Cosmological Constant Problem and Scale Invariance

Taichiro Kugo
Maskawa Institute, Physics Dept., Kyoto Sangyo University

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1 Cosmological Constant Problem

Dark Clouds hanging over the two well-established theories

Quantum Field Theory $\Longleftrightarrow$ Einstein Gravity Theory

We know the recently observed Dark Energy Λ_0, which looks like a small Cosmological Constant (CC):

$$\text{Present observed CC} \quad 10^{-29}\text{gr/cm}^3 \sim 10^{-47}\text{GeV}^4 \equiv \Lambda_0 \tag{1}$$

We do not mind this tiny CC now, which will be explained after our CC problem is solved. However, we use it as the scale unit Λ_0 of our discussion.

What is the true problem?

Essential point: multiple mass scales are involved!

There are several dynamical symmetry breakings and they are necessarily accompanied by vacuum condensation energy:

In particular, we are confident from the success of the Standard Model of the existence of at least two symmetry breakings:

$$\text{Higgs Condensation} \sim \quad (\,200\,\text{GeV}\,)^4 \sim 10^9\text{GeV}^4 \sim 10^{56}\Lambda_0$$
$$\text{QCD Chiral Condensation } \langle \bar{q}q \rangle^{4/3} \sim \quad (\,200\,\text{MeV}\,)^4 \sim 10^{-3}\text{GeV}^4 \sim 10^{44}\Lambda_0$$

Nevertheless, these seem not contributing to the Cosmological Constant!

It is a Super fine tuning problem:

$$c \;:\; \text{initially prepared CC } (> 0)$$

$$c - 10^{56}\Lambda_0 \;:\; \text{should cancell, but leaving 1 part per } 10^{12}\text{; i.e., } \sim 10^{44}\Lambda_0$$

$$c - 10^{56}\Lambda_0 - 10^{44}\Lambda_0 \;:\; \text{should cancell, but leaving 1 part per } 10^{44}\text{; i.e., } \sim \Lambda_0$$

$$c - 10^{56}\Lambda_0 - 10^{44}\Lambda_0 \sim \Lambda_0 \;:\; \text{present Dark Energy}$$

$$c = \text{initially prepared CC}$$

$$\underbrace{654321,098765}_{\text{12 digits}}4321,0987654321,0987654321,0987654321,0987654321 \times \Lambda_0 \sim 10^{56}\Lambda_0$$

$$c + V_{\text{Higgs}} =$$

$$\underbrace{4321,0987654321,0987654321,0987654321,0987654321}_{\text{44 digits}} \times \Lambda_0 \sim 10^{44}\Lambda_0$$

$$c + V_{\text{Higgs}} + V_{\text{QCD}} = \text{present Dark Energy}$$

$$1 \times \Lambda_0 \sim \Lambda_0$$

Note that the vacuum energy is almost totally cancelled at each stage of spontaneous breaking as far as the the relevant energy scale order.

2 Vacuum Energy $\simeq$ vacuum condensation energy

"Two" origins of Cosmological Constant

Vacuum Energy in QFT:

$$\sum_{\boldsymbol{k},s} \frac{1}{2}\hbar\omega_{\boldsymbol{k}} - \sum_{\boldsymbol{k},s} \hbar E_{\boldsymbol{k}} \tag{2}$$

Vacuum Condensation Energy:

$$V(\phi_c) : \text{potential} \tag{3}$$

They are separately stored in our (or my, at least) memory, but actually, almost the same object, as we see now.

"Vacuum energy", which we learn in the beginning of QFT textbook, comes from the normal ordering of the creation and annihiration opeartors of free particles, and is infinite.

