Fermion scattering amplitudes from gauge-invariant actions for open superstring field theory

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Ref: H. Kunitomo, Y. Okawa, H. Sukeno, T. Takezaki
[arXiv:hep-th/161200777]

Introduction

In the world-sheet theory of strings, scattering amplitudes are calculated from the world-sheet path integral, and the integration over the moduli space of Riemann surfaces is crucial for ensuring the decoupling of unphysical degrees of freedom.

In **string field theory**, actions are constructed from **spacetime gauge symmetries** as the first principle. The world-sheet path integral is reproduced from propagators and interaction vertices in string field theory, as in ordinary QFTs.

We now have gauge-invariant actions for open superstring field theory including both the NS sector and the Ramond sector. In this talk, calculations of scattering amplitudes which involve spacetime fermions will be presented.

Open bosonic string field theory

$$S = -\frac{1}{2} \langle\!\langle\, \Psi,\, Q\,\Psi\,\rangle\!\rangle - \frac{g}{3} \langle\!\langle\, \Psi,\, \Psi * \Psi\,\rangle\!\rangle$$

$$\begin{aligned} \Psi &: \text{string field (Grassmann odd)} \\ Q &: \text{BRST charge} \\ \langle\!\langle\ ,\ \rangle\!\rangle &: \text{BPZ inner product} \\ A * B &: \text{star product} \end{aligned}$$

The star product is non-commutative and associative:

$$A * B \neq B * A\,, \qquad A * (B * C) = (A * B) * C\,.$$

The action

$$S = -\frac{1}{2} \langle\!\langle\, \Psi,\, Q\,\Psi \,\rangle\!\rangle - \frac{g}{3} \langle\!\langle\, \Psi,\, \Psi * \Psi \,\rangle\!\rangle$$

is invariant under the transformation

$$\delta\,\Psi = Q\,\Lambda + g\,(\,\Psi * \Lambda - \Lambda * \Psi\,)\ .$$

The gauge invariance follows from

$$\begin{aligned}
&Q\,(A * B) = QA * B + (-1)^{|A|} A * QB\ ,\\
&Q^2 = 0\ ,\\
&\langle\!\langle\, A,\, B \,\rangle\!\rangle = (-1)^{|A|\cdot|B|} \langle\!\langle\, B,\, A \,\rangle\!\rangle\ ,\\
&\langle\!\langle\, QA,\, B \,\rangle\!\rangle = -(-1)^{|A|} \langle\!\langle\, A,\, QB \,\rangle\!\rangle\ ,\\
&\langle\!\langle\, A,\, B * C \,\rangle\!\rangle = \langle\!\langle\, A * B,\, C \,\rangle\!\rangle\ ,\\
&A * (B * C) = (A * B) * C\ .
\end{aligned}$$

We impose Siegel gauge condition:

$$b_0\, \Psi = 0\ .$$

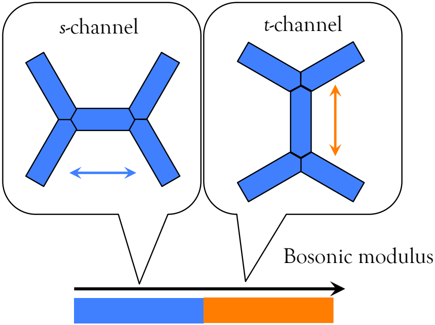
Then the propagator is calculated as

$$|\overbracket{\Psi\rangle\!\rangle\langle\!\langle\Psi}|\ = \frac{b_0}{L_0}\ .$$

The four-point amplitude in open bosonic string field theory is calculated from Feynman diagrams with two cubic vertices and one propagator.

$$\mathcal{A}^{(s)} = \langle\!\langle\, \Psi_A * \Psi_B \,,\, \frac{b_0}{L_0} (\, \Psi_C * \Psi_D \,) \,\rangle\!\rangle$$

$$\mathcal{A}^{(t)} = \langle\!\langle\, \Psi_B * \Psi_C \,,\, \frac{b_0}{L_0} (\, \Psi_D * \Psi_A \,) \,\rangle\!\rangle$$



Open bosonic string field theory was constructed according to the gauge principle, and the scattering amplitudes in the world-sheet path integral are correctly reproduced with the propagator and cubic vertex alone.

