First Integrated Massive Vertex in Pure Spinor Formulation of Superstring

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Discussion Meeting on String Field Theory and String Phenomenology
HRI, Allahabad

Based on arXiv: 1802.*****
in collaboration with
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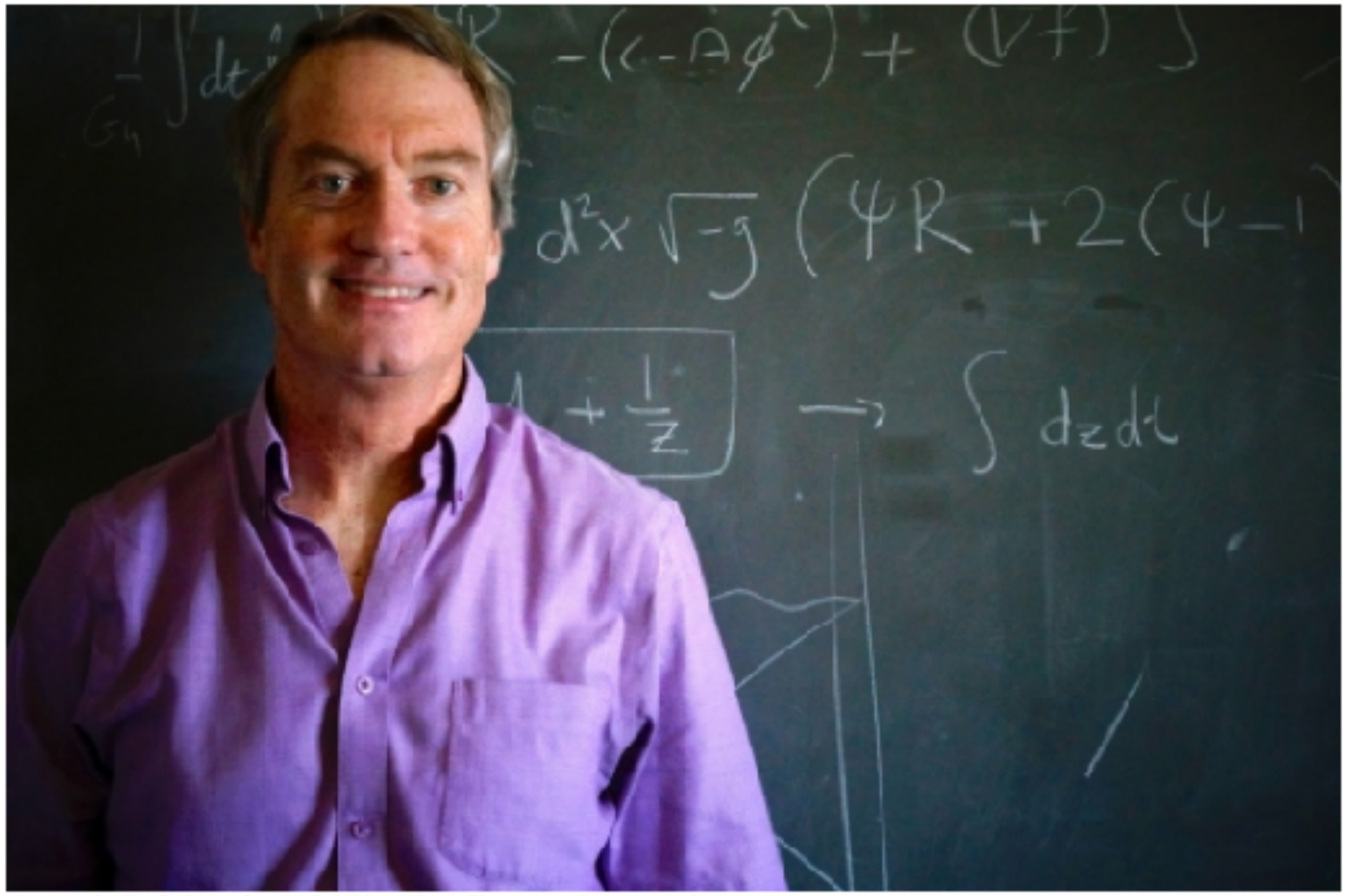

Joseph Polchinski, UCSB's 2014 Faculty Research Lecturer

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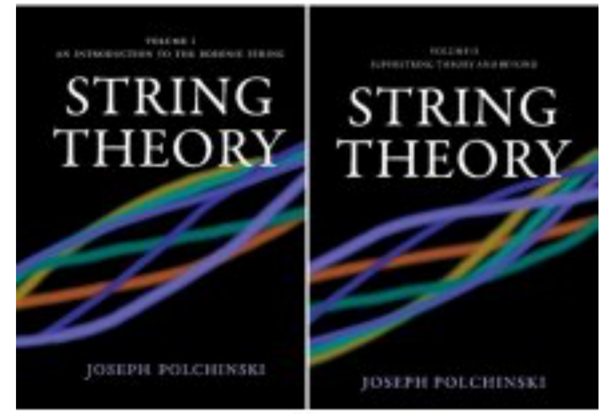

Among the 187 papers with his name on Inspire, 15 are renowned (above 500 citations). It may be interesting to enumerate them:

- Toroidal path integrals for Polyakov strings (1986)
- T-duality between IIA and IIB, with Dai and Leigh (1989)
- Minimal realistic SUGRA, with Alvarez-Gaume and Wise (1983)
- Some exact RG equations (1983)
- D-branes and RR charges (1995, 2,500 citations now)
- Heterotic-type I S-duality, with Witten (1995)
- Consistency conditions on Chan-Paton boundaries and cross caps, with Gimon (1996, about symplectic D-brane gauge groups etc.)
- Notes on D-branes, pedagogic, with Chaudhuri and Johnson (1996) – BTW Chaudhuri later turned into a loon religiously and sexually obsessed with Jo her papers against his will, and Joe wrote me quite something about that abusive relationship
- TASI lectures on D-branes (1996)
- Polchinski-Strassler confining gauge theory, an important AdS/QCD paper (2000)
- Bousso-Polchinski discretuum of vacua with fluxes, an early landscape (2000)
- Stringy Randall-Sundrum, with Kachru and Giddings (2001)
- Hard (power law) scattering in string theory,
- from the holographic warped dual theory, with Strassler (2001)
- Integrability of AdS5 x S5, with Bena and Roiban (2003)
- Black hole firewalls, with Almheiri, Marolf, and Sully (2012)

Quite a list,

GOAL

$$QU = \partial V$$

BRST CHARGE

INTEGRATED VERTEX

WORLDSHEET Derivative

UNINTEGRATED VERTEX

SOLVE FOR U GIVEN THE FIRST EXCITED MASSIVE OPEN STRING UNINTEGRATED VERTEX OPERATOR V (BERKOVITS+CHANDIA (2002))

MOTIVATION

- The tree level scattering amplitude for N external states is given by

$$\mathcal{A}_N = \langle V^1 V^2 V^3 \int U^4 \cdots \int U^N \rangle$$

where, V and U are the unintegrated and integrated vertex operators

- The g-loop scattering amplitude for N external states is given by

$$\mathcal{A} = \int d^{3g-3}\tau \langle \mathcal{N}(y) \prod_{i=1}^{3g-3} (\int dw_i \mu_i(w_j) b(w_j)) \prod_{j=1}^{N} \int dz_j U(z_j) \rangle$$

- INTEGRATED VERTEX OPERATOR IS A MUST SUFFICIENTLY HIGHER POINT TREE LEVEL AND ALL LOOP LEVEL AMPLITUDES

- VERTEX OPERATOR FOR MASSLESS OPEN STRING STATES IN UNINTEGRATED AND INTEGRATED FORM ARE KNOWN

- THE ONLY KNOWN MASSIVE VERTEX OPERATOR IN PURE SPINOR FORMALISM IS AT FIRST EXCITED LEVEL OF OPEN STRING $(Mass)^2 = \frac{1}{\alpha'}$

- WE SHALL PRESENT THE INTEGRATED VERTEX FORM THE ABOVE VERTEX

- WE SHALL SEE THAT OUR CONSTRUCTION SEEMS TO BE GENERALISABLE TO HIGHER MASS LEVELS

SO, HOW DO WE SOLVE $QU = \partial V$?

SIMPLE EXAMPLE

- **CONSIDER**

$$\sum_i^N \hat{B}_i c_i = 0$$

ALONG WITH

$$\boxed{I_i(\hat{B}_1, \hat{B}_2, \cdots, \hat{B}_N) = 0 \quad ; \quad i = 0, 1, 2, \cdots, p}$$

CONSTRAINTS

where, $\{\hat{B}_i\} \in V$

- **QUESTION: WHAT VALUES OF** $\{c_i\}$ **SOLVES** $\sum_i^N \hat{B}_i c_i = 0$ **?**

- **ANSWER: DEPENDS ON NUMBERS OF CONSTRAINTS.**

- IF $p = 0$ THEN $c_i = 0 \quad \forall \quad i$
- IF $p \neq 0$ THEN WE HAVE TWO OPTIONS FOR SOLVING FOR $\{c_i\}$

✦ **OPTION 1:** ELIMINATE SOME $\{\hat{B}_a\}$ IN FAVOUR OF OTHERS USING

$$\boxed{I_i(\hat{B}_1, \hat{B}_2, \cdots, \hat{B}_N) = 0 \quad ; \quad i = 0, 1, 2, \cdots, p}$$