But, once the infinity is renormalized for the massless case, or cancelled between fermions and bosons, for instance, by Supersymmetry, then the rest vacuum energy density observed as the deviation from the massless one can be computed finite and in fact be counted in the vacuum condensation energies $V(\phi_c)$: Consider the chiral quark condensation in QCD. For simplicity, consider NJL model as a parallel model for the realistic QCD:

$$\begin{aligned} \mathcal{L}_{\rm NJL} &= \bar{q}i\gamma^\mu\partial_\mu q + \frac{G}{4}\left[(\bar{q}q)^2 + (\bar{q}i\gamma_5 q)^2\right] \\ &\rightarrow \quad \bar{q}(i\gamma^\mu\partial_\mu - \sigma - i\gamma_5\pi)q - \frac{1}{G}(\sigma^2 + \pi^2) \end{aligned}$$

The effective potential $V(\sigma,\pi)$ is a function of $\sigma^2+\pi^2$ and can be computed at the $\pi=0$ section $V(\sigma)=V(\sigma,\pi=0)$:

$$V(\sigma)=\frac{1}{G}\sigma^2-\int\frac{d^4p}{i(2\pi)^4}\ln\det(\not{p}-\sigma)$$

But the second term is nothing but the vacuum energy

$$-\int\frac{d^4p}{i(2\pi)^4}\ln\det(\not{p}-\sigma)=-\sum_{\boldsymbol{p},s}\hbar\sqrt{\boldsymbol{p}^2+\sigma^2}+(\sigma\text{-independent const})$$

implying that

$$\langle\bar{q}q\rangle\ \text{ condensation energy }\ \simeq\ \text{ Dirac sea vacuum energy} \tag{4}$$

Moreover, in a Shwinger-Dyson approach to realistic QCD, the quark mass is calculated as a mass function $\Sigma(p)$ possessing the support only $\lesssim\Lambda_{\mathrm{QCD}}$, and the condensation energy is computed finite.

3 UV Quantum Gravity is irrelevant

CC problem is to be considered in Einstein Gravity theory.

$$\text{Einstein gravity is a unique Low Energy Effective Theory} \tag{5}$$

Just like Chiral Lagrangian

$$\mathcal{L} = f_\pi^2 \,\mathrm{tr}\left(\partial_\mu U^\dagger \partial^\mu U\right)$$
$$U = \exp(i\pi/f_\pi), \qquad \pi = \pi^a(x)T^a$$

is a unique Effective Theory in the low energy region $E \lesssim f_\pi$, i.e., in the lowest (second) order in the derivative. We know that the fundamental theory describing the strong interaction is QCD. But, whatever the dynamical theory is beyond $E > f_\pi$, the sysytem is described by the the Nambu-Goldstone (NG) bosons $\boldsymbol{\pi}$ based on the coset $SU(3)_L \times SU(3)_R/SU(3)_V$, and the dynamics is uniquely described by this non-linear sigam model. The non-linearly realized chiral symmetry uniquely determines the dynamics of the NG bosons, self-coupling and coupling to other matters in the low energy regime. Moreover, even the quantum correction in this system can be computed by this Lagrangian in the sense of Weinberg.

In exactly the same manner, the general coordinate (GC) invariance uniquely determine the Lagrangian in the lowest (second) order in the derivative; that is, it is the Einstein-Hilbert action. In this analogy, it is worth noticing

Graviton is a NG tensor boson corresponding to GL(4) $\to$ SO(3,1)

Nakanishi-Ojima (1979)

So the Einstein-Hilbert action is exactly analogous to the chiral Lagrangian, and $M_{\rm Pl}$ is the counterpart of the pion decay constant f_π:

$$S_{\rm eff} = \int d^4x\,\sqrt{-g}\Big\{c_0 M_{\rm Pl}^4 + c_1 M_{\rm Pl}^2 R + c_2 R^2 + c_3 R_{\mu\nu}R^{\mu\nu} + \cdots\Big\}$$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}/M_{\rm Pl}$$

The CC term (with no derivatives) is consistent with GC invariance and its natural scale is $O(M_{\rm Pl}^4)$.

