Wrapped magnetized branes in open string field theory

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at Discussion Meeting on String Field Theory and String Phenomenology, HRI, Allahabad.

Since 2014, we can construct any string field solution associated with a given BCFT, as far as it does not depends on X^0 direction.[1)]

Recently, Ishibashi, Kishimoto and Takahashi constructed a solution for constant $F_{\mu\nu}$ flux.[2)] Then the authors $\oplus$ I calculated the A_μ profile of this solution.[3)]

surprisingly, it is continuous over the whole torus, despite the nontrivial configuration of A_μ.

1) Erler-Maccaferri, '14 JHEP **1410** (2014) 029

2) Ishibashi-Kishimoto-Takahashi, '17 PTEP **2017** (2017) no.1, 013B06

3) Ishibashi-Kishimoto-tm-Takahashi '18 (*to appear*)

Today I will talk about a solution for **multiply wrapping D-brane with a constant magnetic flux,** a byproduct of this profile calculation.

Quite recently, I realized that this system is also studied in the context of string phenomenology, so I thought it's a good subject for this conference.

Plan of this talk:

1. review of constant F_{12} flux solution
   - first quantization $\rightarrow$ bcc operators
   - some simple observations

2. solution for multiply-wrapped magnetized D-brane
   - properties
   - calculation of the boundary state

3. concluding remarks

Open string in a constant $F_{12} \neq 0$

We need a bcco connecting Neumann and $F_{12} \neq 0$ b.c.

$$S = \int \partial X \partial \tilde{X} - A_\nu \dot{X}^\nu \Big|_{\sigma=\pi}$$

$$X = (X^1 + iX^2)/\sqrt{2}$$

$$\tilde{X} = (X^1 - iX^2)/\sqrt{2}$$

We concentrate on $\{X^1,\ X^2\}$:

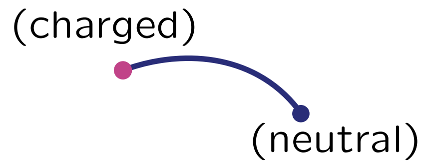
$$X^i \sim X^i + 2\pi R_i$$

$$F_{\mu\nu} = \begin{pmatrix} 0 & F_{12} \\ -F_{12} & 0 \end{pmatrix}$$

the Dirac quantization condition

$$F_{12} = N/(2\pi R_1 R_2)$$

$$\lambda \equiv \arctan 2\pi\alpha' F_{12}$$



Ishibashi-Kishimoto-Takahashi '15

Abouelsaood-Callan-Nappi-Yost '87 Nucl. Phys. B **280** (1987) 599.

Mode expansion

$$X(z,\bar{z}) = x + \sum i \left(\frac{\alpha'}{2}\right)^{1/2} \sum_{k=-\infty}^{\infty} \frac{1}{k+\lambda}(z^{-k-\lambda} + \bar{z}^{-k-\lambda})\alpha_{k+\lambda}$$

$$\tilde{X}(z,\bar{z}) = \tilde{x} + \sum i \left(\frac{\alpha'}{2}\right)^{1/2} \sum_{k=-\infty}^{\infty} \frac{1}{k-\lambda}(z^{-k+\lambda} + \bar{z}^{-k+\lambda})\tilde{\alpha}_{k-\lambda}$$

canonical quantization $\rightarrow$ zeromode is non-commutative

$$\left[x^1, x^2\right] = \frac{1}{F_{12}} = \frac{2\pi R_1 R_2}{N} = -\frac{2\pi\alpha'}{\tan \pi\lambda}$$

Introducing $U = e^{ix^1/R_1}$ and $V = e^{ix^2/R_2}$,

the algebra is expressed by

$$UV = VUe^{\frac{2\pi i}{N}}$$

We need $N \times N$ matrices to represent zeromode algebra:

$$U = \begin{pmatrix} 0 & 1 & \dots & & 0 \\ \vdots & 0 & 1 & & \vdots \\ \vdots & & 0 & 1 & \\ & & & & 1 \\ 1 & & & \dots & 0 \end{pmatrix}, \qquad V = \begin{pmatrix} 1 & & \dots & & 0 \\ \vdots & \omega & & & \vdots \\ \vdots & & \omega^2 & & \\ & & & \ddots & \\ 0 & & & & \omega^{N-1} \end{pmatrix}$$

