

# LOCALIZATION OF EFFECTIVE ACTIONS

## IN OPEN SUPERSTRING FIELD THEORY

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w/A. Merlano  
(Tunis Univ.)

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AIM: COMPUTE THE TREE-LEVEL EFFECTIVE POTENTIAL

FOR MASSLESS OPEN STRING STATES FROM OSFT



DETERMINE THE ALGEBRAIC PART OF NON-ABELIAN DBI.

Given  $\phi \ll Q$  SFT this is in principle straightforward:

$$S_0[\phi] = S_0[\phi^{(e)} + \phi^{(h)}] \Rightarrow \frac{\delta S_0}{\delta \phi^{(h)}} = 0 \rightarrow \phi^{(h)} = \phi^{(h)} / (\phi^{(e)})$$

$$\boxed{S_{\text{eff}}[\phi^{(e)}] = S_0[\phi^{(e)} + \phi^{(h)}(\phi^{(e)})]}$$

$\Rightarrow$  We will consider Berkovits OSFT in the NS sector (sensible at tree-level)

~~REMARK: The ghost field  $\bar{\Phi}$  is decomposed into a ghost part  $\bar{\Phi}^{(g)}$  and a ghostless part  $\bar{\Phi}^{(h)}$ . The ghost part is zero at tree-level.~~

$$\bar{\Phi} = \bar{\Phi}^{(e)} + \bar{\Phi}^{(h)} \quad \begin{cases} \in H_{\text{LARGE}} \quad (\mu, \bar{\Phi} \neq 0) \\ \text{ghost} = 0 \\ \text{ghost} = 0 \end{cases}$$

For our purposes it is useful to rewrite the (2)  
even-like term as

$$\begin{aligned} S[\bar{\Phi}] &= \text{Tr} \left[ (\gamma \bar{\Phi}) \frac{e^{\bar{\Phi}} - 1 + e^{-\bar{\Phi}}}{e^{\bar{\Phi}} + 1} (Q \bar{\Phi}) \right] = \\ &= \cancel{\text{Tr}} - \frac{1}{2} \text{Tr} [\gamma \bar{\Phi} Q \bar{\Phi}] - \sum_{n=1}^{\infty} \frac{(-1)^n}{(n+1)!} \text{Tr} [M \bar{\Phi} e^{\bar{\Phi}} Q \bar{\Phi}] \end{aligned}$$

### VARIATION

$$\begin{aligned} \delta S[\bar{\Phi}] &= \text{Tr} \left[ \bar{e}^{-\bar{\Phi}} \delta e^{\bar{\Phi}} M_0 (e^{-\bar{\Phi}} Q e^{\bar{\Phi}}) \right] = \\ &= \text{Tr} \left[ \delta \bar{\Phi} \frac{e^{\bar{\Phi}} - 1}{e^{\bar{\Phi}} + 1} M_0 \frac{1 - e^{-\bar{\Phi}}}{e^{\bar{\Phi}} + 1} Q \bar{\Phi} \right] \end{aligned}$$

Fix on gauge:  $b_0 \bar{\Phi} = 0$   $\xi_0 \bar{\Phi} = 0$

From the physical massless states are the Kernel  
of  $L_0$ .

$$\bar{\Phi}^{(e)} = P_0 \bar{\Phi} \quad \bar{\Phi}^{(h)} = (1 - P_0) \bar{\Phi} = \bar{P}_0 \bar{\Phi}$$

$P_0 \rightarrow$  Projector  
on the  
Kernel  
of  $L_0$ .

