Localization of effective actions in OSFT - Applications and Future Developments

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Summary of the presentation

- Recap of the general results (see Carlo's talk, 1801.07607)
- Yang-Mills potential from $D(2n)$ branes system
- Yang-Mills instantons from $D3-D(-1)$ branes system
- Future Directions

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Main results (see Carlo's talk)

- From Berkovits OSFT, the effective action for a massless field Φ_A at the quartic level is

$$\begin{aligned} S_4(\Phi_A) &= +\frac{1}{8}\mathrm{Tr}\,[[\eta_0\Phi_A, Q_B\Phi_A]\,\xi_0\frac{b_0}{L_0}\,[\eta_0\Phi_A, Q_B\Phi_A]] \\ &\quad -\frac{1}{24}\mathrm{Tr}\,[[\eta_0\Phi_A, \Phi_A]\,[\Phi_A, Q_B\Phi_A]]\,. \end{aligned}$$

- For consistent and interesting string theory background we can always promote the natural worldsheet $\mathcal{N}=1$ to $\mathcal{N}=2$. (see Sen (1986), Banks and Dixon et al.(1987-88))

- So there is a R-symmetry current j in the matter sector. Assume that the string field is the sum of opposite charged string fields

$$\Phi_A = \Phi_A^{(+)} + \Phi_A^{(-)} \quad , \quad \Phi_A^{(\pm)} = c\gamma^{-1}\mathbb{V}_{\frac{1}{2}}^{(\pm)}.$$

- The conservation of the $R-$charge implies that the effective action localizes in terms of unphysical "auxiliary fields" such as:

$$
\begin{aligned}
P_0[\Phi_A, Q_B\Phi_A] &= +A_\mu A_\nu \, c\gamma^{-1}\psi^\mu \left(\frac{1}{\sqrt{3}}\right)\left[c\left(i\sqrt{2}\partial X^\nu\right) - \gamma\psi^\nu\right]\left(-\frac{1}{\sqrt{3}}\right) \\
&\quad -A_\mu A_\nu \left[c\left(i\sqrt{2}\partial X^\nu\right) - \gamma\psi^\nu\right]\left(\frac{1}{\sqrt{3}}\right) c\gamma^{-1}\psi^\mu \left(-\frac{1}{\sqrt{3}}\right)\Bigg|_0 \\
&= -[A_\mu, A_\nu] c\, \psi^\mu\psi^\nu(0)|0\rangle.
\end{aligned}
$$

where known OPEs of the ghost/superghosts have been used

$$\gamma(z)\gamma^{-1}(-z) = 1 + O(2z),$$

$$c(z)\, c(-z) \sim -2z\, c\partial c(0) + \cdots$$

- The general structure of the auxiliary fields appearing at the quartic order is

$$\begin{aligned} h^{(\pm\pm)} &\equiv P_0[\Phi_A^{(\pm)}, Q_B\Phi_A^{(\pm)}] = -2c\,\mathbb{H}_1^{(\pm)} \\ \widehat{h}^{(\pm\pm)} &\equiv P_0[\Phi_A^{(\pm)}, \eta_0\Phi_A^{(\pm)}] = 2c\partial c\,\xi\, e^{-2\phi}\ \mathbb{H}_1^{(\pm)} \\ g^{(\pm\mp)} &\equiv P_0[\eta_0\Phi_A^{(\pm)}, Q_B\Phi_A^{(\mp)}] = \mp c\,\eta\,\mathbb{H}_0 \\ \widehat{g}^{(\pm\mp)} &\equiv P_0[\Phi_A^{(\pm)}, \Phi_A^{(\mp)}] = \mp c\partial c\,\xi\partial\xi\, e^{-2\phi}\,\mathbb{H}_0. \end{aligned}$$

- The explicit form of the above string fields is given by the leading OPE between the matter superconformal primaries

$$\mathbb{V}_{\frac{1}{2}}^{(\pm)}(z)\mathbb{V}_{\frac{1}{2}}^{(\pm)}(-z) = \mathbb{H}_1^{(\pm)}(0) + \cdots$$

$$\mathbb{V}_{\frac{1}{2}}^{(\mp)}(z)\mathbb{V}_{\frac{1}{2}}^{(\pm)}(-z) - \mathbb{V}_{\frac{1}{2}}^{(\pm)}(z)\mathbb{V}_{\frac{1}{2}}^{(\mp)}(-z) = \frac{1}{2z}\mathbb{H}_0 + \cdots .$$

