

School on String Field Theory
and String Phenomenology

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at Harish-Chandra Research Institute

"Construction of superstring field theory"

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Construction of superstring field theory

The rules of the game

Step 1 Construct a kinetic term.

the equation of motion

→ the physical state condition

a gauge symmetry

→ the equivalence relation

Step 2 Include interaction terms.

invariance under a nonlinearly
extended gauge transformation

§1. Open bosonic string field theory

the physical state condition $Q\Phi = 0$

the equivalence relation $\Phi \sim \Phi + Q\Lambda$

Step 1

$$S = -\frac{1}{2} \langle \Phi, Q\Phi \rangle$$

$$\delta S = 0 \quad \text{under} \quad \delta\Phi = Q\Lambda$$

$$\langle A, B \rangle = (-1)^{AB} \langle B, A \rangle$$

$$Q^2 = 0$$

$$\langle A, QB \rangle = -(-1)^A \langle QA, B \rangle$$

Step 2

$$S = -\frac{1}{2} \langle \Phi, Q\Phi \rangle - \frac{g}{3} \langle \Phi, \Phi * \Phi \rangle$$

g : the open string coupling constant

$$\delta S = 0 \quad \text{under} \quad \delta\Phi = Q\Lambda + g(\Phi * \Lambda - \Lambda * \Phi)$$

$$Q(A * B) = QA * B + (-1)^A A * QB$$

$$(A * B) * C = A * (B * C)$$

$$\langle A, B * C \rangle = \langle A * B, C \rangle$$

associativity

non associative two-string product $V_2(A_1, A_2)$

$$S = -\frac{1}{2} \langle \Xi, Q\Xi \rangle - \frac{g}{3} \langle \Xi, V_2(\Xi, \Xi) \rangle$$

$$- \frac{g^2}{4} \langle \Xi, V_3(\Xi, \Xi, \Xi) \rangle + O(g^3)$$



three-string product

$$V_3(A_1, A_2, A_3)$$

$$(-1)^{A_1} V_2(A_1, A_2) = (-1)^{A_1} A_1 + A_2$$

$$(-1)^{A_1} V_3(A_1, A_2, A_3) = (-1)^{A_1} A_1 + A_2 + A_3 + 1$$

$$\langle A_1, V_2(A_2, A_3) \rangle = \langle V_2(A_1, A_2), A_3 \rangle$$

$$\langle A_1, V_3(A_2, A_3, A_4) \rangle$$

$$= (-1)^{A_1} \langle V_3(A_1, A_2, A_3), A_4 \rangle$$

$$\delta S = O(g^3)$$

under

$$\delta \Xi = Q\Lambda + g(V_2(\Xi, \Lambda) - V_2(\Lambda, \Xi))$$

$$+ g^2(V_3(\Xi, \Xi, \Lambda) - V_3(\Xi, \Lambda, \Xi)$$

$$+ V_3(\Lambda, \Xi, \Xi))$$

$$+ O(g^3)$$

if

$$Q^2 = 0,$$

$$QV_2(A_1, A_2) - V_2(QA_1, A_2) - (-1)^{A_1} V_2(A_1, QA_2) = 0,$$

$$QV_3(A_1, A_2, A_3)$$

$$- V_2(V_2(A_1, A_2), A_3) + V_2(A_1, V_2(A_2, A_3))$$

$$+ V_3(QA_1, A_2, A_3) + (-1)^{A_1} V_3(A_1, QA_2, A_3)$$

$$+ (-1)^{A_1+A_2} V_3(A_1, A_2, QA_3) = 0.$$

• These relations are extended
to higher orders. $\rightarrow A_\infty$ structure

• A_∞ structure
 $\updownarrow$ close relation
the decomposition of the moduli space
of Riemann surfaces

• A_∞ structure
 $\rightarrow$ The Batalin-Vilkovisky quantization
is straightforward.

gauge invariance
as a spacetime theory

$\downarrow$ A_∞ structure
world-sheet picture

⑧ 1時間 35分

(10:20 ~ 11:55)