But the bosonic string contains the tachyon in both the open string channel and the closed string channel.

How about the construction of open superstring field theory?

Gauge-invariant actions including **the Ramond sector** were constructed. [Kunitomo-Okawa (2015)] [Sen (2015)] [Erler-Okawa-Takezaki(2016)] [Konopka-Sachs (2016)]

In the approach in [Kunitomo-Okawa (2015)], the Ramond string field **in the small Hilbert space** is combined with the Berkovits formulation for the NS sector **in the large Hilbert space**.

We regard Ramond string fields characterized by the *restriction*

$$\mathcal{X}\mathcal{Y}\,\Psi = \Psi$$

as fundamental. Here,

$$\mathcal{X} = \delta(\beta_0)\, G_0 + \delta'(\beta_0)\, b_0\ , \quad \mathcal{Y} = -c_0\, \delta'(\gamma_0)\ ,$$

and an appropriate inner product for the kinetic term of the Ramond sector is given by

$$S^{(0)} = -\frac{1}{2}\langle\!\langle\, \Psi,\, \mathcal{Y}Q\, \Psi\, \rangle\!\rangle\ .$$

Cf. The operator $\mathcal{X}$ satisfies

$$\mathcal{X}\mathcal{Y}\mathcal{X} = \mathcal{X}\ , \quad \mathcal{X} = \{\, Q,\, \Xi\, \}\ , \quad \Xi = \Theta(\beta_0)\ ,$$

and Ξ satisfies

$$\{\, \eta_0,\, \Xi\, \} = 1\ , \quad \Xi^2 = 0\ .$$

The gauge-invariant action is written in a closed form.

$$S = -\frac{1}{2}\langle\!\langle\, \Psi,\, \mathcal{Y} Q\Psi \,\rangle\!\rangle - \int_0^1 \mathrm{d}t \,\langle\, A_t(t),\, QA_\eta(t) + (\, F(t)\Psi\,)^2 \,\rangle\ ,$$

where

$$A_t(t) = \mathrm{e}^{\phi(t)}\,\partial_t \mathrm{e}^{-\phi(t)}, \quad A_\eta(t) = \mathrm{e}^{\phi(t)}\,\eta_0 \mathrm{e}^{-\phi(t)},$$
$$F(t)\Psi = \Psi + \Xi\{\, A_\eta(t),\, \Psi\,\} + \Xi\{\, A_\eta(t),\, \Xi\{\, A_\eta(t),\, \Psi\,\}\} + \cdots\ .$$

ϕ : NS string field in the large Hilbert space

$\langle\, ,\, \rangle$: inner product in the large Hilbert space

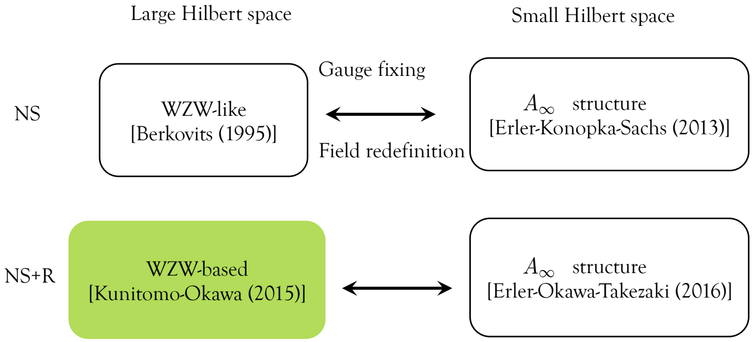
$$\langle\, \xi_0 A,\, B\,\rangle = \langle\!\langle\, A,\, B\,\rangle\!\rangle$$