Correspondingly there are n degenerate 'ground states' of open string vibration

$$|k\rangle \quad k = 1, ..., N.$$

bcc operators

by state-operator mapping, we naturally have N pairs of bccos $\{\sigma_*^i, \bar{\sigma}_*^i\}$, which are eigen states of V:

$$V\sigma_*^k = \omega^{(k-1)}\sigma_*^k, \qquad U\sigma_*^k = \sigma_*^{k-1}$$

$$\langle ...\bar{\sigma}_*^l(\xi_2)...\sigma_*^l(\xi_1)...\rangle \quad (\xi_2 < \xi_1) \ \rightarrow \begin{cases} \underline{\mathsf{N}} \text{ for } z < \xi_2,\ \xi_1 < z \\ \underline{F_{12} \neq 0} \text{ for } \xi_2 < z < \xi_1 \end{cases}$$

The conformal weight of σ_* or $\bar{\sigma}_*$ is $\frac{1}{2}\lambda(1-\lambda)$.

through boundary 3pt function, we can derive ope:

$$\sigma_*^k(s)\bar{\sigma}_*^l(0) \sim \frac{s^{-\lambda(1-\lambda)}}{(2\pi)^2 R_1 R_2} \sum_{n_1,\ n_2} (s\delta)^{\alpha' k_\mu k^\mu} \times \omega^{\frac{n_1 n_2}{2} + n_2 l} \delta_{k-l,\, n_1 (\mathrm{mod}\ N)} e^{-ik_\mu X^\mu}(0)$$

$$\bar{\sigma}_*^k(s)\sigma_*^l(0) \sim \frac{|\cos\pi\lambda| s^{-\lambda(1-\lambda)}}{(2\pi)^2 R_1 R_2} \sum_{n_1,\ n_2} (s\delta)^{\alpha' \cos^2(\pi\lambda)\, k_\mu k^\mu} \times \omega^{\frac{n_1 n_2}{2} + n_2 l} \delta_{k-l,\, n_1 (\mathrm{mod}\ N)} e^{-ik_\mu X^\mu}(0)$$

$$k^\mu = \frac{n_\mu}{R_\mu}, \qquad \mu = 1, 2$$

$\rightarrow$ some terms have nonzero momentum.

where

$$\delta \equiv \exp(2\psi(1) - \psi(\lambda) - \psi(1-\lambda))$$

with ψ a digamma function

$$\psi(z) = \frac{\Gamma'(z)}{\Gamma(z)}.$$

mapping: BCFT $\mapsto$ classical solution

Construct a solution to eom of open SFT

$$Q\Psi + \Psi * \Psi = 0$$

by defining a regularize bccos

$$\sigma = \sigma_* \otimes e^{i\sqrt{h}X^0} \qquad \bar{\sigma} = \bar{\sigma}_* \otimes e^{-i\sqrt{h}X^0} \qquad (\times\text{const.})$$

so that ope is non-singular: $0 < s$

$$\bar{\sigma}(s)\sigma(0) \sim 1 \qquad \sigma(s)\bar{\sigma}(0) \sim g_*/g_0$$

From bccos σ, $\bar{\sigma}$ with non-singular ope, we can make a classical solution for BCFT$_*$

cf. $g_*/g_0 = 1$ case: Kiermaier-Okawa-Solar '10 JHEP **1103** (2011) 122

The EM solution is given by

$$\Psi := \Psi_{\mathrm{tv}} - \Sigma\Psi_{\mathrm{tv}}\bar{\Sigma}$$

where

$$\Sigma = Q_{\mathrm{tv}}\left(\frac{1}{\sqrt{1+K}}B\sigma\frac{1}{\sqrt{1+K}}\right)$$
$$\bar{\Sigma} = Q_{\mathrm{tv}}\left(\frac{1}{\sqrt{1+K}}B\bar{\sigma}\frac{1}{\sqrt{1+K}}\right)$$
$$Q_{\mathrm{tv}}\cdots = Q + [\Psi_{\mathrm{tv}}, \ldots]$$
$$\Psi_{\mathrm{tv}} = \frac{1}{\sqrt{1+K}}c(1+K)Bc\frac{1}{\sqrt{1+K}}.$$

This construction is valid for any time-independent BCFT.

