha - w_\alpha\partial\lambda^\alpha - 2\bar{b}\partial\bar{c} - \bar{c}\partial\bar{b} + T_C. \end{aligned}$$

The BRST charge of the model is defined to be

$$Q = \oint \{\lambda^\alpha d_\alpha + \bar{c}T_+ - \bar{b}\bar{c}\partial\bar{c}\}, \tag{26}$$

and naturally incorporates the sector splitting.

Cohomology

In addition to super Yang-Mills, encoded in

$$U_{SYM} = \lambda^\alpha \bar{c} A^I_\alpha J_I, \tag{27a}$$

$$\gamma^{\alpha\beta}_{mnpqr} D_\alpha A^I_\beta = 0, \tag{27b}$$

$$\delta_\Sigma A^I_\alpha = D_\alpha \Sigma^I, \tag{27c}$$

the $\mathcal{N} = 1$ supergravity states are correctly described:

$$U_{SG} = \lambda^\alpha \bar{c} A^m_\alpha P^+_m, \tag{28a}$$

$$\gamma^{\alpha\beta}_{mnpqr} D_\alpha A^s_\beta = 0, \tag{28b}$$

$$\partial^n \partial_n A^m_\alpha - \partial^m \partial_n A^n_\alpha = 0, \tag{28c}$$

$$\delta_\Sigma A^m_\alpha = D_\alpha \Sigma^m + \partial^m \Sigma_\alpha. \tag{28d}$$

- Dimensionful parameters? Initially, the sectorized model was viewed as an infinite tension string;
- Amplitude computations provide terms with different powers of momenta;
- "hidden"length parameter in the model:

$$P_m^+ = P_m + \partial X_m \rightarrow P_m^+ = P_m + \frac{1}{\ell^2}\partial X_m$$

- Can there be extra (massive) states in the cohomology?

Massive cohomology

The sectorized heterotic model is two-fold. Depending on the choice of the supersymmetric sector (+ or -), there is a massive state denoted by $U = \bar{c}U_{\rm open}$, with

$$\begin{aligned} U_{\rm open} \quad &\equiv \quad \lambda^\alpha \partial\theta^\beta B_{\alpha\beta} + \lambda^\alpha d_\beta C_\alpha^\beta + J\lambda^\alpha E_\alpha \\ &\quad +\lambda^\alpha N^{mn} F_{\alpha mn} + \partial\lambda^\alpha G_\alpha + \lambda^\alpha P_m^+ H_\alpha^m . \end{aligned} \tag{29}$$

This vertex was shown by Chandia and Berkovits to describe the first massive level of the open string: S^{mn}, A_{mnp} and $\psi_{m\alpha}$.

$$\begin{aligned} \eta^{mn} G_{mn} &= 0, & \partial^m \psi_{m\alpha} &= 0, \\ \partial^m G_{mn} &= 0, & (\gamma^m \psi_m)^\alpha &= 0. \\ \partial^m A_{mnp} &= 0, \end{aligned}$$

The infinite length limit

It is straightforward to reintroduce the dimensionful parameter ℓ in the sectorized model (global scale symmetry of the action):

$$\begin{aligned} &X^m \to \ell^{-1} X^m, \quad \theta^\alpha \to \ell^{-\frac{1}{2}}\theta^\alpha, \quad \lambda^\alpha \to \ell^{-\frac{1}{2}}\lambda^\alpha, \quad \bar{c} \to \ell^{-2}\bar{c}, \\ &P_m \to \ell^{+1} P_m, \quad p_\alpha \to \ell^{+\frac{1}{2}} p_\alpha, \quad w_\alpha \to \ell^{+\frac{1}{2}} w_\alpha, \quad \bar{b} \to \ell^{+2}\bar{b}. \end{aligned}$$

The BRST charge, for example, is rewritten as:

$$\begin{aligned} Q &= \oint \{\lambda^\alpha [p_\alpha - \frac{1}{2} P_m (\gamma^m \theta)_\alpha - \frac{1}{2\ell^2} \Pi^m (\gamma_m \theta)_\alpha]\} \\ &\quad + \oint \{\frac{1}{4}\bar{c}\eta^{mn} P_m^- P_n^- + \frac{1}{\ell^2}\bar{c} T_C + \frac{1}{\ell^2}\bar{b}\bar{c}\partial\bar{c}\}, \\ &= \oint \{\lambda^\alpha [p_\alpha - \frac{1}{2} P_m (\gamma^m \theta)_\alpha] + \frac{1}{4}\bar{c} P^m P_m\} + \mathcal{O}(\ell^{-2}). \end{aligned}$$

The infinite length limit

Surprisingly, the ambitwistor string can be seen as a tensionless (infinite length) limit of the sectorized model.

And what about the massive state?

In this case, it is more convenient to identify explicitly the physical degrees of freedom.

This can be achieved by using a DDF construction ($M^2 = 2k^+k^-$):

$$\begin{cases} G^{mn}, A_{mnp} & \to U_{j,i}, U_{a,\dot{a}}, \\ \psi_{m\alpha} & \to U_{j,\dot{a}}, U_{a,i}. \end{cases}$$

The infinite length limit

$SO(8)$ decomposition of the physical d.o.f. illustrating the origin of the extra states of Mason and Skinner:

$$
\begin{tikzcd}
U_{j,i} \arrow[r, leftrightarrow, "\bar{q}_{\dot{b}}"'] \arrow[d, leftrightarrow, "q_b"] & U_{a,i} \arrow[d, leftrightarrow, "q_b"'] \\
U_{j,\dot{a}} \arrow[r, leftrightarrow, "\bar{q}_{\dot{b}}"] & U_{a,\dot{a}}
\end{tikzcd}
\quad \xrightarrow[\ell\to\infty]{} \quad
\begin{tikzcd}
U_{j,i} \arrow[d, leftrightarrow, "q_b"] & U_{a,i} \arrow[l, dashed, "\bar{q}_{\dot{b}}"] \arrow[d, leftrightarrow, "q_b"'] \\
U_{j,\dot{a}} & U_{a,\dot{a}} \arrow[l, dashed, "\bar{q}_{\dot{b}}"']
\end{tikzcd}
$$

$SO(8)$ indices: i, j (vector) and $a, \dot{a}$ (spinor).

Summary and prospects

Summary and prospects

- Mason and Skinner: ambitwistor string is a string model for the CHY ampltudes;
- Pure spinor construction (explicitly supersymmetric);
- Generalization: sectorized model (can be extended to all superstring formalisms);
- Physical motivation?
- Open problems?
- Connection to twistor string theory;