COLLECT ALL THE COEFFICIENTS OF LEFTOVER $\{\hat{B}_j| \quad j \neq a\}$

AND SET THEIR COEFFICIENTS TO 0 AND SOLVE FOR $\{c_i\}$

✦ **OPTION 2:** INTRODUCE LAGRANGE MULTIPLIERS $\{K_i|i = 1, 2, \cdots, p\}$

$$\downarrow$$

$$\sum_i^N \hat{B}_i c_i + \sum_{j=1}^p I_j K_j = 0$$

COLLECT COEFFICIENTS OF ALL THE $\{\hat{B}_i\}$

AND SET THEIR COEFFICIENTS TO 0 AND SOLVE FOR $\{c_i\}$

✦ **OPTION 3** **USE OPTION 1 AND OPTION 2 IN A MIXED WAY.**

OUR CASE

FEATURES

- **THERE ARE CONSTRAINTS.**

- **CONSTRAINTS NOT KNOWN IN LITERATURE** → **DISCOVER THEM**

SUPERFIELDS

$$\sum_{i}^{N} \hat{B}_i c_i + \sum_{j=1}^{p} I_j K_j = 0$$

PURE SPINOR WORLDSHEET OPERATORS

RESULT

$$
\begin{aligned}
U \;=\;& :\Pi^m\Pi^n F_{mn}: \;+\; :\Pi^m d_\alpha F_m{}^\alpha: \;+\; :\Pi^m\partial\theta^\alpha G_{m\alpha}: \;+\; :\Pi^m N^{pq} F_{mpq}: \\
&+ \;:d_\alpha d_\beta K^{\alpha\beta}: \;+\; :d_\alpha\partial\theta^\beta F^\alpha{}_\beta: \;+\; :d_\alpha N^{mn} G^\alpha{}_{mn}: \;+\; :\partial\theta^\alpha\partial\theta^\beta H_{\alpha\beta}: \\
&+ \;:\partial\theta^\alpha N^{mn} H_{mn\alpha}: \;+\; :N^{mn}N^{pq}G_{mnpq}:
\end{aligned}
$$

WHERE

$$
F_{mn} = -\frac{18}{\alpha'}G_{mn} \quad , \quad F_m{}^\alpha = \frac{288}{\alpha'}(\gamma^r)^{\alpha\beta}\partial_r\Psi_{m\beta} \quad , \quad G_{m\alpha} = -\frac{432}{\alpha'}\Psi_{m\alpha}
$$

$$
F_{mpq} = \frac{12}{(\alpha')^2}B_{mpq} - \frac{36}{\alpha'}\partial_{[p}G_{q]m} \quad , \quad K^{\alpha\beta} = -\frac{1}{(\alpha')^2}\,\gamma^{\alpha\beta}_{mnp}B^{mnp}
$$

$$
F^\alpha{}_\beta = -\frac{4}{\alpha'}(\gamma^{mnpq})^\alpha{}_\beta\partial_m B_{npq} \quad , \quad G^\alpha_{mn} = \frac{48}{(\alpha')^2}\gamma^{\alpha\sigma}_{[m}\Psi_{n]\sigma} + \frac{192}{\alpha'}\gamma^{\alpha\sigma}_r\partial^r\partial_{[m}\Psi_{n]\sigma}
$$

$$
H_{\alpha\beta} = \frac{2}{\alpha'}\gamma^{mnp}_{\alpha\beta}B_{mnp} \quad , \quad H_{mn\alpha} = -\frac{576}{\alpha'}\,\partial_{[m}\Psi_{n]\alpha} - \frac{144}{\alpha'}\partial^q(\gamma_{q[m})_\alpha{}^\sigma\Psi_{n]\sigma}
$$

$$
G_{mnpq} = \frac{4}{(\alpha')^2}\partial_{[m}B_{n]pq} + \frac{4}{(\alpha')^2}\partial_{[p}B_{q]mn} - \frac{12}{\alpha'}\partial_{[p}\partial_{[m}G_{n]q]}
$$

UNINTEGRATED VERTEX

- **FIRST EXCITED STATE OF OPEN STRING FORMS A SPIN 2 MULTIPLET COMPRISING**

$$\psi_{s\alpha} = \Psi_{s\alpha}|_{\theta=0} \qquad g_{mn} = G_{mn}|_{\theta=0} \qquad b_{mnp} = B_{mnp}|_{\theta=0}$$

128 fermionic d.o.f

44 + 84 bosonic d.o.f

- **IN THE UNINTEGRATED VERTEX THESE APPEAR AS**

$$B_{\alpha\beta} = (\gamma^{mnp})B_{mnp} \qquad C^{\alpha}{}_{\beta} = (\gamma^{mnpq})^{\alpha}{}_{\beta}C_{mnpq} \qquad H_{m\alpha} = -72\Psi_{m\alpha}$$

$$V = :\partial\theta^{\beta}\lambda^{\alpha}B_{\alpha\beta}: + :d_{\beta}\lambda^{\alpha}C^{\beta}_{\alpha}: + :\Pi^{m}\lambda^{\alpha}H_{m\alpha}: + :N^{mn}\lambda^{\alpha}F_{\alpha mn}:$$

$$H_{s\alpha} = -72\Psi_{s\alpha} = \frac{3}{7}(\gamma^{mn})_{\alpha}{}^{\beta}D_{\beta}B_{mns}\,, \quad C_{mnpq} = \frac{1}{2}\partial_{[m}B_{npq]}\,,$$

$$F_{\alpha mn} = \frac{1}{8}\left(7\partial_{[m}H_{n]\alpha} + \partial^{q}(\gamma_{q[m})_{\alpha}{}^{\beta}H_{n]\beta}\right)$$

CONSTRUCTION

STEP 1

WRITE DOWN THE MOST GENERAL OPERATOR CONSTRUCTED OUT OF BASIS WITH CONFORMAL WEIGHT 2 AND GHOST NUMBER 0.