§ 2 Closed bosonic string field theory

Step 1

$$S = -\frac{1}{2} \langle \Phi, c_0^- Q \Phi \rangle$$



$$c_0^- = \frac{1}{2} (c_0 - \tilde{c}_0)$$

Φ : ghost number 2

subject to the constraints

$$(b_0 - \tilde{b}_0) \Phi = 0$$

$$(L_0 - \tilde{L}_0) \Phi = 0$$

$$\delta S = 0 \quad \text{under} \quad \delta \Phi = Q \Lambda$$

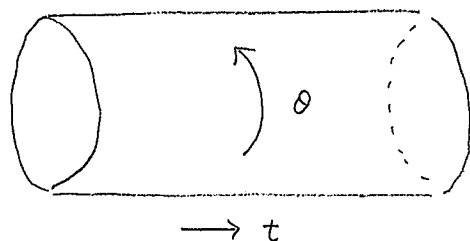


$$(b_0 - \tilde{b}_0) \Lambda = 0$$

$$(L_0 - \tilde{L}_0) \Lambda = 0$$

the origin of the constraints

propagator surface



$$e^{-t(L_0 + \tilde{L}_0) + i\theta(L_0 - \tilde{L}_0)}$$

t, θ : moduli

integration over t ← propagator

$$\frac{b_0^+}{L_0^+} = \int_0^\infty dt \, b_0^+ e^{-tL_0^+}$$

$$\begin{aligned} L_0^+ &= L_0 + \tilde{L}_0 \\ b_0^+ &= b_0 + \tilde{b}_0 \end{aligned}$$

integration over θ

$$B \equiv b_0^- \int_0^{2\pi} \frac{d\theta}{2\pi} e^{i\theta L_0^-}$$

$$L_0^- = L_0 - \tilde{L}_0$$

$$b_0^- = b_0 - \tilde{b}_0$$

$$\sim \delta(b_0^-) \delta(L_0^-)$$

implemented by the constraints

$$B = -i \int_0^{2\pi} \frac{d\theta}{2\pi} \int d\tilde{\theta} e^{i\theta L_0 + i\tilde{\theta} b_0}$$

$\tilde{\theta}$: Grassmann-odd variable

(cf. extended BRST transformation)
Witten, arXiv:1209.5461

$$B c_0 B = B \Rightarrow B c_0 : \text{projector}$$

The constraints can be characterized as

$$B c_0 \mathbb{I} = \mathbb{I}$$

① HRI 1.5 時間

Step 2 completed by Zwiebach in 1992

$$S = -\frac{1}{2} \langle \mathbb{I}, c_0 Q \mathbb{I} \rangle$$

$$- \sum_{n=1}^{\infty} \frac{g^n}{n+2} \langle \mathbb{I}, c_0 \underbrace{V_{n+1}(\mathbb{I}, \mathbb{I}, \dots, \mathbb{I})}_{n+1} \rangle$$

the relations among the multi-string products for the closed string

$\rightarrow L_\infty$ structure

② NCTU

(約 1.5 時間)

§3 Open superstring

bosonic string



spacetime bosons only

superstring

bosons and fermions

conformal field theory



superconformal field theory

$T_F(z)$, $\tilde{T}_F(\bar{z})$: supercurrents

$$T(z) T_F(0) \sim \frac{3}{2z^2} T_F(0) + \frac{1}{z} \partial T_F(0)$$

$$T_F(z) T_F(0) \sim \frac{2c}{3z^3} + \frac{2}{z} T(0)$$

the superconformal ghost sector
the $\beta\gamma$ ghosts

β, γ : commuting fields
weights

$$\beta: \left(\frac{3}{2}, 0\right), \quad \gamma: \left(-\frac{1}{2}, 0\right)$$

the operator products

$$\beta(z_1) \gamma(z_2) \sim -\frac{1}{z_1 - z_2}$$

$$\gamma(z_1) \beta(z_2) \sim \frac{1}{z_1 - z_2}$$

two possible periodicity conditions
in the cylinder coordinate.