Below the Planck energy scale $M_{\rm Pl}$, the dynamics is uniquely described by the E-H action plus interaction terms with matter fields. The UV quantum gravity is quite irrelevant to any problem in much lower energy region than Planck sacale, $E \ll M_{\rm Pl}$, in particular, to the CC problem associated with the spontaneous breaking of Electro-weak symmetry and chiral symmetry.

4 Scale Invariance solves the problem!

Our world is almost scale invariant: that is, the standard model Lagrangian is scale invariant except for the Higgs mass term!

If the Higgs mass term comes from the spontaneous breaking of scale invariance at higher energy scale physics, the total system can be really be scale invariant.

4.1 Classical Scale Invariance

Suppose that our world has no dimensionful parameters.

Suppose that the effective potential V of the total system looks like

$$\begin{array}{ccccccc} V(\phi) = & V_0(\Phi) & + & V_1(\Phi, h) & + & V_2(\Phi, h, \varphi) \\ & \downarrow & & \downarrow & & \downarrow \\ & M & \gg & \mu & \gg & m \end{array}$$

and it is scale invariant. Then, classically, it satisfies the scale invariance relation :

$$\sum_i \phi^i \frac{\partial}{\partial \phi^i} V(\phi) = 4V(\phi), \tag{6}$$

so that the vacuum energy vanishes at any stationary point $\langle \phi^i \rangle = \phi_0^i$:

$$V(\phi_0) = 0.$$

Important point is that this holds at every stages of spontaneous symmetry breaking.

In the above potential V, we can retain only $V_0(\Phi)$ when discussing the physics at scale M, since h and φ are expected to get VEVs of order μ or lower. Then the scale invariance guarantees $V_0(\Phi_0) = 0$.

If we discuss the next stage spontaneous breaking at energy scale μ, we should take $V_0(\Phi) + V_1(\Phi, h)$, and can conclude $V_0(\Phi_0) + V_1(\Phi_0, h_0) = 0$.

Similarly, at scale m, we have the potential $V_0(\Phi) + V_1(\Phi, h) + V_2(\Phi, h, \varphi)$, and can conclude $V_0(\Phi_0) + V_1(\Phi_0, h_0) + V_2(\Phi_0, h_0, \varphi_0) = 0$.

This miracle is realized since the scale invariance holds at each energy scale of spontaneous symmetry breaking.

For the help of understanding, we now write a toy model of potentials.

$$V_0(\Phi) = \frac{1}{2}\lambda_0(\Phi_1^2 - \varepsilon_0\Phi_0^2)^2,$$

in terms of two real scalars Φ_0, Φ_1, to realize a VEV

$$\langle\Phi_0\rangle = M \quad \text{and} \quad \langle\Phi_1\rangle = \sqrt{\varepsilon_0}M \equiv M_1. \tag{7}$$

This M is totally spontaneous and we suppose it be Planck mass giving the Newton coupling constant via the scale invariant Einstein-Hilbert term

$$S_{\text{eff}} = \int d^4x\, \sqrt{-g}\Big\{c_1\Phi_0^2\, R + c_2 R^2 + c_3 R_{\mu\nu}R^{\mu\nu} + \cdots\Big\}$$

If GUT stage exists, ε_0 may be a constant as small as 10^{-4} and then Φ_1 gives the scalar field breaking GUT symmetry; e.g., $\Phi_1 : \mathbf{24}$ causing $SU(5) \to SU(3) \times SU(2) \times U(1)$.

$V_1(\Phi, h)$ part causes the electroweak symmetry breaking:

$$V_1(\Phi, h) = \frac{1}{2}\lambda_1 \left(h^\dagger h - \varepsilon_1\Phi_1^2\right)^2 ,$$

with very small parameter $\varepsilon_1 \simeq 10^{-28}$. This reproduces the Higgs potential when h is the Higgs doublet field and $\varepsilon_1\Phi_1^2$ term is replaced by the VEV $\varepsilon_1 M_1^2 = \mu^2/\lambda_1$.