PRODUCTS AND WORLDSHEET DERIVATIVE OF CONFORMAL WEIGHT 1 BASIS

NO λ^α

$$\{\Pi^m, d_\alpha, \partial\theta^\alpha, N^{mn}, J\}$$

$$\begin{aligned}
U \;=\; & :\partial^2\theta^\alpha C_\alpha: \;+\; :\partial\Pi^m C_m: \;+\; :\partial d_\alpha E^\alpha: \;+\; :(\partial J)C: \;+\; :\partial N^{mn}C_{mn}: \\
& + \; :\Pi^m\Pi^n F_{mn}: \;+\; :\Pi^m d_\alpha F_m{}^\alpha: \;+\; :\Pi^m N^{pq}F_{mpq}: \;+\; :\Pi^m J F_m: \;+\; :\Pi^m\partial\theta^\alpha G_{m\alpha}: \\
& + \; :d_\alpha d_\beta K^{\alpha\beta}: \;+\; :d_\alpha N^{mn}G^\alpha{}_{mn}: \;+\; :d_\alpha J F^\alpha: \;+\; :d_\alpha\partial\theta^\beta F^\alpha{}_\beta: \\
& + \; :N^{mn}N^{pq}G_{mnpq}: \;+\; :N^{mn}JP_{mn}: \;+\; :N^{mn}\partial\theta^\alpha H_{mn\alpha}: \\
& + \; :JJH: \;+\; :J\partial\theta^\alpha H_\alpha: \;+\; :\partial\theta^\alpha\partial\theta^\beta H_{\alpha\beta}:
\end{aligned}$$

STEP 2 **RULE OUT SUPERFIELDS THAT CANNOT HAVE THE PHYSICAL DEGREE OF FREEDOM BY DOING REST FRAME ANALYSIS**

$$\Downarrow$$

$$C_\alpha = C_m = E^\alpha = C = C_{mn} = F_m = F^\alpha = P_{mn} = H = H_\alpha = 0$$

EXAMPLE

$$\begin{tikzcd} C_\alpha \arrow[d] \\ \mathbf{16} \end{tikzcd} \qquad \begin{tikzcd} & C_m \arrow[dl] \arrow[dr] & \\ C_0 \arrow[d] & & C_i \arrow[d] \\ \mathbf{0} & & \mathbf{16} \end{tikzcd} \qquad \begin{tikzcd} & C_{mn} \arrow[dl] \arrow[dr] & \\ C_{0i} \arrow[d] & & C_{ij} \arrow[d] \\ \mathbf{0} & & \mathbf{36} \end{tikzcd}$$

✦ **A SUPERFIELD WITH ONE INDEX VANISHES.**

✦ **A SUPERFIELD WITH TWO ANTI-SYMMETRIC VECTOR INDICES VANISHES.**

STEP 3 **COMPUTE QU**

A FEW TERMS OF THE ABOVE COMPUTATION ARE

1. $\underline{\Pi^m \Pi^n F_{mn}}$

$$Q\,(:\Pi^m\Pi^n F_{mn}:) \;=\; \frac{\alpha'}{2}\Big[:\Pi^m\Pi^n\lambda^\alpha D_\alpha F_{mn}: \;+\; :\Pi^m(\gamma^n_{\alpha\beta})\partial\theta^\beta\lambda^\alpha\Big(F_{mn}+F_{nm}\Big):\Big]$$

2. $\underline{\Pi^m d_\alpha F_m{}^\alpha}$

$$\begin{aligned} Q\,\big(:\Pi^m d_\beta F_m{}^\beta:\big) \;&=\; -\frac{\alpha'}{2}\Big[:\Pi^m d_\beta\lambda^\alpha D_\alpha F_m{}^\beta: \;+\; :d_\beta(\gamma^m_{\alpha\sigma})\partial\theta^\sigma\lambda^\alpha F_m{}^\beta: \\ &\quad + \; :\Pi^m(\gamma^n_{\alpha\beta})\Pi_n\lambda^\alpha F_m{}^\beta:\Big] - \frac{1}{2}\left(\frac{\alpha'}{2}\right)^2\partial^2\lambda^\alpha\gamma^m_{\alpha\sigma}F_m{}^\sigma \\ &\quad + \frac{(\alpha')^2}{2}:\Pi^m(\gamma^n_{\alpha\beta})\partial\lambda^\alpha\partial_n F_m{}^\beta: \end{aligned}$$

3. $\underline{\Pi^m N^{pq} F_{mpq}}$

$$\begin{aligned} Q\left(:\Pi^m N^{pq} F_{mpq}:\right) &= \frac{\alpha'}{2}\Big[:\Pi^m N^{pq}\lambda^\alpha D_\alpha F_{mpq}: \; + \; :\partial\theta^\sigma N^{pq}(\gamma^m_{\alpha\sigma})\lambda^\alpha F_{mpq}:\Big] \\ &\quad -\frac{\alpha'}{4}\;:\Pi^m d_\alpha(\gamma^{pq})^\alpha{}_\beta\lambda^\beta F_{mpq}: -\frac{1}{2}\left(\frac{\alpha'}{2}\right)^2 :\Pi^m\partial\lambda^\beta(\gamma^{pq})^\alpha{}_\beta D_\alpha F_{mpq}: \\ &\quad -\frac{1}{2}\left(\frac{\alpha'}{2}\right)^2\Big[\partial^2\theta^\sigma\lambda^\beta\gamma^m_{\alpha\sigma}(\gamma^{pq})^\alpha{}_\beta F_{mpq} + \partial\theta^\sigma\partial\lambda^\beta\gamma^m_{\alpha\sigma}(\gamma^{pq})^\alpha{}_\beta F_{mpq}\Big] \end{aligned}$$