The localized effective action

- The ghost/superghost CFT is universal and can be computed. Then we remain with

$$S^{(4)}_{\text{eff}}(\Phi_A) = \mathrm{tr}\left[\langle \mathbb{H}^{(+)}_1 | \, \mathbb{H}^{(-)}_1 \rangle + \frac{1}{4}\langle \mathbb{H}_0 | \, \mathbb{H}_0 \rangle\right],$$

- This is the main result of 1801.07607: *Choose your string background; the effective action at the quartic level for the massless fields is given by the BPZ product of the matter part of the auxiliary field.*

- If not for this term, the effective action would be zero. This matter vertex depends on the chosen string background. So let us analyze two important examples.

Yang-Mills potential:the worldsheet theory

- The worldsheet theory of a system of N coincident $D(2n)$ euclidean branes contains the ψ^μ superconformal fields which we rearrange according to a $U(n) \in SO(2n)$ decomposition

$$\psi^{\jmath} = \frac{1}{\sqrt{2}}(\psi^{2j-1} + i\psi^{2j}) = e^{ih_j} \qquad J_0\psi^{\jmath} = +\psi^{\jmath},$$

$$\psi^{\bar{\jmath}} = \frac{1}{\sqrt{2}}(\psi^{2j-1} - i\psi^{2j}) = e^{-ih_j} \qquad J_0\psi^{\bar{\jmath}} = -\psi^{\bar{\jmath}}.$$

with $(\jmath, \bar{\jmath}, j) = 1, ..., n$. The localizing current is given

$$J(z) = -i\sum_{j=1}^{n} \partial h_j(z).$$

Yang-Mills potential: the auxiliary fields

- The matter vertex is then given

$$\mathbb{V}^{(+)}_{\frac{1}{2}}(z) = A_{\mathsf{J}}\,\psi^{\mathsf{J}}(z), \qquad \mathbb{V}^{(-)}_{\frac{1}{2}}(z) = A_{\bar{\mathsf{J}}}\,\psi^{\bar{\mathsf{J}}}(z),$$

- The auxiliary fields are easily extracted from the leading term in the OPE

$$\begin{aligned}
\mathbb{H}^{(+)}_{1}(z) &= \frac{1}{2}[A_{\mathsf{I}}, A_{\mathsf{J}}]\,\psi^{\mathsf{IJ}}(z), \\
\mathbb{H}^{(-)}_{1}(z) &= \frac{1}{2}[A_{\bar{\mathsf{I}}}, A_{\bar{\mathsf{J}}}]\,\psi^{\overline{\mathsf{IJ}}}(z), \\
\mathbb{H}_{0}(z) &= [A_{\bar{\mathsf{J}}}, A_{\mathsf{J}}].
\end{aligned}$$

where $\psi^{\mathsf{IJ}}(z) \equiv :\psi^{\mathsf{I}}\psi^{\mathsf{J}}:(z)$. The effective action is then easily computed from

$$S^{(4)}_{\mathrm{eff}}(\Phi_A) = \mathrm{tr}\left[\langle \mathbb{H}^{(+)}_{1} | \, \mathbb{H}^{(-)}_{1}\rangle + \frac{1}{4}\langle \mathbb{H}_{0} | \, \mathbb{H}_{0}\rangle\right]$$

Yang-Mills potential

$$S^{(4)}_{\text{eff}}(\Phi_A) \;=\; \mathrm{tr}\left[-\frac{1}{2}[A_{\mathrm{I}}, A_{\mathrm{J}}][A_{\bar{\mathrm{I}}}, A_{\bar{\mathrm{J}}}] + \frac{1}{4}[A_{\bar{\mathrm{J}}}, A_{\mathrm{J}}][A_{\bar{\mathrm{I}}}, A_{\mathrm{I}}]\right].$$