In our example

$$\bar{\Phi}^{(e)} = c \gamma^{-1} V_{1/2} \xrightarrow{\text{superconformal primary of}} h = \frac{1}{2}$$

$$\begin{cases} M Q \bar{\Phi}^{(e)} = 0 \\ L_0 \bar{\Phi}^{(e)} = 0 \end{cases}$$

$$\begin{cases} Q \bar{\Phi}^{(e)} = c V_1 - \gamma V_{1/2} \\ M \bar{\Phi}^{(e)} = c V_{1/2} \bar{e}^{-\phi} s(\phi) \end{cases}$$

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com for  $\phi^{(4)}$ :

$$\bar{P}_0 \frac{e^{\omega T} - 1}{\omega T} \gamma_0 \frac{1 - e^{-\omega T}}{\omega T} Q \underline{\Phi} = 0 \quad (1)$$

$$\underline{\Phi} = \underline{\Phi}^{(e)} + \underline{\Phi}^{(h)}$$

Solve perturbatively:

$$\underline{\Phi} = g \underline{\Phi}^{(e)} + \sum_{m=2}^{\infty} g^m \underline{\Phi}_m^{(h)} \rightarrow \text{and plug in He action} \longrightarrow$$

expanding (1) in powers of  $g$  one finds equations for

$$\gamma Q \underline{\Phi}_2^{(h)} = \bar{P}_0 \frac{1}{2} [\gamma \underline{\Phi}^{(e)}, Q \underline{\Phi}^{(e)}]$$

$$\gamma Q \underline{\Phi}_3^{(h)} = \bar{P}_0 \left( \dots \text{longer} \dots \right)$$

$$\vdots \qquad \vdots$$

$$\Rightarrow \underline{\Phi}_m^{(h)} = \gamma_0 \frac{b_0}{L_0} \left( \text{stuff determined at lower orders} \right)$$

$$\text{ex: } \boxed{\underline{\Phi}_2^{(h)} = \frac{1}{2} \frac{b_0}{L_0} \bar{P}_0 [\gamma_0 \underline{\Phi}^{(e)}, Q \underline{\Phi}^{(e)}]}$$

etc. . .

At quartic order we find

$$S_{\text{eff}}^{(4)} = \frac{1}{g} \text{Tr} \left[ [Y \bar{\Phi}, Q \bar{\Phi}] \otimes \frac{b_0}{L_0} \bar{P} [Y \bar{\Phi}, Q \bar{\Phi}] \right] \text{prop.}$$

$$- \frac{1}{24} \text{Tr} \left[ [Y \bar{\Phi}, \bar{\Phi}] [\bar{\Phi}, Q \bar{\Phi}] \right] \text{contact}$$

In case  $\bar{\Phi} = c \gamma^{-1} A_\mu \Psi^\mu(0) |0\rangle$  this has been  
computed by Brkouits-Schmid 03.07.019

$$= \left( -\frac{1}{4} \text{tr} [A_\mu A^\mu A_\nu A^\nu] - \frac{1}{8} [A_\mu A_\nu A^\mu A^\nu] \right) \text{prop}$$

$$+ \left( -\frac{1}{8} \text{tr} [A_\mu A_\nu A^\mu A^\nu] + \frac{1}{2} [A_\mu A^\mu A_\nu A^\nu] \right) \text{contact}$$

$$= -\frac{1}{8} \text{tr} [[A_\mu, A_\nu] [A^\mu, A^\nu]]$$

$\Rightarrow$  We will pay attention to doing  
the computation which significantly simplifies  
the effort.

$$\text{The computation can be simplified assuming } \quad (5)$$

$$V_{1/2} = V_{1/2}^{(+)} + V_{1/2}^{(-)} \quad J_0 V_{1/2}^{(\pm)} = \pm V_{1/2}^{(\pm)}$$

In general this is realized by there with

an  $N=1 \rightarrow N=2$  enhancement (typically needed to have spin one fermions)

$$T_F = T_F^{(+)} + T_F^{(-)}$$

$$\left\{ \begin{array}{l} T_F^{(+)}(z) T_F^{(-)}(\omega) = \frac{z^{\frac{c}{3}}}{(z-\omega)^3} + \frac{J(\omega)}{(z-\omega)^2} + \dots \\ J(z) T_F^{(\pm)}(\omega) = \pm \frac{T_F^{\mp}(\omega)}{z-\omega} \end{array} \right.$$

part of the  
 $N=2$  SC algebra

$\rightarrow$  Realized  $\emptyset$  by gauge field, transverse scalars to D-branes,  $\emptyset$  DP(D(p+q)) open strings  $\rightarrow$  see Alberto Hebecker

$\Rightarrow$  Advantage the net  $\emptyset$  charge inside a container must vanish.