4. $\underline{\Pi^m\partial\theta^\beta G_{m\beta}}$

$$\begin{aligned} &Q\left(:\Pi^m\partial\theta^\beta G_{m\beta}:\right) \\ &\quad = \; -\frac{\alpha'}{2}:\Pi^m\partial\theta^\beta\lambda^\alpha D_\alpha G_{m\beta}: \; + \frac{\alpha'}{2}:\partial\theta^\sigma\partial\theta^\beta\lambda^\alpha\gamma^m_{\alpha\sigma}G_{m\beta}: \; + \frac{\alpha'}{2}\;:\Pi^m\partial\lambda^\beta G_{m\beta}: \end{aligned}$$

5. $\underline{d_\alpha d_\beta K^{\alpha\beta}}$

$$\begin{aligned} Q\left(:d_\alpha d_\beta K^{\alpha\beta}:\right) &= \frac{\alpha'}{2}:d_\sigma d_\beta\lambda^\alpha D_\alpha K^{\sigma\beta}: -\frac{\alpha'}{2}:\Pi_m d_\beta(x)\lambda^\alpha\gamma^m_{\alpha\sigma}\left[K^{\sigma\beta}(z)-K^{\beta\sigma}\right]: \\ &\quad +\frac{\alpha'^2}{2}:d_\beta\partial\lambda^\alpha\gamma^m_{\alpha\sigma}\partial_m\left[K^{\sigma\beta}-K^{\beta\sigma}\right]: + \left(\frac{\alpha'}{2}\right)^2\partial\theta^\delta\partial\lambda^\alpha\gamma_{m\beta\delta}\gamma^m_{\alpha\sigma}K^{\sigma\beta} \\ &\quad +\left(\frac{\alpha'}{2}\right)^2 :\gamma_{n\sigma\rho}\partial^2\theta^\rho(x)\lambda^\alpha(z)\gamma^n_{\alpha\beta}K^{\sigma\beta} \end{aligned}$$

FIVE MORE SUCH TERMS

STEP 4 **COMPUTE ∂V USING**

$$\partial S(X,\theta) = 2\Pi^m \partial_m S(X,\theta) + \partial\theta^\alpha D_\alpha S(X,\theta)$$

WORLDSHEET DERIVATIVE

SPACETIME DERIVATIVE

$$\begin{aligned}
\partial V \;=\; & :\partial\theta^\beta \partial\lambda^\alpha B_{\alpha\beta}: \;+\; :\Pi^m \partial\lambda^\alpha H_{m\alpha}: \;+\; :\partial^2\theta^\alpha \lambda^\beta \left(B_{\beta\alpha} + \alpha' \gamma^m_{\sigma\alpha} \partial_m C^\sigma{}_\beta\right): \\
& + \; :\partial\theta^\beta \partial\theta^\delta \lambda^\alpha D_\delta B_{\alpha\beta}: \;+\; :\Pi^m \partial\theta^\beta \lambda^\alpha \left(2\partial_m B_{\alpha\beta} + D_\beta H_{m\alpha}\right): \;+\; :\partial d_\beta \lambda^\alpha C^\beta{}_\alpha: \\
& + \; :d_\beta \partial\lambda^\alpha C^\beta{}_\alpha: \;+\; :d_\beta \partial\theta^\sigma \lambda^\alpha D_\sigma C^\beta{}_\alpha: \;+\; :2\Pi^m d_\beta \lambda^\alpha \partial_m C^\beta{}_\alpha: \;+\; :\partial\Pi^m \lambda^\alpha H_{m\alpha}: \\
& + \; :2\Pi^m \Pi^n \lambda^\alpha \partial_n H_{m\alpha}: \;+\; :\partial N^{mn} \lambda^\alpha F_{\alpha mn}: \;+\; :N^{mn} \partial\lambda^\alpha F_{\alpha mn}: \\
& + \; :\partial\theta^\beta N^{mn} \lambda^\alpha D_\beta F_{\alpha mn}: \;+\; :2\Pi^p N^{mn} \lambda^\alpha \partial_p F_{\alpha mn}:
\end{aligned}$$