- The $N \times N$ matrices $A_{\mathrm{J}}, A_{\bar{\mathrm{J}}}$ are related to the original A_μ's by

$$A_{\mathrm{J}} = \tau^{\mu}_{\mathrm{J}}\, A_\mu \qquad A_{\bar{\mathrm{J}}} = \bar{\tau}^{\mu}_{\bar{\mathrm{J}}}\, A_\mu,$$

where non-vanishing entries are

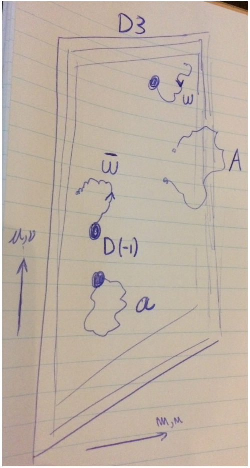
$$\tau^{2j-1}_{\mathrm{J}} = \frac{1}{\sqrt{2}} = \bar{\tau}^{2j-1}_{\bar{\mathrm{J}}} \qquad \tau^{2j}_{\mathrm{J}} = \frac{i}{\sqrt{2}} = -\bar{\tau}^{2j}_{\bar{\mathrm{J}}}.$$

$$\sum_{\mathrm{J}=1}^{n} \tau^{\mu}_{\mathrm{J}} \bar{\tau}^{\nu}_{\bar{\mathrm{J}}} = \frac{1}{2}\left(\delta^{\mu\nu} - i\epsilon^{\mu\nu}\right).$$

- Thus, the Y-M potential is correctly reproduced

$$S^{(4)}_{\text{eff}} \;=\; -\frac{1}{8}\mathrm{tr}\left[[A_\mu, A_\nu][A^\mu, A^\nu]\right].$$

Y-M Instantons: the worldsheet theory of $D3$–$D(-1)$



The total massless string field is given by

$$\Phi_A(z) = c\gamma^{-1} \begin{pmatrix} A & \omega \\ \bar{\omega} & a \end{pmatrix} (z),$$

where

$$A(z) = A_\mu \psi^\mu(z) + \phi_p \psi^p(z),$$

$$\omega(z) = \omega_\alpha^{N\times k}\, \Delta S^\alpha(z),$$

$$\bar{\omega}(z) = \bar{\omega}_\alpha^{k\times N}\, \bar{\Delta} S^\alpha(z),$$

$$a(z) = a_\mu \psi^\mu(z) + \chi_p \psi^p(z).$$

- Greek indices μ label the directions along the D3 branes, roman indices p label the transverse directions while $SO(4)$ spinor indices α are $(\frac{1}{2}, \frac{1}{2})$ and $(-\frac{1}{2}, -\frac{1}{2})$.

Benefits of the Localization

- Certainly we can compute this effective action using the Berkovits quartic effective action, but for the instanton potential it requires 4−point functions of (excited) twist fields which are not even well known in the literature. The most challenging one is roughly given:

$$\mathrm{Tr}\left[[\eta_0\Phi_A, Q_B\Phi_A]\,\xi_0\frac{b_0}{L_0}\,[\eta_0\Phi_A, Q_B\Phi_A]\right] \sim \langle\tau^\mu(z_1)\bar{\tau}^\nu(z_2)\Delta(z_3)\bar{\Delta}(z_4)\rangle.$$

- So it is easy to appreciate how localization avoids this problems reducing all the computation to a 2−point function.

Yang-Mills Instantons: Gauge Boson

- The worldsheet theory contains the ψ^μ superconformal fields which we rearrange as before

$$A = A^{(+)} + A^{(-)} + \phi^{(+)} + \phi^{(-)},$$

$$a = a^{(+)} + a^{(-)} + \chi^{(+)} + \chi^{(-)},$$

where using the same notation of previous example

$$A^{(+)} = A_{\jmath}\,\psi^{\jmath} \qquad , \qquad A^{(-)} = A_{\bar{\jmath}}\,\psi^{\bar{\jmath}},$$

$$\phi^{(+)} = \phi_m\,\psi^m \qquad , \qquad \phi^{(-)} = \phi_{\bar{m}}\,\psi^{\bar{m}},$$

The same also for a. Here $\jmath = 1, 2$ and $\bar{\jmath} = \bar{1}, \bar{2}$ indices denote respectively the fundamental and antifundamental representation of $SU(2) \subset SO(4)$, while $m = 1, 2, 3$ and $\bar{m} = \bar{1}, \bar{2}, \bar{3}$ indices denote respectively the fundamental and antifundamental representation of $SU(3) \subset SO(6)$.