$\Rightarrow$  This essentially picks up only terms

$S^{(q)} [\emptyset^{(+)} + \emptyset^{(-)}]$  only terms with an equal number of  $\emptyset^{(+)}$  and  $\emptyset^{(-)}$  survive.

| idea from  
| Ben  
| 1508.02481  
| Sect. 8.1

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Computing we have

$$S^{(4)} [\bar{\Phi}^{(+)} + \bar{\Phi}^{(-)}] = S_{\text{prop}}^{(4)} [\bar{\Phi}^{(+)} + \bar{\Phi}^{(-)}] + S_{\text{cont}}^{(4)} [\bar{\Phi}^{(+)} + \bar{\Phi}^{(-)}]$$

Working on  $S_{\text{prop}}^{(4)}$  and using

- $\gamma Q \bar{\Phi}^{(\pm)} = 0$
- $\langle \gamma(\dots) \rangle = \langle Q(\dots) \rangle = 0$
- Conservation of J-charge
- $[Q, \frac{b_0}{L_0} \bar{P}] = \bar{P} = 1 - P_0$

we find :

$$S_{\text{prop}}^{(4)} = \frac{1}{4} \text{Tr} \left[ [\bar{\Phi}^{(-)}, \gamma \bar{\Phi}^{(-)}] P_0 [\bar{\Phi}^{(+)}, Q \bar{\Phi}^{(+)}] + [\bar{\Phi}^{(+)}, \bar{\Phi}^{(-)}] P_0 [\gamma \bar{\Phi}^{(-)}, Q \bar{\Phi}^{(+)}] \right]$$

$$- S_{\text{cont}}^{(4)} [\bar{\Phi}^{(+)} + \bar{\Phi}^{(-)}]$$

$\hookrightarrow$  cancels with the boundary vertex

What remains is localized at the boundary  
of the strip

$$P_0 = \lim_{t \rightarrow \infty} e^{-t L_0} = \underline{\underline{e^{-t L_0}}}$$

$$\text{Tr}[(A \otimes B) P_0 (C \otimes D)] = \begin{array}{c} A \\ \diagup \quad \diagdown \\ B \end{array} \xrightarrow[t \rightarrow \infty]{} \begin{array}{c} C \\ \diagup \quad \diagdown \\ D \end{array}$$

$\Rightarrow$  only the non-zero contribution  
of  $A \otimes B$  and  $C \otimes D$  is participating

explicitly we find

$$P_0 [\Phi^{(+)}, Q \Phi^{(+)}] = 2c H_1^{(+)}(0) |0\rangle \quad (J=2)$$

$$P_0 [\Phi^{(-)}, Q \Phi^{(-)}] = 2c \partial c \xi e^{-\omega \phi} H_1^{(-)}(0) |0\rangle \quad (J=-2)$$

$$P_0 [Y \Phi^{(-)}, Q \Phi^{(+)}] = 2c Y H_0(0) |0\rangle \quad (J=0)$$

$$P_0 [\Phi^{(-)}, \Phi^{(+)}] = c \partial c \xi \partial \xi e^{-\omega \phi} H_0(0) |0\rangle \quad (J=0)$$

The "AUXILIARY FIELDS"  $H$  are defined as

$$V_{1/2}^{(\pm)}(x) V_{1/2}^{(\pm)}(-x) = H_1^{(\pm)}(0) + \text{reg} \dots$$

$$V_{1/2}^{(\pm)}(x) V_{1/2}^{(\mp)}(-x) - V_{1/2}^{(\mp)}(x) V_{1/2}^{(\pm)}(-x) = \frac{1}{2x} H_0$$

↳ proportional  
to the identity.

Then, computing the trivial ghost condensates  
one finds

$$S^{(4)}(\Phi) = \text{tr} \left[ \langle H_1^{(+)}, H_1^{(-)} \rangle + \frac{1}{4} \langle H_0, H_0 \rangle \right]$$

↳ Simple matter 2-point functions!

↳ Continues on A. Henriques  
slides.