- **NOTE THAT OPERATION WITH BRST CHARGE AND WORLDSHEET DERIVATIVE GIVES RISE TO 26 BASIS ELEMENTS**

$$\Pi^m\Pi^n\lambda^\alpha \;,\; \Pi^m d_\alpha\lambda^\beta \;,\; \Pi^m\partial\theta^\beta\lambda^\gamma \;,\; \Pi^m J\lambda^\alpha \;,\; \Pi^m N^{np}\lambda^\alpha \;,\; \partial\Pi^m\lambda^\alpha \;,\; \Pi^m\partial\lambda^\alpha$$

$$d_\alpha d_\beta\lambda^\gamma \;,\; d_\alpha\partial\theta^\beta\lambda^\gamma \;,\; d_\alpha J\lambda^\alpha \;,\; d_\alpha N^{mn}\lambda^\alpha \;,\; \partial d_\alpha\lambda^\beta \;,\; d_\alpha\partial\lambda^\beta$$

$$\partial\theta^\alpha\partial\theta^\beta\lambda^\gamma \;,\; \partial\theta^\alpha J\lambda^\beta \;,\; \partial\theta^\alpha N^{mn}\lambda^\alpha \;,\; \partial^2\theta^\alpha\lambda^\beta \;,\; \partial\theta^\alpha\partial\lambda^\beta$$

$$N^{mn}N^{pq}\lambda^\alpha \;,\; N^{mn}J\lambda^\alpha \;,\; \partial N^{mn}\lambda^\alpha \;,\; N^{mn}\partial\lambda^\alpha$$

$$JJ\lambda^\alpha \;,\; \partial J\lambda^\alpha \;,\; J\partial\lambda^\alpha$$

$$\partial^2\lambda^\alpha$$

CONFORMAL WEIGHT 2, GHOST NUMBER 1

STEP 5 **ADD SPECIAL ZEROS OF THE FORM** $\displaystyle\sum_{A=1}^{6} I_A K^A$

WHERE,

$$
\begin{aligned}
(I_1)^n_\beta &\equiv\ :N^{mn}J\lambda^\alpha:(\gamma_m)_{\alpha\beta}-\frac{1}{2}:JJ\lambda^\alpha:(\gamma^n)_{\alpha\beta}-\alpha':J\partial\lambda^\alpha:\gamma^n_{\alpha\beta}\ =\ 0\\
(I_2)^{mnq}_\beta &\equiv\ :N^{mn}N^{pq}\lambda^\alpha:(\gamma_p)_{\alpha\beta}-\frac{1}{2}:N^{mn}J\lambda^\alpha:(\gamma^q)_{\alpha\beta}-\alpha':N^{mn}\partial\lambda^\alpha:\gamma^q_{\alpha\beta}\ =\ 0\\
(I_3)^n_{\sigma\beta} &\equiv\ :d_\sigma N^{mn}\lambda^\alpha:(\gamma_m)_{\alpha\beta}-\frac{1}{2}:d_\sigma J\lambda^\alpha:(\gamma^n)_{\alpha\beta}-\alpha':d_\sigma\partial\lambda^\alpha:\gamma^n_{\alpha\beta}\ =\ 0\\
(I_4)^{pn}_\beta &\equiv\ :\Pi^p N^{mn}\lambda^\alpha:(\gamma_m)_{\alpha\beta}-\frac{1}{2}:\Pi^p J\lambda^\alpha:(\gamma^n)_{\alpha\beta}-\alpha':\Pi^p\partial\lambda^\alpha:\gamma^n_{\alpha\beta}\ =\ 0\\
(I_5)^{\sigma n}_\beta &\equiv\ :\partial\theta^\sigma N^{mn}\lambda^\alpha:(\gamma_m)_{\alpha\beta}-\frac{1}{2}:\partial\theta^\sigma J\lambda^\alpha:(\gamma^n)_{\alpha\beta}-\alpha':\partial\theta^\sigma\partial\lambda^\alpha:\gamma^n_{\alpha\beta}\ =\ 0\\
(I_6)^n_\beta &\equiv\ :\partial N^{mn}\lambda^\alpha:(\gamma_m)_{\alpha\beta}+:N^{mn}\partial\lambda^\alpha:(\gamma_m)_{\alpha\beta}-\frac{1}{2}:\partial J\lambda^\alpha:(\gamma^n)_{\alpha\beta}-\frac{1}{2}:J\partial\lambda^\alpha:(\gamma^n)_{\alpha\beta}\\
&\quad-\alpha'\gamma^n_{\alpha\beta}\partial^2\lambda^\alpha\ =\ 0
\end{aligned}
$$

$$\boxed{QU=\partial V+\sum_{a=1}^{6} I_a K_a}$$

STEP 6 COLLECT ALL THE TERMS WITH SAME BASIS

1. $\underline{\Pi^m \Pi^n \lambda^\alpha}$

$$\frac{\alpha'}{2}\left[D_\alpha F_{mn} - \gamma_{n\alpha\beta} F_m{}^\beta\right] = 2\partial_n H_{m\alpha}$$

2. $\underline{\Pi^m \partial\theta^\beta \lambda^\alpha}$

$$\frac{\alpha'}{2}\left[\gamma^n_{\alpha\beta}(F_{mn} + F_{nm}) - D_\alpha G_{m\beta} - \gamma^m_{\alpha\delta} F^\delta{}_\beta\right] = 2\partial_m B_{\alpha\beta} + D_\beta H_{m\alpha}$$

3. $\underline{d_\alpha \partial\theta^\beta \lambda^\sigma}$

$$\frac{\alpha'}{2}\left[-\gamma^m_{\sigma\beta} F_m{}^\alpha + D_\sigma F^\alpha_\beta - \frac{1}{2}(\gamma^{mn})^\alpha{}_\sigma H_{mn\beta}\right] = D_\beta C^\alpha{}_\sigma$$