Erler-Maccaferri '13

See also: Ted's lecture

$$z = \frac{2}{\pi}\arctan\xi$$
$$K = \int\frac{dz}{2\pi i}T(z)\mathcal{I}$$
$$B = \int\frac{dz}{2\pi i}b(z)\mathcal{I}$$
$$c = c(1)\mathcal{I}$$
$$\sigma = \sigma(1)\mathcal{I}$$
$$\bar{\sigma} = \bar{\sigma}(1)\mathcal{I}$$

For KBc subalgebra, see

Okawa '06 JHEP **0604** (2006) 055

Erler '06 JHEP **0705** (2007) 083

Summary: property of the solution

energy density= DBI action

$$S(\Psi_i) = \sqrt{1 + (2\pi\alpha' F_{12})^2}$$

the Ellwood Invariant reproduces coupling to closed string:

$$O_V(\Psi_i) \propto \frac{\delta}{\delta V} S_{DBI}, \;\; V = G^{\mu\nu}, B^{\mu\nu}$$

important: Σ^ks are orthogonal $\bar{\Sigma}^i \Sigma^j = \delta^{ij}$

$\rightarrow$naturally multi-brane solution arise:

$$\Psi_{\mathrm{multi}} = \Psi_{\mathrm{tv}} - \Sigma_1 \Psi_{\mathrm{tv}} \bar{\Sigma}_1 \cdots - \Sigma_N \Psi_{\mathrm{tv}} \bar{\Sigma}_N$$

for N D-branes, each magnetized with $F_{12} = \frac{N}{2\pi R_1 R_2}$.

profile of constant $F_{\mu\nu}$ solution

$$\Psi = \sum_{\vec{p}} t(\vec{p}) c_1 |\vec{p}\rangle + A_\nu(p) \alpha^\nu_{-1} c_1 |\vec{p}\rangle + \ldots$$

Here

$$\vec{p} = (p_1, p_2)$$

$$|\vec{p}\rangle = e^{i\vec{p}\cdot X}(0)|0\rangle$$

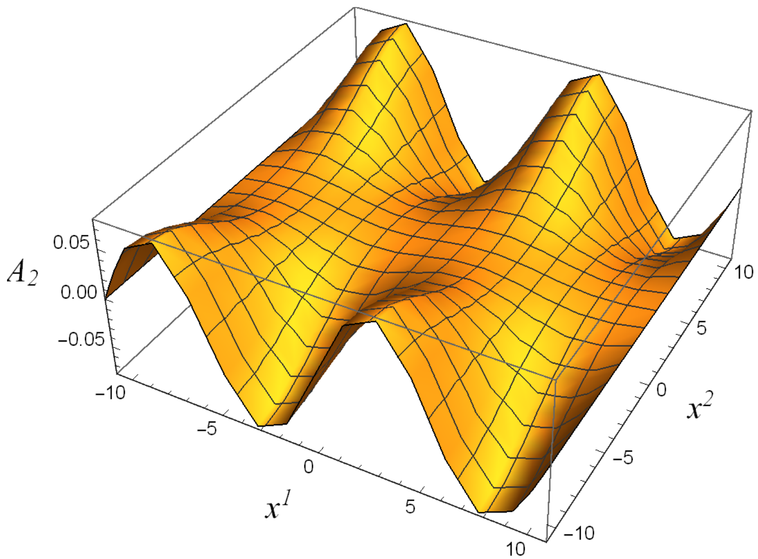
Ishibashi-Kishimoto-tm-Takahashi '18 (*to appear*)

We calculated the following profiles

$$t(x^1, x^2) \equiv \sum_p t(\vec{p}) e^{ip_\mu x^\mu},$$

$$A^\mu(x^1, x^2) \equiv \sum_p A^\mu(\vec{p}) e^{p_\mu x^\mu}$$

$\rightarrow$ they are continuous



A Numerical plot of $A_2(x^1, x^2)$, with $N = 2$, $R_1 = R_2 = 2\sqrt{3}$, $\alpha' = 1$.