In the complemental approach for open superstring field theory based on an algebraic structure known as **A_∞ structure**, a gauge-invariant action for open bosonic string field theory, for example, can be constructed with a set of interaction vertices

$$S = \sum \frac{1}{n!} \langle\!\langle \, \Psi, \, V_n(\underbrace{\Psi, \, \cdots \, , \, \Psi}_{n-1}) \, \rangle\!\rangle$$

that satisfies the A_∞ relations,

$$\begin{aligned}
&QA_1 = 0 \\
&QV_2(A_1, A_2) - V_2(QA_1, A_2) - (-1)^{|A_1|} V_2(A_1, QA_2) = 0 \\
&QV_3(A_1, A_2, A_3) - V_2(V_2(A_1, A_2), A_3) + V_2(A_1, V_2(A_2, A_3)) \\
&\quad + V_3(QA_1, A_2, A_3) + (-1)^{|A_1|} V_3(A_1, QA_2, A_3) \\
&\quad + (-1)^{|A_1|+|A_2|} V_3(A_1, A_2, QA_3) = 0
\end{aligned}$$



Quantization	**Complicated**	**Straightforward**
Explicit vertices	**Just expand a closed form**	**Recursion relation**
S-matrix	**Explicit calculation**	**Mathematical proof**

The A_∞ structure of open string field theory in the small Hilbert space is closely related to the decomposition of moduli space of Riemann surfaces. It also makes the Batalin-Vilkovisky quantization of open string field theory straightforward.

On the other hand, use of the large Hilbert space obscures the relation to the supermoduli space, and the Batalin-Vilkovisky quantization of the Berkovits formulation seems to be formidably complicated.

It is generally believed in the framework of string field theory that the extension from a free theory to an interacting theory is unique up to field redefinition if the gauge invariance in the free theory is nonlinearly extended and if the interacting theory is invariant under the nonlinearly extended gauge transformation.

We do not have sufficient understandings as to why the use of large Hilbert space is so effective that we can have closed-form expressions for gauge-invariant actions for string field theory.

Our motivation for the calculations is **not only to confirm the consistency** but also to see how the correct results are reproduced.

In the rest of this talk, I would like to elucidate **how** gauge-invariant actions in OSFT describe **the moduli space of disks**.

Our calculation is a generalization of that for Berkovits formulation in the NS sector[Iimori-Noumi-Okawa-Torii (2013)], and we incorporated the contributions from the Ramond sector.

Fermion scattering amplitudes

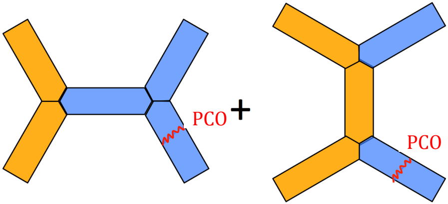
In **the world-sheet theory**, the correct four-point scattering amplitude is given by

$$\mathcal{A}^{WS}_{FFBB} = \langle\!\langle\, \Psi_A * \Psi_B \,,\, \frac{b_0}{L_0}\,(\, X_0 \Phi_C * \Phi_D)\rangle\!\rangle + \langle\!\langle\, \Psi_B * X_0 \Phi_C \,,\, \frac{b_0}{L_0}\,(\, \Phi_D * \Psi_A)\rangle\!\rangle,$$

$$\#_{\rm pic}(\, \Phi\,) = -1 \;, \quad \#_{\rm pic}(\, \Psi\,) = -\frac{1}{2} \;,$$

$$\#(\text{even moduli}) = n_{NS} + n_R - 3 \quad (= 1) \;,$$

$$\#(\text{odd moduli}) = \#(\text{PCO } X) = n_{NS} + \frac{n_R}{2} - 2 \quad (= 1) \;.$$



Now we calculate the propagators. We introduce a source J and add a source term to the kinetic term as

$$S_{NS}^{(0)}[J] = -\frac{1}{2}\langle\, \phi,\, Q\eta_0\phi\,\rangle + \langle\, \phi,\, J\,\rangle\ .$$