Yang-Mills Instantons: Stretched strings

- The worldsheet theory contains also the composite ΔS^α and $\bar{\Delta} S^\alpha$ which are superconformal primaries of weight $\frac{1}{2}$. Spin fields are defined through bosonization by the scalars h_1, h_2

$$S^{(\frac{1}{2},\frac{1}{2})} = e^{\frac{i}{2}(h_1+h_2)} \quad , \quad S^{(-\frac{1}{2},-\frac{1}{2})} = e^{-\frac{i}{2}(h_1+h_2)}$$

$$J_0\, S^{(\frac{1}{2},\frac{1}{2})} = +S^{(\frac{1}{2},\frac{1}{2})} \quad , \quad J_0\, S^{(-\frac{1}{2},-\frac{1}{2})} = -S^{(-\frac{1}{2},-\frac{1}{2})}.$$

Finally we can decompose the off-diagonal part of $\mathbb{V}_{\frac{1}{2}}$ as

$$\omega = \omega^{(+)} + \omega^{(-)} \quad , \quad \bar{\omega} = \bar{\omega}^{(+)} + \bar{\omega}^{(-)}$$

where

$$\omega^{(+)} = \omega_1\, \Delta S^{(\frac{1}{2},\frac{1}{2})} \quad , \quad \omega^{(-)} = \omega_2\, \Delta S^{(-\frac{1}{2},-\frac{1}{2})}$$

$$\bar{\omega}^{(+)} = \bar{\omega}_1\, \bar{\Delta} S^{(\frac{1}{2},\frac{1}{2})} \quad , \quad \bar{\omega}^{(-)} = \bar{\omega}_2\, \bar{\Delta} S^{(-\frac{1}{2},-\frac{1}{2})}.$$

Yang-Mills Instantons - Auxiliary fields

- The auxiliary fields $\mathbb{H}_0, \mathbb{H}_1^{(+)}, \mathbb{H}_1^{(-)}$ along the D3 branes are

$$\mathbb{H}_1^{(+)D3} = \begin{pmatrix} [A_1, A_2] + \omega_1\bar{\omega}_1 & 0 \\ 0 & [a_1, a_2] - \bar{\omega}_1\omega_1 \end{pmatrix} \psi_{1\,2}|0\rangle,$$

$$\mathbb{H}_1^{(-)D3} = \begin{pmatrix} [A_{\bar{1}}, A_{\bar{2}}] + \omega_2\bar{\omega}_2 & 0 \\ 0 & [a_{\bar{1}}, a_{\bar{2}}] - \bar{\omega}_2\omega_2 \end{pmatrix} \psi_{\bar{1}\,\bar{2}}|0\rangle,$$

$$\mathbb{H}_0^{D3} = \begin{pmatrix} [A_{\bar{J}}, A_J] - (\omega_1\bar{\omega}_2 + \omega_2\bar{\omega}_1) & 0 \\ 0 & [a_{\bar{J}}, a_J] + (\bar{\omega}_1\omega_2 + \bar{\omega}_2\omega_1) \end{pmatrix} |0\rangle.$$

- Note the absence of off-diagonal terms due to the OPE

$$\psi^\mu(z)\, S^\alpha(w) \quad \sim \quad -\frac{1}{\sqrt{2}}\, \frac{(\gamma^\mu)^\alpha{}_{\dot{\beta}}\; S^{\dot{\beta}}(w)}{(z-w)^{\frac{1}{2}}}.$$

Yang-Mills Instantons - Covariant auxiliary fields

- At first sight maybe they do not look significant, but after algebraic manipulations involving the ladder t'Hooft symbols:

$$\eta_+^{\mu\nu} \equiv \eta_1^{\mu\nu} + i\eta_2^{\mu\nu} \qquad , \qquad \eta_-^{\mu\nu} \equiv \eta_1^{\mu\nu} - i\eta_2^{\mu\nu},$$

it is possible to see that they carry a SU(2) representation

$$\mathbb{H}_1^{(+)D3} = -\frac{i}{4}\,\eta_-^{\mu\nu}\;T_{\mu\nu}\,\psi_{1\,2}|0\rangle, \quad , \quad \mathbb{H}_1^{(-)D3} = +\frac{i}{4}\,\eta_+^{\mu\nu}\;T_{\mu\nu}\,\psi_{\bar{1}\,\bar{2}}|0\rangle,$$