(quasi-)periodicity

A^μ profiles are periodic with respect to the space-time;

for i-th solution Ψ^i,

$$A_i{}^\mu\left(x^1 + \frac{2\pi R_1}{N}, x^2\right) = A_i{}^\mu\left(x^1, x^2\right)$$

$$A_i{}^\mu\left(x^1, x^2 + \frac{2\pi R_2}{N}\right) = A_{i+1}{}^\mu\left(x^1, x^2\right)$$

Actually this holds for all the component fields, i.e.

$$\Psi^i\left(x^1 + \frac{2\pi R_1}{N},\ x^2\right) = \Psi^i\left(x^1,\ x^2\right)$$

$$\Psi^i\left(x^1,\ x^2 + \frac{2\pi R_2}{N}\right) = \Psi^{i+1}\left(x^1,\ x^2\right)$$

Comment 1) Notice the resemblance to

$$U\Psi^i = \Psi^{i+1}, \quad V\Psi^i = \Psi^i. \qquad (*)$$

Since $[x^1, x^2] = \frac{2\pi R_1 R_2}{N}$

$$\rightarrow \quad x^1 \sim \Delta\partial_2 \quad \left(\Delta = \frac{2\pi R_1 R_2}{N}\right)$$

$$U = e^{\frac{ix^1}{R_1}} \sim e^{\frac{i2\pi R_2}{N}\partial_2}$$

U implements a translation. (similar holds for V)

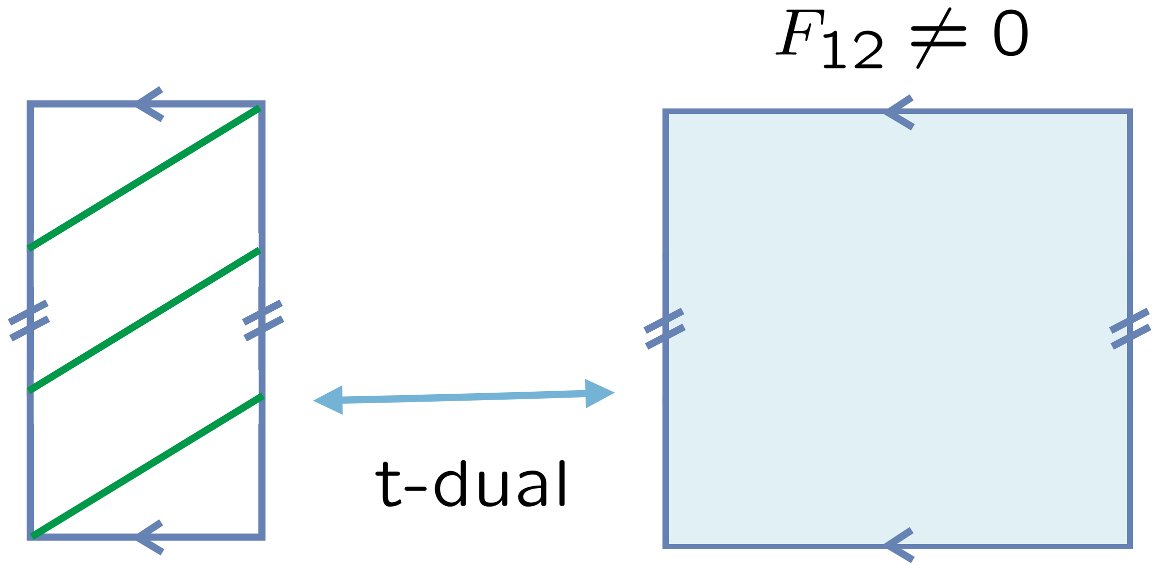
$(*)$ already imply the periodicity.

Comment 2) A rigorous proof of the (quasi-) periodicity follows from the form of boundary three point functions, which is used to calculate profiles:

$$\langle \tilde{\phi}(x_1)\sigma(x_2)\sigma(x_3)\rangle$$

$\rightarrow$ see our paper.

Comment 3) T-dual of this system wrt X^1 is slanted D1-branes, which exhibits periodicity for X^2 clearly.