the Neveu - Schwarz (NS) sector

$$T_F(\omega + 2\pi) = -T_F(\omega)$$

$$\beta(\omega + 2\pi) = -\beta(\omega)$$

$$\gamma(\omega + 2\pi) = -\gamma(\omega)$$

the Ramond sector

$$T_F(\omega + 2\pi) = T_F(\omega)$$

$$\beta(\omega + 2\pi) = \beta(\omega)$$

$$\gamma(\omega + 2\pi) = \gamma(\omega)$$

We use the doubling trick
for the open superstring.

the NS sector $\rightarrow$ spacetime bosons

the Ramond sector $\rightarrow$ spacetime fermions

§4 Open superstring field theory: the NS sector

mode expansions

$$\beta(z) = \sum_r \frac{\beta_r}{z^{r+\frac{3}{2}}}, \quad \gamma(z) = \sum_r \frac{\gamma_r}{z^{r-\frac{1}{2}}}$$

$\nwarrow \quad \quad \quad \nearrow$
 $r = \pm \frac{1}{2}, \pm \frac{3}{2}, \dots$

$$[\gamma_r, \beta_s] = \delta_{r, -s}$$

$$\beta_r |0\rangle_{NS} = 0 \quad \text{for } r \geq \frac{1}{2}$$

$$\gamma_r |0\rangle_{NS} = 0 \quad \text{for } r \geq \frac{1}{2}$$

this choice $\rightarrow$ -1 picture

The operator corresponding to $|0\rangle_{NS}$
is denoted by $\delta(\gamma(\sigma))$.

ghost number $\sim$ bosonic moduli
picture number $\sim$ fermionic moduli

Step 1

$$S = -\frac{1}{2} \langle \mathbb{F}, Q\mathbb{F} \rangle$$

$\mathbb{F}$: ghost number 1

picture number -1

Q : the BRST operator

in the open superstring

$$\delta S = 0 \quad \text{under} \quad \delta \mathbb{F} = Q\Lambda$$

Step 2

$$S = -\frac{1}{2} \langle \mathbb{F}, Q\mathbb{F} \rangle - \frac{2}{3} \langle \mathbb{F}, \mathbb{F} * \mathbb{F} \rangle ?$$

↑

total picture -3

The total picture for a BPZ
inner product has to be -2

a local insertion of
a picture-changing operator

↓

singularity

~~Step 2~~

The Berkovits formulation

The superconformal ghost sector
can also be described by
 $\xi(z)$, $\eta(z)$, and $\phi(z)$.

anticommuting
fields

weight

linear dilaton CFT

picture

$\xi(z)$	0	1
$\eta(z)$	1	-1
$e^{\ell\phi}(z)$	$-\frac{1}{2}\ell^2 - \ell$	ℓ

$\beta(z), \gamma(z)$: the small Hilbert space

$\xi(z), \eta(z), \phi(z)$: the large Hilbert space

$A \in$ the small Hilbert space

$$\Leftrightarrow \eta A = 0$$

η : the zero mode of $\eta(z)$

BPZ inner products

$$\langle\langle A, B \rangle\rangle = \langle \xi_0 A, B \rangle$$

$\uparrow$

small

$\uparrow$

large

ξ_0 : the zero mode of $\xi(z)$

The total picture
has to be -1.

Step 1

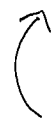
$$S = -\frac{1}{2} \langle \Xi, Q\eta\Xi \rangle$$

$\Xi \in$ the large Hilbert space

ghost number 0.

picture number 0.

$$\delta S = 0 \quad \text{under} \quad \delta \Xi = Q\Lambda + \eta\Omega$$



$$\eta^2 = 0$$

$$\{Q, \eta\} = 0$$

$$\langle \eta A, B \rangle = -(-1)^A \langle A, \eta B \rangle$$

Ξ in the large Hilbert space

satisfying

$$\{\eta, \xi\} = 1$$

(For example, $\xi = \xi_0$.)