$V_2(\Phi, h, \varphi)$ part causes the chiral symmetry breaking, e.g., $\mathrm{SU(2)_L}\times\mathrm{SU(2)_R} \to \mathrm{SU(2)_V}$. Using the 2×2 matrix scalar field $\varphi = \sigma + i\boldsymbol{\tau}\cdot\boldsymbol{\pi}$ (chiral sigma-model field), we may similarly write the potential

$$V_2(\Phi, h, \varphi) = \tfrac{1}{4}\lambda_2 \left(\mathrm{tr}(\varphi^\dagger\varphi) - \varepsilon_2\Phi_1^2\right)^2 + V_{\text{break}}(\Phi, h, \varphi)$$

with another small parameter $\varepsilon_2 \simeq 10^{-34}$. The first term reproduces the linear σ-model potential invariant under the chiral $\mathrm{SU(2)_L}\times\mathrm{SU(2)_R}$ transformation $\varphi \to g_{\mathrm{L}}\varphi g_{\mathrm{R}}$ when $\varepsilon_2\Phi_1^2$ is replaced by the VEV $\varepsilon_2 M_1^2 = m^2/\lambda_2$. The last term V_{break} stands for the chiral symmetry breaking term which is caused by the explicit quark mass terms appearing as the result of tiny Yukawa couplings of u, d quarks, y_u, y_d, to the Higgs doublet h; e.g.,

$$V_{\text{break}}(\Phi, h, \varphi) = \frac{1}{2}\varepsilon_2 \Phi_1^2 \,\text{tr}\left(\varphi^\dagger \left(y_u \epsilon h^* \;\; y_d h\right) + \text{h.c.}\right)$$

4.2 Quantum Mechanically

However, we have neglected the scale invariance anomaly in quantum field theory. Actually, if we take account of the renormalization point μ, we have the RGE

$$\left(\mu\frac{\partial}{\partial\mu} + \sum_a \beta_a(\lambda)\frac{\partial}{\partial\lambda_a} + \sum_i \gamma_i(\lambda)\phi_i\frac{\partial}{\partial\phi_i}\right)V(\phi) = 0$$

and the dimension counting identity

$$\left(\mu\frac{\partial}{\partial\mu} + \sum_i \phi_i\frac{\partial}{\partial\phi_i}\right)V(\phi) = 4V(\phi).$$

From these we obtain

$$\left(\sum_i (1-\gamma_i(\lambda))\phi_i\frac{\partial}{\partial\phi_i} - \sum_a \beta_a(\lambda)\frac{\partial}{\partial\lambda_a}\right)V(\phi) = 4V(\phi)$$

This is the correct equation in place of the above naive one:

$$\sum_i \phi_i\frac{\partial}{\partial\phi_i}\,V(\phi) = 4V(\phi)$$

This shows the anomalous dimension $\gamma_i(\lambda)$ is not the problem.
$\beta_a(\lambda)$ terms may be problematic:

$$\longrightarrow \qquad 4\,V(\phi_0) = -\sum_a \beta_a(\lambda)\frac{\partial}{\partial\lambda_a}\,V(\phi_0)$$

So, an obvious possibility is that all the coupling constants go to the Infrared Fixed Points: $\beta_a(\lambda_{\rm IR}) = 0$. But,

What does this equation really imply?

We argue that the potential value $V(\phi_0)$ at the stationary point $\phi = \phi_0$, $\left.\dfrac{dV}{d\phi}\right|_{\phi=\phi_0} = 0$, is zero at any μ, even before reaching the IR limit $\mu = 0$; that is, *The vanishing property of the stationary potential value $V(\phi)$ is not injured by the scale-inv anomaly.*

The potential value $V(\phi_0) = V_0(\lambda;\mu^2)$ at stationary points satisfies the RGE:

$$\left(\mu\frac{\partial}{\partial\mu} + \sum_a \beta_a(\lambda)\frac{\partial}{\partial\lambda_a}\right) V_0(\lambda;\mu^2) = 0$$

(The first term $\mu\partial/\partial\mu$ may be replaced by 4 since $V_0(\lambda;\mu^2) = \mu^4 v(\lambda)$.)