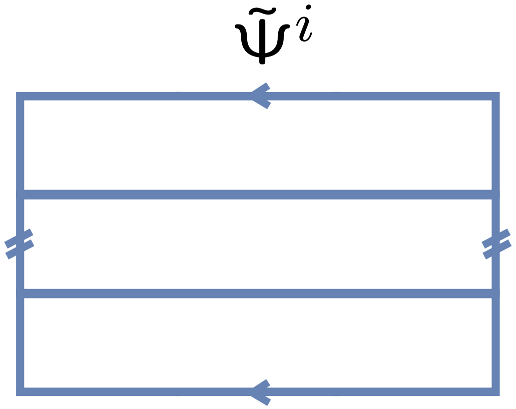


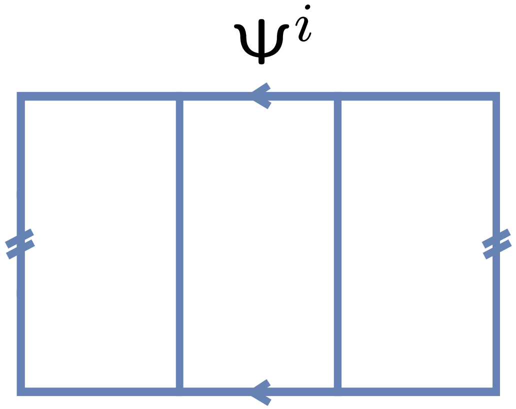
Comment 4) if we diagonalized U instead of V, we obtain $\tilde{\Psi}^k$ s.t.

$$U\tilde{\Psi}^k = \tilde{\Psi}^k, \quad V\tilde{\Psi}^k = \tilde{\Psi}^{k+1}$$

or

$$\tilde{\Psi}^k\left(x^1 + \frac{2\pi R_1}{N}, x^2\right) = \tilde{\Psi}^{k+1}\left(x^1, x^2\right)$$
$$\tilde{\Psi}^k\left(x^1, x^2 + \frac{2\pi R_2}{N}\right) = \tilde{\Psi}^k\left(x^1, x^2\right)$$

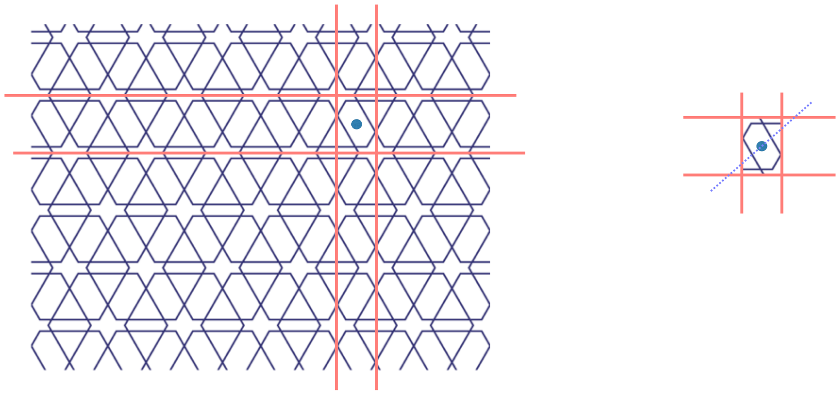

$\tilde{\Psi}^i$


Ψ^i

tiling/dividing a solution

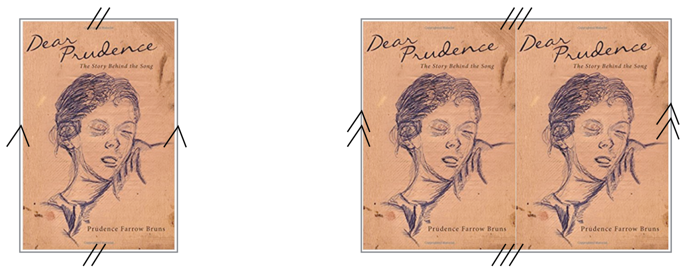
For some special case, we can trivially map a solution to another solution of another theory:

(1) by deviding Ψ; if a solution has a symmetry with respect to space time coordinates, we can divide Ψ to obtain a solution on a smaller torus etc.



$$\text{e.g. } \mathcal{H}\left(\mathrm{OSFT}_{nR_1\times mR_2}\right)\Big|_{\text{periodic}} \to \mathcal{H}\left(\mathrm{OSFT}_{R_1\times R_2}\right)$$

(2) tiling: the inverse procedure of (1):



for the case of the flux solution:

$(n$-unit flux solution)

$\leftrightarrow$ n time reputation of (1-unit flux solution).