$$\Xi = \eta\xi\Xi + \xi\eta\Xi$$



$$\delta\Xi = \eta\Omega$$

$$\Xi = \xi\Pi \quad \text{with} \quad \Pi = \eta\Xi$$

in the small Hilbert space

$$Q\eta\Xi = Q\eta\xi\Pi = Q\{\eta, \xi\}\Pi = Q\Pi = 0 \quad \text{!}$$

② 1時間45分

(13:05 ~ 14:50)

② HRI 約1.5時間

Algebraic relations in the large Hilbert space

$$\langle B, A \rangle = (-1)^{AB} \langle A, B \rangle ,$$

$$Q^2 = 0, \quad \eta^2 = 0, \quad \{Q, \eta\} = 0$$

$$\langle QA, B \rangle = -(-1)^A \langle A, QB \rangle ,$$

$$\langle \eta A, B \rangle = -(-1)^A \langle A, \eta B \rangle ,$$

$$\langle A, B * C \rangle = \langle A * B, C \rangle ,$$

$$(A * B) * C = A * (B * C) ,$$

$$Q(A * B) = QA * B + (-1)^A A * QB ,$$

$$\eta(A * B) = \eta A * B + (-1)^A A * \eta B .$$

Step 2($A * B \rightarrow AB$ in what follows) $O(g^2)$

$$S = -\frac{1}{2} \langle \Phi, Q \eta \Phi \rangle - \frac{g}{6} \langle \Phi, Q [\Phi, \eta \Phi] \rangle$$

$$- \frac{g^2}{24} \langle \Phi, Q [\Phi, [\Phi, \eta \Phi]] \rangle + O(g^3)$$

$$\delta \Phi = Q \Lambda + \eta \Omega - \frac{g}{2} [\Phi, Q \Lambda] + \frac{g}{2} [\Phi, \eta \Omega]$$

$$+ \frac{g^2}{12} [\Phi, [\Phi, Q \Lambda]] + \frac{g^2}{12} [\Phi, [\Phi, \eta \Omega]]$$

$$+ O(g^3)$$

the WZW-like form

Berkovits, hep-th/9503099

$$S = \frac{1}{2} \langle e^{-\Phi} Q e^{\Phi}, e^{-\Phi} \eta e^{\Phi} \rangle$$

$$- \frac{1}{2} \int_0^1 dt \langle e^{-\Phi(t)} \partial_t e^{\Phi(t)},$$

$$\{ e^{-\Phi(t)} Q e^{\Phi(t)}, e^{-\Phi(t)} \eta e^{\Phi(t)} \} \rangle$$

$$\Phi(1) = \Phi$$

$$\Phi(0) = 0$$

$$g = 1$$

Note: no A_∞ structure

Another form

$$S = - \int_0^1 dt \langle A_t(t), Q A_\eta(t) \rangle$$

$$A_\eta(t) = (\eta e^{\Phi(t)}) e^{-\Phi(t)}$$

$$A_t(t) = (\partial_t e^{\Phi(t)}) e^{-\Phi(t)}$$

t -dependence : topological

the gauge transformations

$$A_\delta = Q\Lambda + D_\eta \Omega$$

$$A_\delta = (\delta e^{\Phi}) e^{-\Phi}$$

$$D_\eta A = \eta A - A_\eta A + (-1)^A A A_\eta$$

$$\uparrow \quad \text{with } A_\eta = (\eta e^{\Phi}) e^{-\Phi}$$

"covariant derivative"

$$\eta A_\eta(t) - A_\eta(t) A_\eta(t) = 0$$

$$\partial_t A_\eta(t) = \eta A_t(t) - A_\eta(t) A_t(t) + A_t(t) A_\eta(t)$$

→ gauge invariance,

topological t -dependence

The partial gauge fixing $\Phi \rightarrow \xi \Phi$
 can be extended to the interacting theory.
 $\rightarrow$ a regular formulation
 in the small Hilbert space

Iimori, Noumi, Okawa and Torii,
 arXiv: 1312.1677

$\chi = \{Q, \xi\}$: a line integral
 of the picture-changing
 operator
 $\rightarrow$ no singularities

an action with an A_∞ structure
 using ξ as a new ingredient

Erler, Konopka and Sachs,
 (in Munich)

arXiv: 1312.2948

The Munich construction

$$V_2(A_1, A_2) \\ = \frac{1}{3} \left[\chi(A_1 A_2) + (\chi A_1) A_2 + A_1 (\chi A_2) \right]$$

non associative

→ We need V_3 , but the construction is complicated.