The equation of motion is

$$Q\eta_0\phi = J\ ,$$

and, under the gauge conditions $b_0\phi = 0$ and $\xi_0\phi = 0$, the solution to the equation of motion is

$$\phi = \frac{\xi_0 b_0}{L_0} J\ .$$

Evaluating the kinetic term for this solution, we find

$$S_{NS}^{(0)}[J] = -\frac{1}{2}\langle\, J,\, \frac{\xi_0 b_0}{L_0} J\,\rangle\ .$$

Therefore the propagator is given by

$$|\overbracket{\phi\rangle\langle\phi}| = \frac{\xi_0 b_0}{L_0}$$

$$|\overbracket{\Phi\rangle\!\rangle\langle\!\langle\Phi}| = \frac{b_0}{L_0}$$

We introduce a source J and add a source term to the kinetic term as

$$S_R^{(0)}[J] = -\frac{1}{2}\langle\!\langle \Psi, \mathcal{Y} Q \Psi \rangle\!\rangle + \langle\!\langle \Psi, J \rangle\!\rangle \ .$$

The equation of motion is

$$\mathcal{Y} Q \Psi = J \ , \quad Q\Psi = \mathcal{X} J$$

and, under the gauge conditions $b_0 \Psi = 0$, the solution to the equation of motion is

$$\Psi = \frac{b_0 \mathcal{X}}{L_0} J \ .$$

Evaluating the kinetic term for this solution, we find

$$S_{NS}^{(0)}[J] = \frac{1}{2}\langle\!\langle J, \frac{b_0 \mathcal{X}}{L_0} J \rangle\!\rangle \ .$$

Therefore the propagator is given by

$$|\overbrace{\Psi\rangle\!\rangle\langle\!\langle\Psi}| = \frac{b_0 \mathcal{X}}{L_0} \quad .$$

In the Kunitomo-Okawa construction,

$$\mathcal{P}_{NS} \sim \frac{b_0}{L_0}\ , \qquad \mathcal{P}_R \sim \frac{b_0 \mathcal{X}}{L_0}\ ,$$

where $\mathcal{X}$ is a PCO-like operator used to characterize the Ramond spectrum, and

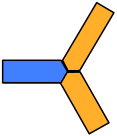
$$S_{BFF} = -\langle\, \phi,\, \Psi * \Psi\, \rangle\ .$$

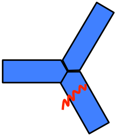
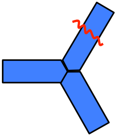
$$S_{BBB} = \frac{g}{3!}\Big\langle\, \phi,\, (\, Q\phi *\, \eta_0\phi - \eta_0\phi * Q\phi\,)\Big\rangle\ .$$

In the Kunitomo-Okawa construction,

$$\mathcal{P}_{NS} \sim \frac{b_0}{L_0} \ , \qquad \mathcal{P}_R \sim \frac{b_0 \mathcal{X}}{L_0} \ ,$$

where $\mathcal{X}$ is a PCO-like operator used to characterize the Ramond spectrum, and

$S_{BFF} \sim$ 

$S_{BBB} \sim$  $+$ 

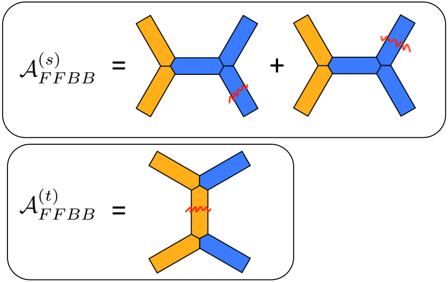
 : picture-changing operator X_0

From Feynman diagrams with one propagator and two cubic vertices, we obtain

$$\mathcal{A}^{(s)}_{FFBB} \;= \frac{1}{2} \langle\!\langle\, \Psi_A\, \Psi_B\, \frac{b_0}{L_0}\, (\, X_0 \Phi_C\, \Phi_D + \Phi_C\, X_0 \Phi_D\,)\, \rangle\!\rangle \;,$$