(Remember that, the eom of open SFT is a collection of infinite number of spacetime DEs)

wrapped magnetized D-branes

Consider N multi-brane solution: periodic

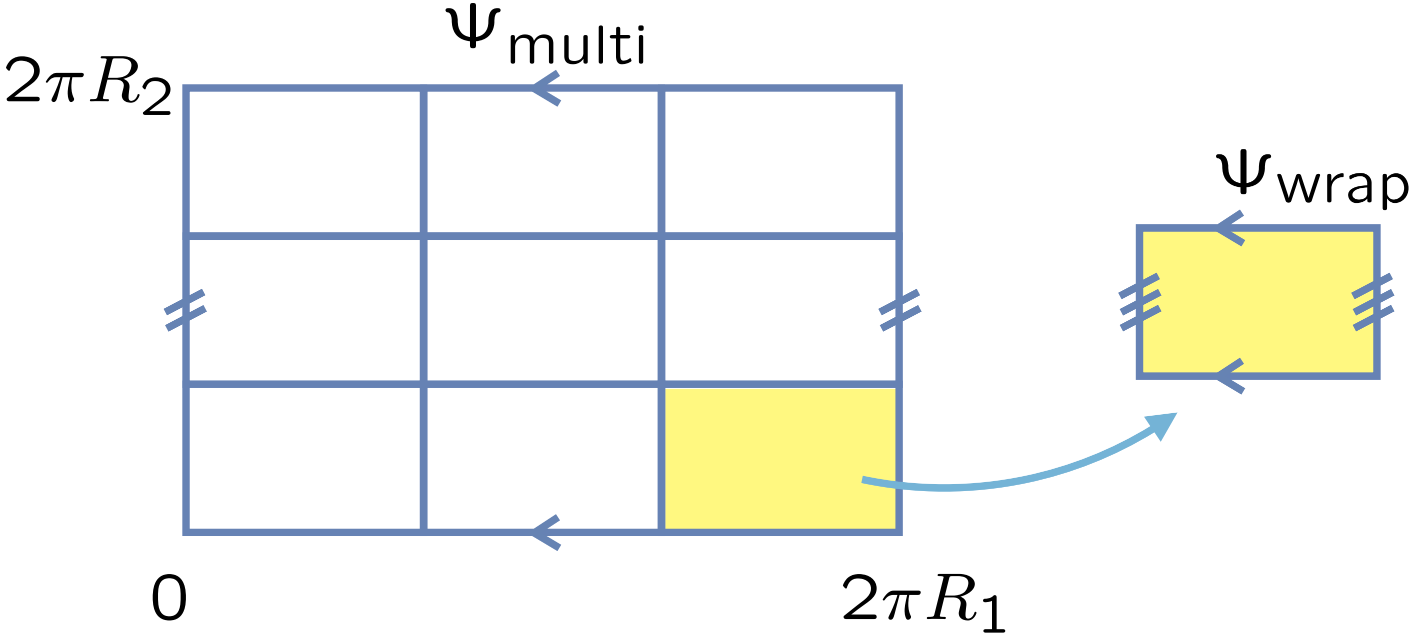
$$\Psi_{\text{multi}}\left(x^1 + \frac{2\pi R_1}{N},\ x^2\right) = \Psi_{\text{multi}}\left(x^1,\ x^2 + \frac{2\pi R_2}{N}\right)$$
$$= \Psi_{\text{multi}}\left(x^1,\ x^2\right)$$

Then divide it to obtain

$$\Psi_{\text{wrap}} \in \mathcal{H}\left(\text{OSFT}_{R'_1 \times R'_2}\right)$$

with $R'_i = R_i/N$.