Idea

Consider the following field redefinition of the free theory:

$$S = -\frac{1}{2} \ll \tilde{\Xi}, Q \tilde{\Xi} \gg,$$

$$\tilde{\Xi} = \Xi + g \tilde{V}_2(\Xi, \Xi) + O(g^2).$$

If $V_2(A_1, A_2)$ satisfies

$$(-1)^{\tilde{V}_2(A_1, A_2)} = (-1)^{A_1 + A_2 + 1}, \\ \ll A_1, \tilde{V}_2(A_2, A_3) \gg = (-1)^{A_1} \ll \tilde{V}_2(A_1, A_2), A_3 \gg,$$

the action can be written as

$$S = -\frac{1}{2} \ll \tilde{\Xi}, Q \tilde{\Xi} \gg \\ = -\frac{1}{2} \ll \Xi, Q \Xi \gg - \frac{g}{3} \ll \Xi, V_2(\Xi, \Xi) \gg \\ + O(g^2),$$

where

$$V_2(A_1, A_2) \\ = Q \tilde{V}_2(A_1, A_2) + \tilde{V}_2(Q A_1, A_2) + (-1)^{A_1} \tilde{V}_2(A_1, Q A_2).$$

We can show that the resulting two-string product $V_2(A_1, A_2)$ satisfies

$$(-1)^{V_2(A_1, A_2)} = (-1)^{A_1 + A_2},$$

$$\ll A_1, V_2(A_2, A_3) \gg = \ll V_2(A_1, A_2), A_3 \gg,$$

and one of the A_∞ relations

$$Q V_2(A_1, A_2) - V_2(QA_1, A_2) - (-1)^{A_1} V_2(A_1, QA_2) = 0.$$

This way we can generate a set of multi-string products satisfying the A_∞ relations by field redefinition, but this is just a complicated way of describing the free theory.

The two-string product

$$V_2(A_1, A_2) = \frac{1}{3} \left[\chi(A_1, A_2) + (\chi A_1) A_2 + A_1 (\chi A_2) \right]$$

can be written as

$$V_2(A_1, A_2) = Q \tilde{V}_2(A_1, A_2) + \tilde{V}_2(QA_1, A_2) + (-1)^{A_1} \tilde{V}_2(A_1, QA_2)$$

with

$$\tilde{V}_2(A_1, A_2) = \frac{1}{3} \left[\xi(A_1, A_2) + (\xi A_1) A_2 + (-1)^{A_1} A_1 (\xi A_2) \right].$$

This choice of $V_2(A_1, A_2)$ satisfying the A_∞ relation can be formally understood as being generated from the following field redefinition:

$$\begin{aligned} \widetilde{\Phi} = \Phi + \frac{g}{3} \left[\xi(\Phi\Phi) + (\xi\Phi)\Phi - \Phi(\xi\Phi) \right] \\ + O(g^2). \end{aligned}$$

However, this is actually an "illegal" field redefinition because

$$\eta \widetilde{\Phi} \neq 0.$$

This is why the resulting theory can be interacting.

While $\eta \tilde{\Xi} \neq 0$, the multi-string products have to be in the small Hilbert space.

We can show

$$\eta \tilde{\Xi} - \tilde{\Xi}^2 = 0$$

↓

$\forall n \in$ the small Hilbert space

the same relation as

$$\eta A_n(t) - A_n(t)^2 = 0$$

In fact, the Munich action can be brought to the WZW-like form where $A_n(t)$ and $A_t(t)$ are parameterized in a complicated way.