The solution is given by

$$V_0(\lambda;\mu^2) = V(\bar{\lambda}(t);\mu^2 e^{2t}),$$

where $t = \ln \mu$,

$$\frac{d\bar{\lambda}_a(t)}{dt} = \beta_a(\bar{\lambda}(t)) \quad \text{with} \quad \bar{\lambda}_a(t=0) = \lambda_a.$$

Or, writing $V_0(\lambda;\mu^2) = \mu^4 v(\lambda)$, we have

$$\mu^4 v(\lambda) = (\mu^2 e^{2t})^2 v(\bar{\lambda}(t)) \quad \rightarrow \quad v(\bar{\lambda}(t)) = e^{-4t} v(\lambda).$$

We now assume that the theory has an IR fixed point $\lambda_{\rm IR}$.
Taking the IR limit $t \rightarrow -\infty$ ($\mu \rightarrow 0$) gives

$$v(\lambda_{\rm IR}) = e^{+\infty} v(\lambda).$$

If $v(\lambda_{\rm IR})$ is finite, then we must have

$$v(\lambda) = 0 \quad \rightarrow \quad V_0(\lambda;\mu^2) = 0.$$

That is, provided that IR fixed point $\lambda_{\rm IR}$, as well as the theory on top of that point, exist, then, the vanishing property of the potential value at stationary point is not injured by the anomaly!

5 Example Calculation in $\lambda\phi^4$ theory

One-loop RGE-improved tree potential:

$$V(\phi, \lambda; \mu^2) = \frac{\lambda}{4!} \frac{1}{1 - \dfrac{3\lambda}{32\pi^2} \ln \dfrac{\frac{1}{2}\lambda\phi^2}{\mu^2}} \phi^4$$

Or, denoting $4\pi\phi = \varphi$, $\frac{1}{2}\lambda\phi^2 = \alpha\varphi^2$, $\frac{\lambda}{32\pi^2} = \alpha$,

$$192\pi^2 V = \frac{\alpha\varphi^4}{1 - 3\alpha \ln(\alpha\varphi^2/\mu^2)}$$

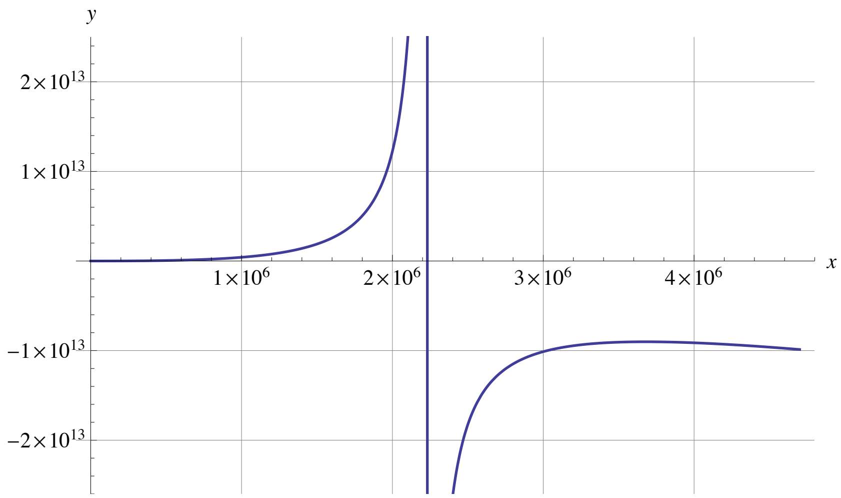

図 1: $y = \alpha x^2/(1 - 3\alpha \ln(\alpha x))$, $\alpha = 3/100$, $\mu = 1$

V has two stationary points φ_0^2 at the origin and φ_1^2, beyond the Landau pole:

$$\varphi_0^2 = 0 \qquad \text{and} \qquad \varphi_1^2 = \frac{\mu^2}{\alpha} \exp\left(\frac{1}{2} + \frac{1}{3\alpha}\right) \;\rightarrow\; \infty\, e^{\infty} \;\; \text{as } \alpha \rightarrow 0+\ . \tag{8}$$