4. $\underline{\Pi^m d_\beta \lambda^\alpha}$

$$\frac{\alpha'}{2}\left[-D_\alpha F_m{}^\beta - \frac{1}{2}(\gamma^{pq})^\beta{}_\alpha F_{mpq} - \gamma^m_{\alpha\sigma}\left(K^{\sigma\beta} - K^{\beta\sigma}\right)\right] = 2\partial_m C^\beta{}_\alpha$$

5. $\underline{\partial\theta^\alpha \partial\theta^\beta \lambda^\sigma}$

$$\frac{\alpha'}{2}\left[\gamma^m_{\sigma[\alpha} G_{m\beta]} + D_\sigma H_{\alpha\beta}\right] \quad = \quad D_{[\beta} B_{|\sigma|\alpha]}$$

6. $\underline{\partial\Pi_m \lambda^\alpha}$

$$\frac{(\alpha')^2}{8}(\gamma_m \gamma^{pq})_{\beta\alpha} G^\beta_{pq} = H_{m\alpha}$$

7. $\underline{d_\alpha d_\beta \lambda^\sigma}$

$$\frac{\alpha'}{2}\left[D_\sigma K^{\alpha\beta} + \frac{1}{2}(\gamma^{mn})^\beta{}_\sigma G^\alpha_{mn}\right] = 0$$

8. $\underline{\partial^2\theta^\beta\lambda^\alpha}$

$$\frac{\alpha'}{2}\left[-\frac{\alpha'}{4}\gamma^m_{\beta\sigma}(\gamma^{pq})^\sigma{}_\alpha F_{mpq} + \frac{\alpha'}{2}\gamma^m_{\delta\beta}\gamma_{m\alpha\sigma}K^{\delta\sigma}\right] = B_{\alpha\beta} + \alpha'\gamma^m_{\sigma\beta}\partial_m C^\sigma{}_\alpha$$

9. $\underline{\Pi^m N^{pq}\lambda^\alpha}$

$$\frac{\alpha'}{2}\left[D_\alpha F_{mpq} - \gamma_{m\alpha\beta}G^\beta{}_{pq}\right] = 2\partial_m F_{\alpha pq} + (\gamma_{[p})_{\alpha\beta}(K_4)^\beta{}_{|m|q]}$$

10. $\underline{\Pi^m J\lambda^\alpha}$

$$0 = -\frac{1}{2}\gamma^q{}_{\alpha\beta}(K_4)^\beta{}_{mq}$$

11. $\underline{\Pi^m\partial\lambda^\alpha}$

$$\frac{\alpha'}{2}\left[\alpha'\gamma^n_{\alpha\beta}\partial_n F_m{}^\beta - \frac{\alpha'}{4}(\gamma^{pq})^\beta{}_\alpha D_\beta F_{mpq} + G_{m\alpha} + \frac{\alpha'}{4}(\gamma_m\gamma^{pq})_{\beta\alpha}G^\beta_{pq}\right]$$

$$= H_{m\alpha} - \alpha'\gamma^q_{\alpha\beta}(K_4)^\beta{}_{mq}$$

12. $\underline{\partial\theta^\alpha N^{mn}\lambda^\beta}$

$$\frac{\alpha'}{2}\left[\gamma^p_{\alpha\beta}F_{pmn} - D_\beta H_{mn\alpha}\right] \quad = \quad D_\alpha F_{\beta mn} + (\gamma_{[m})_{\beta\sigma}(K_5)^\sigma{}_{\alpha n]}$$

14 MORE SUCH TERMS

STEP 7

WRITE DOWN THE ANSATZ FOR SUPERFIELDS OF INTEGRATED VERTEX AND THE LAGRANGE MULTIPLIERS

✤ SUPERFIELDS APPEARING IN INTEGRATED VERTEX

$$F_{mn} = f_1 G_{mn} \quad , \quad G_{m\alpha} = g_1 \Psi_{m\alpha}$$

$$K^{\alpha\beta} = a\, \gamma^{\alpha\beta}_{mnp} B^{mnp} \quad , \quad H_{\alpha\beta} = h_1 \gamma^{mnp}_{\alpha\beta} B_{mnp}$$

$$F^{\alpha}{}_{\beta} = f_5 (\gamma^{mnpq})^{\alpha}{}_{\beta} k_m B_{npq} \quad , \quad F^{\alpha}_{m} = f_2 k^r (\gamma_r)^{\alpha\beta} \Psi_{m\beta}$$

$$F_{mpq} = f_3 G_{m[p} k_{q]} + f_4 B_{mpq} \quad , \quad G^{\beta}_{pq} = g_2 \gamma^{\beta\sigma}_{[p} \Psi_{q]\sigma} + g_3 k^r \gamma^{\beta\sigma}_{r} k_{[p} \Psi_{q]\sigma}$$

$$H_{mn\alpha} = h_2\, k_{[m} \Psi_{n]\alpha} + h_3 k^q (\gamma_{q[m})_{\alpha}{}^{\sigma} \Psi_{n]\sigma}$$

$$G_{mnpq} = g_4 k_{[m} B_{n]pq} + g_5 k_{[p} B_{q]mn} + g_6 k_{[m} G_{n][p} k_{q]} + g_7\, \eta_{[m[p} G_{q]n]}$$