	The Berkovits formulation	The Munich construction
action	beautifully written in the WZW-like form	not written in a closed form
A_∞	No	Yes

equivalence

Erler, Okawa and Takezaki,

arXiv: 1505.01659

Erler, arXiv: 1505.02069

1510.00364

③ NCTU

(約2時間)

§5 Open superstring field theory: the Ramond sector

mode expansion

$$\beta(z) = \sum_r \frac{\beta_r}{z^{r+\frac{3}{2}}}, \quad \gamma(z) = \sum_r \frac{\gamma_r}{z^{r-\frac{1}{2}}}$$

$\nwarrow \quad \quad \quad \nearrow$
 $r = 0, \pm 1, \pm 2, \dots$

$$[\gamma_r, \beta_s] = \delta_{r, -s}$$

$$\beta_r |0\rangle_R = 0 \quad \text{for } r \geq 0$$

$$\gamma_r |0\rangle_R = 0 \quad \text{for } r \geq 1$$

this choice $\rightarrow -\frac{1}{2}$ picture

③ HRI 1.5 時間

Consider a state A of picture number $-1/2$ in the small Hilbert space.

We say that A is in the restricted space when A satisfies

$$XYA = A,$$

where

$$X = G_0 \delta(\beta_0) + b_0 \delta'(\beta_0),$$

$$Y = -c_0 \delta'(\gamma_0)$$

with G_0 being the zero mode of $T_F(z)$.

$$XYX = X \Rightarrow XY: \text{projector}$$

$$[Q, X] = 0$$

Step 1

$$S = -\frac{1}{2} \langle\langle \Phi, \gamma Q \Phi \rangle\rangle$$

$\Phi \in$ the small Hilbert space

ghost number 1

picture number $-1/2$

$$X\gamma\Phi = \Phi$$

$$\delta S = 0 \quad \text{under} \quad \delta\Phi = Q\lambda$$

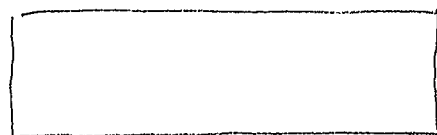
$\uparrow$

$$X\gamma\lambda = \lambda$$

the origin of the restriction

propagator strip

for the Ramond sector



$$e^{-tL_0 + \xi G_0}$$

$\rightarrow t$

bosonic modulus t

+

fermionic modulus ξ

integration over ξ

$$X = \int d\xi \int d\tilde{\xi} e^{\xi G_0 - \tilde{\xi} \beta_0}$$

$\tilde{\xi}$: Grassmann-even variable

(not an ordinary number!)

Witten, arXiv: 1209.5461

Ohmori and Okawa,

arXiv: 1703.08214

$$= G_0 \delta(\beta_0) + b_0 \delta'(\beta_0)$$

$$X \Upsilon \bar{\Psi} = \bar{\Psi}$$



$$B C_0 \bar{\Psi} = \bar{\Psi} \quad \text{for the closed string field} \quad \perp$$

⑩ 1時間 25分

(9:05 ~ 10:30)

Step 2

Let us choose the free theory to be

$$S^{(0)} = -\frac{1}{2} \langle \Phi, \Omega \eta \Phi \rangle - \frac{1}{2} \langle \Phi, \Upsilon \Omega \Phi \rangle$$

$$\delta S^{(0)} = 0$$

$$\text{under } \delta^{(0)} \Phi = \Omega \Lambda + \eta \Omega,$$

$$\delta^{(0)} \Xi = \Omega \lambda.$$

We want to write $X = \{ \Omega, \Xi \}$.

Roughly speaking, $\Xi = \mathbb{H}(\beta_0)$.

See Erler, Okawa and Takezaki,

arXiv: 1602.02582

for a refined definition.