We claim that Ψ_{wrap} is a wrapped D-brane (N-fold) with $F_{12} = \frac{1}{N}\frac{1}{2\pi R'_1 R'_2}$. $\left(\sim \frac{1}{N} \text{ unit of flux}\right)$



$$\Psi_{\text{multi}} = \Psi_{\text{tv}} - \Sigma_1 \Psi_{\text{tv}} \bar{\Sigma}_1 \cdots - \Sigma_N \Psi_{\text{tv}} \bar{\Sigma}_N$$

$$F_{12} = \frac{N}{2\pi R_1 R_2} = \frac{1}{N} \frac{1}{2\pi R_1' R_2'}$$

How do we know the D-brane is multiply-wrapping ?

1) energy

2) excitation around Ψ_{wrap}:

$$e^{2\pi R_1' \partial_2} \psi^{(i,j)} = \psi^{(i,j)}$$

$$e^{2\pi R_2' \partial_1} \psi^{(i,j)} = \psi^{(i+1,j+1)}$$

3) coupling to $G_{\mu\nu}$ and $B_{\mu\nu}$,
from Ellwood invariants

Excitation around Ψ:

$$\Psi + \Phi$$

$$\psi_{ij} \sim \bar{\Sigma}^i \Phi \Sigma^j$$

Erler-Maccaferri '13

Kishimoto-tm-Takahashi-Takemoto '14 PTEP **2015** (2015) no.3, 033B05

Can we calculate more non-trivial quantity?

→ boundary state

Comment: wrapping N-times along the other cycle;

(1) diagonalise U, instead of V
(2) take another set of regularized bccos $\{\Xi^i,\ \bar{\Xi}^i\}_{i=1}^N$

$$U\Xi^i = \omega^{i-1}\Xi^i, \quad V\Xi^i = \Xi^{i+1}$$

and

$$\tilde{\Psi}_{\text{multi}} = \Psi_{\text{tv}} - \Xi^1\Psi_{\text{tv}}\bar{\Xi}^1 \cdots - \Xi^N\Psi_{\text{tv}}\bar{\Xi}^N$$

(3) divide it to obtain $\tilde{\Psi}_{\text{wrap}}$.

Actually, these two are the same classical solution

$$\tilde{\Psi}_{\text{wrap}} = \Psi_{\text{wrap}},$$

because $\Xi^j = W_i^j \Sigma^i$ with W_i^j a unitary matrix.

boundary state

Non-SFT derivation: BS for wrapped magnetized D-branes is derived about 10 years ago. It is not a trivial calculation.

- winding# & momentum related
- a consistent phase factor
- delicate coupling to closed string vertex operators

Di Vecchia-Liccardo-Marotta-Pezzella-Pesando '07
JHEP **0711** (2007) 100

Duo-Russo-Sciuto '07
JHEP **0712** (2007) 042

Pesando '09
JHEP **1002** (2010) 064

Can we reproduce this result from open SFT?

SFT derivation: there are two ways to calculate BS from a classical solution: KOZ or KMS formalism

KMS formalism

- assume $\text{CFT}_{\text{auxiliary}}$ sector
- uplift classical solution
- assume the Ellwood conjecture in this larger theory

Kiermaier-Okawa-Zwiebach '08
0810.1737 [hep-th]

Kudrna-Maccaferri-Schnabl '12
JHEP **1307** (2013) 033

$\rightarrow \left\langle V_{(h,h)} \middle| B \right\rangle$ for any matter (h, h) primary V is calculable.

Since boundary state is a linear combination of Ishibashi states, we can read off all the components if the couplings $\left\langle V_{(h,h)} \middle| B \right\rangle$ are known.