$$\mathbb{H}_0^{D3} = -\frac{i}{2}\,\eta_3^{\mu\nu}\;T_{\mu\nu}\,|0\rangle.$$

The covariant tensor $T^{\mu\nu}$ is given by

$$T^{\mu\nu} = \begin{pmatrix} [A^\mu,A^\nu]+\frac{1}{2}\,\omega_\alpha(\gamma^{\mu\nu})^{\alpha\beta}\bar{\omega}_\beta & 0 \\ 0 & [a^\mu,a^\nu]-\frac{1}{2}\,\bar{\omega}_\alpha(\gamma^{\mu\nu})^{\alpha\beta}\omega_\beta \end{pmatrix}.$$

The ADHM constraint and the effective action

- On the D3-branes' worldvolume, we get the quartic potential given by a perfect square

$$S_T = \mathrm{tr}\left[\langle \mathbb{H}_1^{(+)D3} | \, \mathbb{H}_1^{(-)D3}\rangle + \frac{1}{4}\langle \mathbb{H}_0^{D3} | \, \mathbb{H}_0^{D3}\rangle\right] = -\frac{1}{16}\,\mathrm{tr}\,[D_a D_a]\,,$$

where the auxiliary field

$$D_a = \eta_a^{\mu\nu}\; T_{\mu\nu},$$

contains through its EOM the 3 ADHM constraints (on the $D(-1)$ slot).

$$\eta_a^{\mu\nu}\; T_{\mu\nu} = 0 \quad \rightarrow \quad \eta_a^{\mu\nu}\; \left[[a_\mu, a_\nu] - \frac{1}{2}\,\bar{\omega}_\alpha\,(\gamma_{\mu\nu})^{\alpha\beta}\,\omega_\beta\right] = 0$$

Yang-Mills Instantons transverse auxiliary fields

- The auxiliary fields in the transverse directions are less interesting and have more complicated structure:

$$\mathbb{H}_1^{(+)T} = \begin{pmatrix} \frac{1}{2}[\phi_m,\phi_n]\ \psi^{m\,n}+[A_{\mathrm{J}},\phi_m]\ \psi^{\mathrm{J}\,m} & (\phi_m\omega_1-\bar{\omega}_1\chi_m)\ \psi^m\Delta S^{(\frac{1}{2},\frac{1}{2})} \\ (\chi_m\bar{\omega}_1-\bar{\omega}_1\phi_m)\ \psi^m\bar{\Delta}S^{(\frac{1}{2},\frac{1}{2})} & \frac{1}{2}[\chi_m,\chi_n]\ \psi^{m\,n}+[a_{\mathrm{J}},\chi_m]\ \psi^{\mathrm{J}\,m} \end{pmatrix}|0\rangle,$$

$$\mathbb{H}_1^{(-)T} = \mathbb{H}_1^{(+)T}(m \to \bar{m}, 1 \to 2),$$

$$\mathbb{H}_0^{T} = \begin{pmatrix} [\phi_{\bar{m}},\phi_m] & 0 \\ 0 & [\chi_{\bar{m}},\chi_m] \end{pmatrix}|0\rangle,$$

Yang-Mills Instantons effective action

- The complete effective action is

$$S^{(4)}_{\rm eff}(\Phi_A) \;=\; {\rm tr}\left[\langle \mathbb{H}_1^{(+)} | \, \mathbb{H}_1^{(-)} \rangle + \frac{1}{4}\langle \mathbb{H}_0 | \, \mathbb{H}_0 \rangle \right] =$$