✤ LAGRANGE MULTIPLIER SUPERFIELDS

$$
\begin{aligned}
(K_1)^\alpha_m &= c_1 k^r (\gamma_r)^{\alpha\beta}\Psi_{m\beta} \\
(K_2)^\alpha_{mnq} &= c_2 k_{[m}\gamma^{\alpha\beta}_{n]}\Psi_{q\beta} + c_3 k_q \gamma^{\alpha\beta}_{[m}\Psi_{n]\beta} + c_4 \gamma^{\alpha\beta}_q k_{[m}\Psi_{n]\beta} + c_5 k^r \gamma^{\alpha\beta}_{rmn}\Psi_{q\beta} + c_6 k^r \gamma^{\alpha\beta}_{rq[m}\Psi_{n]\beta} \\
&\quad + c_7 k^r k_q \gamma^{\alpha\beta}_r k_{[m}\Psi_{n]\beta} + c_8 k^r \gamma^{\alpha\beta}_r \eta_{q[m}\Psi_{n]\beta} \\
(K_3)^{\alpha\beta}_m &= c_9 G_{mn}(\gamma^n)^{\alpha\beta} + c_{10} k_m B_{stu}(\gamma^{stu})^{\alpha\beta} + c_{11} k_s B_{tum}(\gamma^{stu})^{\alpha\beta} + c_{12} k_s B_{tuv}(\gamma_m{}^{stuv})^{\alpha\beta} \\
(K_4)^\alpha_{mn} &= c_{13}(\gamma_n)^{\alpha\beta}\Psi_{m\beta} + c_{14}(\gamma_m)^{\alpha\beta}\Psi_{n\beta} + c_{15} k^r k_m (\gamma_r)^{\alpha\beta}\Psi_{n\beta} + c_{16} k^r k_n (\gamma_r)^{\alpha\beta}\Psi_{m\beta} \\
(K_5)^\alpha{}_{\beta m} &= c_{17} k_p G_{qm}(\gamma^{pq})^\alpha{}_\beta + c_{18} B_{mpq}(\gamma^{pq})^\alpha{}_\beta + c_{19} B_{pqr}(\gamma_m{}^{pqr})^\alpha{}_\beta + c_{20} k_m k_p B_{qrs}(\gamma^{pqrs})^\alpha{}_\beta \\
(K_6)^\alpha_m &= c_{21} k^r (\gamma_r)^{\alpha\beta}\Psi_{m\beta}
\end{aligned}
$$

SUBSTITUTE THESE ANSATZ IN THE 26 EQUATIONS

STEP 8 **ELIMINATE THE BASES FOR THE CONSTRAINTS FOR WHICH THE LAGRANGE MULTIPLIERS ARE NOT INTRODUCED.**

EXAMPLE

✤ **CONSIDER THE CONSTRAINT** $:d_\alpha d_\beta: \; + \; :d_\beta d_\alpha: \; + \; \frac{\alpha'}{2}\partial\Pi^t(\gamma_t)_{\alpha\beta} \; = \; 0$

✤ **THIS RELATES**

6. $\underline{\partial\Pi_m\lambda^\alpha}$

$$\frac{(\alpha')^2}{8}(\gamma_m\gamma^{pq})_{\beta\alpha}G^\beta_{pq} = H_{m\alpha}$$

7. $\underline{d_\alpha d_\beta\lambda^\sigma}$

$$\frac{\alpha'}{2}\left[D_\sigma K^{\alpha\beta} + \frac{1}{2}(\gamma^{mn})^\beta{}_\sigma G^\alpha_{mn}\right] = 0$$

✤ **RE-EXPRESS 6 COMPLETELY IN TERMS OF 7** → **ONE EQUATION LESS**

STEP 9

SUBSTITUTE THE ANSATZ AND SET COEFFICIENTS OF ALL THE BASIS TO ZERO.

EQUATIONS RELATING

$$a, \{f_1, f_2, \cdots, f_5, \}, \{g_1, g_2, \cdots, g_7\}, h_1, h_2, h_3, \{c_1, c_2, \cdots, c_{21}\}$$

STEP 10

SOLVE FOR THE ABOVE EQUATIONS

$$a = -\frac{1}{\alpha'^2} \quad , \quad f_1 = -\frac{18}{\alpha} \quad , \quad f_2 = \frac{288i}{\alpha} \quad , \quad f_3 = \frac{36i}{\alpha'}$$

$$f_4 = \frac{12}{\alpha'^2} \quad , \quad f_5 = -\frac{4i}{\alpha'} \quad , \quad g_1 = -\frac{432}{\alpha'} \quad , \quad g_2 = \frac{48}{\alpha'^2}$$

$$g_3 = -\frac{192}{\alpha'} \quad , \quad g_4 = \frac{4i}{\alpha'^2} \quad , \quad g_5 = \frac{4i}{\alpha'^2} \quad , \quad g_6 = -\frac{12}{\alpha'}$$

$$g_7 = \frac{3}{2\alpha'^2} \quad , \quad h_1 = \frac{2}{\alpha'} \quad , \quad h_2 = -\frac{576i}{\alpha'} \quad , \quad h_3 = -\frac{144i}{\alpha'}$$

HOW DO WE KNOW ITS THE CORRECT SOLUTION?

Thank You