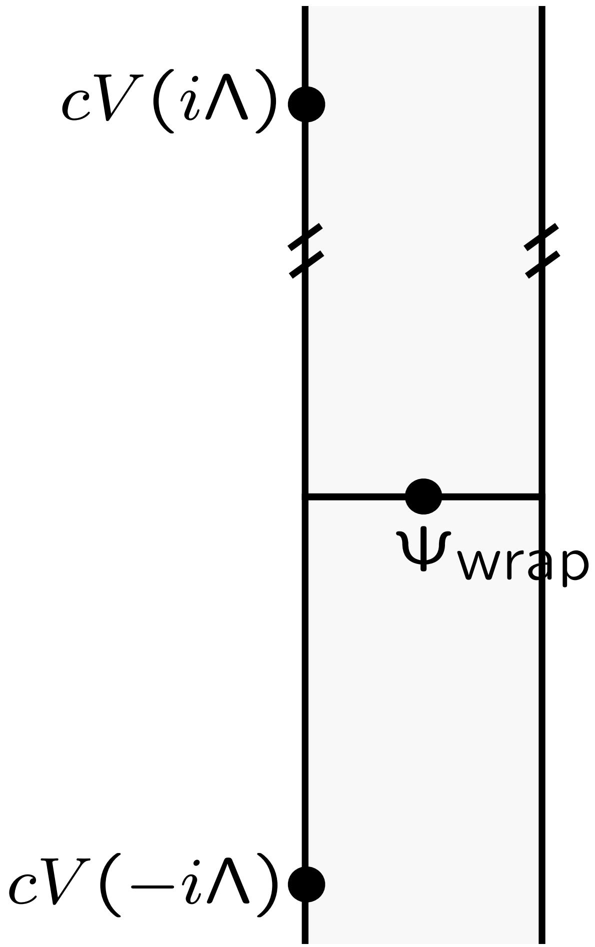
definition/evalulation of the Ellwood invariant:

for $V_{n,m} = \exp(k_\mu X^\mu)$ with $k^\mu = \left(\frac{n_1}{R_1}, \frac{n_2}{R_2}\right)$,

$$\begin{aligned}\langle n, m | B \rangle =& \mathcal{V}_{n,m}(\Psi_{\text{wrap}}) \\ =& \lim_{\Lambda\to\infty} \text{tr}\left[V_{n,m}(i\Lambda, -i\Lambda)\Psi_{\text{wrap}}\right] \times \text{tr}\left[V_{\text{aux}}(i\Lambda, -i\Lambda)\right]\end{aligned}$$

$$n = \begin{pmatrix} n_1 \\ n_2 \end{pmatrix} : \text{momentum\#},\ \ m = \begin{pmatrix} m_1 \\ m_2 \end{pmatrix} : \text{winding\#},$$

$\rightarrow$ we need the bulk-boundary 3pt function of a closed string vertex operator and bccos.


$cV(i\Lambda)$
Ψ_{wrap}
$cV(-i\Lambda)$

The 3 point function can be derived by the stress-tensor method; for $0 \leq s$

$$\left\langle e^{ik_\mu X^\mu}(z,\bar{z})\bar{\sigma}^l(0)\sigma_l(s)\right\rangle_{\mathsf{UHP}} = C\left|\frac{s^h}{z^h(s-z)^h}\right|^2 g\left(\frac{-2iys}{z\bar{z}-zs}\right)$$

with

$$g(z) = \exp\left[\int_1^z dz\left(\frac{\alpha' k\tilde{k}'}{2(1-z)^{1-\lambda_z}} + \frac{\alpha' k'\tilde{k}}{2(1-z)^{\lambda_z}}\right)\right]$$

$$C = \frac{(2\pi)^2 R_1 R_2}{|\cos\pi\lambda|} \times C(k^\mu) \times e^{-i\pi N n_1 n_2}.$$

After some calculation, we find that

$$\langle n, m|B\rangle = \begin{cases} \sqrt{G + 2\pi F} \times e^{-i\pi n_1 n_2 N} & n = Rm \\ 0 & \text{otherwise} \end{cases}$$

with

$$R = \begin{pmatrix} 0 & -\frac{1}{N} \\ +\frac{1}{N} & 0 \end{pmatrix}$$

which is consistent with the previous works:

- ∗ relation between n and m: ok
- ∗ absolute value: ok
- ∗ phase factor: consistent with some of previous works

(Summary:) a classical solution for multiply-wrapped magnetized D-brane on a torus is presented.

We calculated the boundary state from the solution.

Concluding remarks

- it will be straightforward to consider similar system on non-rectangular torus or orbifolds etc.

- it will be also interesting to study properties of numerical solutions.

- open SFT might be useful to clarify understanding of mysterious features of this system.