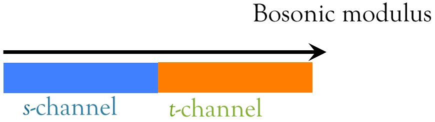
$$\mathcal{A}^{(t)}_{FFBB} \;= \langle\!\langle\, \Psi_B\, \Phi_C\, \frac{b_0 \mathcal{X}}{L_0}\, \Phi_D\, \Psi_A\, \rangle\!\rangle \;.$$

In other words, the assignment of the picture-changing operator is different between the s-channel and the t-channel:



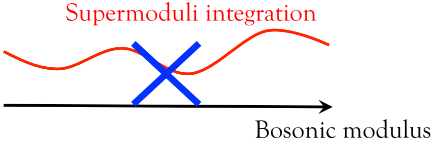
Bosonic moduli

The bosonic modulus is **correctly** covered by Feynman diagrams with one propagator and two cubic vertices.



Supermoduli

The wrong assignment of PCO's is interpreted as **wrong integration over the supermoduli space**.



We will use the following relations:

$$X_0 = \{\, Q,\, \xi_0 \,\} \qquad \mathcal{X} = \{\, Q,\, \Xi \,\}$$

$$X_0 \Phi_a = \{\, Q,\, \xi_0 \,\}\, \Phi_a = Q \xi_0 \Phi_a \quad (\, a = A, B, C, D \,)$$

$$\langle\!\langle\, A,\ B \,\rangle\!\rangle = \langle\, \xi_0 A,\ B \,\rangle \qquad \langle\!\langle\, A,\ B \,\rangle\!\rangle = \langle\, \Xi A,\ B \,\rangle$$

$$\left\{\, Q,\, \frac{b_0}{L_0} \,\right\} = 1$$

$$Q(A * B) = QA * B + (-1)^{|A|} A * QB$$

$$\eta_0(A * B) = \eta_0 A * B + (-1)^{|A|} A * \eta_0 B$$

$$\{\, \eta_0,\, \xi_0 \,\} = 1 \qquad \{\, \eta_0,\, \Xi \,\} = 1$$

$$\mathcal{A}^{(s)}_{FFBB} = \frac{1}{2}\langle\!\langle\, \Psi_A \,\Psi_B \,\frac{b_0}{L_0}\,(\, X_0\Phi_C\,\Phi_D\,)\,\rangle\!\rangle \underbrace{+\frac{1}{2}\langle\!\langle\, \Psi_A \,\Psi_B \,\frac{b_0}{L_0}\,(\, \Phi_C\, X_0\Phi_D\,)\,\rangle\!\rangle}$$

$$\langle\!\langle\, \Psi_A \,\Psi_B \,\frac{b_0}{L_0}\,(\, \Phi_C\, X_0\Phi_D\,)\,\rangle\!\rangle$$

$$= -\langle\, \Psi_A \,\Psi_B \,\frac{b_0}{L_0}\,(\, \xi_0\Phi_C\, Q\xi_0\Phi_D\,)\,\rangle \qquad \boxed{\text{Uplift to the large Hilbert space}}$$

$$\underbrace{=}_{\boxed{\text{Integration by parts of } Q}} -\langle\, \Psi_A \,\Psi_B \,\frac{b_0}{L_0}\,Q\,(\, \xi_0\Phi_C\, \xi_0\Phi_D\,)\,\rangle + \langle\, \Psi_A \,\Psi_B \,\frac{b_0}{L_0}\,(\, Q\xi_0\Phi_C\, \xi_0\Phi_D\,)\,\rangle$$

$$= -\langle\, \Psi_A \,\Psi_B \,(\, \xi_0\Phi_C\, \xi_0\Phi_D\,)\,\rangle + \langle\, \Psi_A \,\Psi_B \,\frac{b_0}{L_0}\,(\, X_0\Phi_C\, \xi_0\Phi_D\,)\,\rangle$$