$$S = S^{(0)} + g S^{(1)} + g^2 S^{(2)} + O(g^3)$$

$$\delta \Phi = \delta^{(0)} \Phi + g \delta^{(1)} \Phi + O(g^2)$$

$$\delta \Xi = \delta^{(0)} \Xi + g \delta^{(1)} \Xi + O(g^2)$$

$$S^{(1)} = -\frac{1}{6} \langle \Phi, \Omega [\Phi, \eta \Phi] \rangle - \langle \Phi, \Xi^2 \rangle$$

$$S^{(2)} = -\frac{1}{24} \langle \Phi, \Omega [\Phi, [\Phi, \eta \Phi]] \rangle$$

$$-\frac{1}{2} \langle \Phi, \{ \Xi, \Xi \{ \eta \Phi, \Xi \} \} \rangle$$

$$\delta^{(1)} \Phi = -\frac{1}{2} [\Phi, \Omega \Lambda] + \frac{1}{2} [\Phi, \eta \Omega] - \{ \Xi, \Xi \Lambda \}$$

$$\delta^{(1)} \Xi = X \eta \{ \Xi, \Lambda \} - X \eta \{ \eta \Phi, \Xi \Lambda \}$$

↖ consistent with $X \Upsilon \delta \Xi = \delta \Xi$

Complete action

Kunitomo and Okawa,

arXiv: 1508.00366

$$S = -\frac{1}{2} \langle \Xi, \gamma Q \Xi \rangle$$

$$- \int_0^1 dt \langle A_t(t), Q A_\eta(t) + (F(t) \Xi)^2 \rangle,$$

where

$$A_\eta(t) = (\eta e^{\Xi(t)}) e^{-\Xi(t)},$$

$$A_t(t) = (\partial_t e^{\Xi(t)}) e^{-\Xi(t)},$$

$$\begin{aligned} F(t) \Xi &= \Xi + \Xi \{A_\eta(t), \Xi\} \\ &+ \Xi \{A_\eta(t), \Xi \{A_\eta(t), \Xi\}\} \\ &+ \dots \end{aligned}$$

We can also use the Munich construction for the NS sector.

→ a complete action

with an A_∞ structure

Erler, Okawa and Takezaki,

arXiv: 1602.02582

Konopka and Sachs,

arXiv: 1602.02583

The approach to the Ramond sector by Sen

Sen, arXiv: 1508.05387

originally in closed superstring field theory
 $\rightarrow$ open superstring field theory

Step 1

$$S = \frac{1}{2} \langle\langle \tilde{\Phi}, Q X_0 \tilde{\Phi} \rangle\rangle - \langle\langle \tilde{\Phi}, Q \Phi \rangle\rangle$$

Φ : ghost number 1
 picture number $-1/2$
 $\tilde{\Phi}$: ghost number 1
 picture number $-3/2$
 no constraints

$$\beta_r |0\rangle_r = 0 \quad \text{for } r \geq 1$$

$$\gamma_r |0\rangle_r = 0 \quad \text{for } r \geq 0$$

X_0 : the zero mode of
 the picture-changing operator

$\Phi \rightarrow$ correct spectrum

$\tilde{\Phi} \rightarrow$ extra free fields

Step 2

Use the interactions
of the complete actions
with $\tilde{\Xi} \rightarrow \tilde{\Xi}_0$.

The equations of motion

$$Q \tilde{\Xi} = \mathcal{F}(\Xi, \bar{\Xi}) \quad \Xi: \text{NS sector}$$

$$Q \Xi - Q X_0 \tilde{\Xi} = 0$$

$$\Rightarrow Q \Xi = X_0 \mathcal{F}(\Xi, \bar{\Xi})$$

fluctuations of $\tilde{\Xi}$: free fields

Everything can be described
by $\xi(z)$, $\eta(z)$, and $\phi(z)$.

closed superstring field theories

§ 6. Future directions

Improve open superstring field theory.

key ingredients

- the supermoduli
- A_∞ structure
- the large Hilbert space

scattering amplitudes

Iimori, Noumi, Okawa and Torii,

arXiv: 1312.1677

Kunitomo, Okawa, Sueno and Takezaki,

arXiv: 1612.00777

the large Hilbert space

→ the supermoduli

the $\beta\gamma$ ghosts

without using the large Hilbert space

Ohmori and Okawa,

arXiv: 1703.08214

the relation to the approach by Sen

Okawa and Sakaguchi, to appear

⑪ 1時間 15分 (10:40 ~ 11:55)

④ NCTU (約 1時間 45分)

④ HRI 約 1.5時間