Stationary Values:

$$V(\varphi_0) = 0 \quad \text{and} \quad V(\varphi_1) = -\frac{2}{3}\varphi_1^4 \;\rightarrow\; -\infty^2 e^{\infty} \qquad \text{as } \alpha \rightarrow 0+$$

According to our general argument, the dimensionless $v(\lambda) \equiv V(\varphi_0^2)/\mu^4$ must vanish or oterwise divergent in the infrared limit. In this case, IR fixed point is $\lambda_{\rm IR} = 0$, and $v(\lambda) = 0$ for the φ_1 point, and for the fake φ_0 point,

$$\lim_{\mu \rightarrow 0} v(\lambda) = -\frac{2}{3} \lim_{\alpha \rightarrow 0+} \frac{1}{\alpha^2} \exp\left(1 + \frac{2}{3\alpha}\right) \;\rightarrow\; -\infty^2 e^{\infty}. \tag{9}$$

6 Gauge Hierarchy

The gauge hierarchy problem has two aspects:

1. Origin: to explain the origin why the hierarchy exist.

2. Stability: to explain its satability against radiative corrections, once it exists anyway.

As for the stability against the radiative correction, it is guaranteed, for instance, by the SUSY as a well-known example. In the present classically scale invariant theory, there exist only the logarithmic divergences but no quadratic divergences, so that the stability is automatic, as was emphasized by Bardeen in 1980's.

(Note that $h^\dagger h \Phi_1^2 \times$ Log term appears always with ε_1.)

W. A. Bardeen, "On naturalness in the standard model," FERMILAB-CONF-95-391-T.

In the above toy model, we have "realized" large gauge hierarchies simply by assuming tiny parameters $\varepsilon_1 \simeq 10^{-28}$, $\varepsilon_2 \simeq 10^{-34}$:

$$V_1 = \frac{1}{2}\lambda_2 \left(h^\dagger h - \varepsilon_1 \Phi_1^2\right)^2 \quad \text{and} \quad V_2 \supset \tfrac{1}{4}\lambda_2 \left(\mathrm{tr}(\varphi^\dagger \varphi) - \varepsilon_2 \Phi_1^2\right)^2$$

However, the chiral symmetry breaking scale ε_2 is, for instance, determined as follows; if GUT is assumed, the SU(3) gauge coupling $\alpha_3 = g_3^2/4\pi$ at scale $\sqrt{\varepsilon_0}M \equiv M_1$ evolves a la RGE, $\bar{\alpha}_3(\mu)$, as the scale μ, and reaches to the $O(1)$ critical coupling $\alpha_3^{\rm cr} \simeq 1$ at scale $\mu \simeq \Lambda_{\rm QCD}$ to break the chiral symmetry, so that $\sqrt{\varepsilon_2}M_1 \simeq \Lambda_{\rm QCD}$. Thus the relation between the GUT scale M_1 and QCD scale $\Lambda_{\rm QCD}$ is fixed by the gauge coupling constant $\alpha_3(M_1)$ at scale M_1 as

$$\mu \frac{d}{d\mu}\alpha_3(\mu) = 2b_3\, \alpha_3^2(\mu) \quad \to \quad \frac{1}{\alpha_3(\mu)} = \frac{1}{\alpha_3(M_1)} - b_3 \ln \frac{\mu^2}{M_1^2}$$
$$\to \quad \varepsilon_2 = \frac{\Lambda_{\rm QCD}^2}{M_1^2} = \exp \frac{1}{b_3}\Big(\frac{1}{\alpha_3(M_1)} - \frac{1}{\alpha_3^{\rm cr}}\Big). \qquad (10)$$

The parameter ε_1 determining electroweak breaking scale would also be determined similarly, for instance, if there is a sub-level gauge interactions like Technicolor.