- and plugging all the auxiliary fields, the action is given by

$$\begin{aligned} = \; & -\frac{1}{8}\,{\rm tr}\left[[A_\mu, A_\nu]^2 + [a_\mu, a_\nu]^2 + [\phi_p, \phi_q]^2 + [\chi_p, \chi_q]^2 \right] \\ & -\frac{1}{4}\,{\rm tr}\left[[A_\mu, \phi_p]^2 + [a_\mu, \chi_p]^2 + \frac{1}{4}\left(\omega\gamma^{\mu\nu}\bar{\omega}\right)^2 + \frac{1}{4}\left(\bar{\omega}\gamma^{\mu\nu}\omega\right)^2 \right] \\ & -\frac{1}{4}\,{\rm tr}\left[[A_\mu, A_\nu]\,\omega_\alpha\gamma^{\mu\nu}\bar{\omega}_\beta - [a_\mu, a_\nu]\,\bar{\omega}_\alpha\gamma^{\mu\nu}\omega_\beta \right] \\ & +\frac{1}{2}{\rm tr}\left[\phi^2\omega_\alpha\epsilon^{\alpha\beta}\bar{\omega}_\beta - \chi^2\bar{\omega}_\alpha\epsilon^{\alpha\beta}\omega_\beta \right] - {\rm tr}\left[\phi_p\,\omega_\alpha\,\chi^p\,\bar{\omega}_\beta\epsilon^{\alpha\beta} \right]. \end{aligned}$$

- The full effective action is correctly reproduced (see Lerda, Frau, Billo, Pesando and Liccardo 2002).

Yang-Mills Instantons effective action

- The complete effective action is

$$S^{(4)}_{\text{eff}}(\Phi_A) \quad = \quad \mathrm{tr}\left[\langle \mathbb{H}^{(+)}_1 | \, \mathbb{H}^{(-)}_1 \rangle + \frac{1}{4}\langle \mathbb{H}_0 | \, \mathbb{H}_0 \rangle\right] =$$

- and plugging all the auxiliary fields, the action is given by

$$\begin{aligned} = \quad & -\frac{1}{8}\,\mathrm{tr}\left[[A_\mu, A_\nu]^2 + [a_\mu, a_\nu]^2 + [\phi_p, \phi_q]^2 + [\chi_p, \chi_q]^2\right] \\ & -\frac{1}{4}\,\mathrm{tr}\left[[A_\mu, \phi_p]^2 + [a_\mu, \chi_p]^2 + \frac{1}{4}\,(\omega\gamma^{\mu\nu}\bar{\omega})^2 + \frac{1}{4}\,(\bar{\omega}\gamma^{\mu\nu}\omega)^2\right] \\ & -\frac{1}{4}\,\mathrm{tr}\,[[A_\mu, A_\nu]\,\omega_\alpha\gamma^{\mu\nu}\bar{\omega}_\beta - [a_\mu, a_\nu]\,\bar{\omega}_\alpha\gamma^{\mu\nu}\omega_\beta] \\ & +\frac{1}{2}\mathrm{tr}\left[\phi^2\omega_\alpha\epsilon^{\alpha\beta}\bar{\omega}_\beta - \chi^2\bar{\omega}_\alpha\epsilon^{\alpha\beta}\omega_\beta\right] - \mathrm{tr}\left[\phi_p\,\omega_\alpha\,\chi^p\,\bar{\omega}_\beta\epsilon^{\alpha\beta}\right]. \end{aligned}$$

- The full effective action is correctly reproduced (see Lerda, Frau, Billo, Pesando and Liccardo 2002).

Higher Orders - Some comments on the future directions

While the quintic order is trivially zero(by $c-$ghost counting for example), the sixth order is definitely more challenging for several reasons:

- The number of involved terms considerably grows.
- The contact term is identically zero (c-ghost counting) and there is a clear hierarchy of propagator terms but it is not likewise evident how to proceed "efficiently".
- The combinatorial nature of the vertices gets complicated due to presence of 1,2 or 3 propagators and ξ_0 in the amplitudes.

Higher Orders - Some comments on the future directions

- However from brute force computation one can observe that at any order g^n in the expansion $\Phi = g\Phi_A + \sum_{m=2}^{\infty} g^m \; R_m$ we have schematically that

$$Tr\left[R_m\left(EOM(R_{n-m})\right)\right], \qquad \frac{n}{2} < m \leq n-1$$

  and then it is zero since we solve the equations of motion perturbatively. For $m = \frac{n}{2}$ we remain with only the kinetic term of $R_{\frac{n}{2}}$:

$$Tr\left[(\eta_0\, R_{\frac{n}{2}})(Q_B\, R_{\frac{n}{2}})\right]$$

- This fact reduces a lot the number of involved terms. We think this is a good starting point and we hope to report on these progress soon.

Thank you for your Attention!