The idea that scale invariance may play an important role for solving the cosmological constant problem, was also proposed by several authors:

E. Rabinovici, B. Saering and W. A. Bardeen, Phys. Rev. D **36** (1987) 562.

M. Shaposhnikov and D. Zenhausern, Phys. Lett. B **671** (2009) 162

C. Wetterich, Nucl. Phys. B **302** (1988) 668.

However, the former two require exact scale invariance also in quantum theory, and no one emphasized the miraculous mechanism of cancelling vacuum condensation energies associated with the multi-step spontaneous symmetry breaking.

The latter two also require the asymptotic safety property for gravity, but the UV property beyond Planck scale should be irrelevant to the essential point of the cosmological constant problem as emphasized here.

7 What should be done next?

7.1 Flat Direction, Or Dimensional Transmutation

We are considering very **nontrivial** possibilities.

Two stationary points $\boldsymbol{\phi}_0 \equiv (\phi_{0i})$, $\frac{\partial V(\boldsymbol{\phi})}{\partial \phi_i}|_{\boldsymbol{\phi}=\boldsymbol{\phi}_0} = 0$ at least:

$$\begin{aligned} \boldsymbol{\phi}_0 = \mathbf{0} &: \text{trivial point at origin} \\ \exists \boldsymbol{\phi}_0 \neq \mathbf{0} &: \text{non-trivial point we need, giving the scale of the world} \end{aligned} \tag{11}$$

Classically, $V(\boldsymbol{\phi})$ is scale invariant, so

$\rho \boldsymbol{\phi}_0$ with $\forall \rho \in \boldsymbol{R}$ also give degenerate stationary points. $\longrightarrow$ Flat Direction

Quantum Mechanically, consider the function

$$f(\rho) \equiv V(\rho \boldsymbol{\phi}_0), \quad \text{satisfying} \quad f(0) = f(1) = 0, \quad f'(0) = f'(1) = 0 \tag{12}$$

We have two possibilities:

1) Flat direction is kept: $f(\rho) \equiv 0$. Choosing $\rho = 1$ is totally spontaneous.

But, the flat direction of V does not necessarily survive the radiative corrections

with classical scale-invariance alone!

We need, e.g.,

1-1. Quantum scale invariance (Shaposhnikov-Zenhausern's Exact SI prescription):
Englert-Truffin-Gastmans, Nuc. Phys. B177(1976)407.

1-2. Additional Symmetry, like SUSY.

2) Dimensional transmutation: $f(\rho) = f'(\rho) = 0$ only at $\rho = 0$ and $\rho = 1$ (or $\rho = \pm 1$)

That is, the scale is fixed by anomaly, explicit breaking.

I prefer this, and suppose it occurs for QCD. (Higashijima-Miransky's SD approach)

Any case is Very non-trivial!

7.2 Other Problems

1. More evidence, or proof, for the claim that
   $V(\phi_0) = 0$ is not injured by quantum anomally for scale-invariance.
2. Gauge hierarchies; how do those potentials appear possessing tiny ε_i's?
3. Global or Local scale invariance?
4. If global, What is $\exists$Dilaton? $\rightarrow$ Higgs ?
5. The fate of dilaton? $\rightarrow$ quantum anomaly would make it massive!
6. How is the present CC value Λ_0 explained?
7. How does the inflation occur in this scale invariant scenario?
8. Thermal effects.
9. Construct scale invariant Beyond Standard Model.
10. (Super)Gravity theory with (local or global) scale invariance.