$$= -\langle\, \Psi_A \,\Psi_B \,(\, \xi_0\Phi_C\, \xi_0\Phi_D\,)\,\rangle + \langle\!\langle\, \Psi_A \,\Psi_B \,\frac{b_0}{L_0}\,(\, X_0\Phi_C\, \Phi_D\,)\,\rangle\!\rangle$$

$$\mathcal{A}^{(s)}_{FFBB} = \frac{1}{2}\langle\!\langle\, \Psi_A\, \Psi_B\, \frac{b_0}{L_0}\,(\, X_0\Phi_C\, \Phi_D\,)\,\rangle\!\rangle \underbrace{+\frac{1}{2}\langle\!\langle\, \Psi_A\, \Psi_B\, \frac{b_0}{L_0}\,(\, \Phi_C\, X_0\Phi_D\,)}\,\rangle\!\rangle$$

$$\langle\!\langle\, \Psi_A\, \Psi_B\, \frac{b_0}{L_0}\,(\, \Phi_C\, X_0\Phi_D\,)\,\rangle\!\rangle$$

$$= -\langle\, \Psi_A\, \Psi_B\, \frac{b_0}{L_0}\,(\, \xi_0\Phi_C\, Q\xi_0\Phi_D\,)\,\rangle$$

Uplift to the large Hilbert space

$$\underbrace{=} -\langle\, \Psi_A\, \Psi_B\, \frac{b_0}{L_0}\, Q\,(\, \xi_0\Phi_C\, \xi_0\Phi_D\,)\,\rangle + \langle\, \Psi_A\, \Psi_B\, \frac{b_0}{L_0}\,(\, Q\xi_0\Phi_C\, \xi_0\Phi_D\,)\,\rangle$$

Integration by parts of Q

$$= -\langle\, \Psi_A\, \Psi_B\,(\, \xi_0\Phi_C\, \xi_0\Phi_D\,)\,\rangle + \langle\, \Psi_A\, \Psi_B\, \frac{b_0}{L_0}\,(\, X_0\Phi_C\, \xi_0\Phi_D\,)\,\rangle$$

$$= -\langle\, \Psi_A\, \Psi_B\,(\, \xi_0\Phi_C\, \xi_0\Phi_D\,)\,\rangle + \langle\!\langle\, \Psi_A\, \Psi_B\, \frac{b_0}{L_0}\,(\, X_0\Phi_C\, \Phi_D\,)\,\rangle\!\rangle$$

$$\mathcal{A}^{(s)}_{FFBB} = \frac{1}{2}\langle\!\langle\, \Psi_A\, \Psi_B\, \frac{b_0}{L_0}\,(\, X_0\Phi_C\, \Phi_D\,)\,\rangle\!\rangle \underbrace{+\frac{1}{2}\langle\!\langle\, \Psi_A\, \Psi_B\, \frac{b_0}{L_0}\,(\, \Phi_C\, X_0\Phi_D\,)}\,\rangle\!\rangle$$

$$\langle\!\langle\, \Psi_A\, \Psi_B\, \frac{b_0}{L_0}\,(\, \Phi_C\, X_0\Phi_D\,)\,\rangle\!\rangle$$

$$= -\langle\, \Psi_A\, \Psi_B\, \frac{b_0}{L_0}\,(\, \xi_0\Phi_C\, Q\xi_0\Phi_D\,)\,\rangle \qquad \boxed{\text{Uplift to the large Hilbert space}}$$

$$\underbrace{=}_{\boxed{\text{Integration by parts of } Q}} -\langle\, \Psi_A\, \Psi_B\, \frac{b_0}{L_0}\, Q\,(\, \xi_0\Phi_C\, \xi_0\Phi_D\,)\,\rangle + \langle\, \Psi_A\, \Psi_B\, \frac{b_0}{L_0}\,(\, Q\xi_0\Phi_C\, \xi_0\Phi_D\,)\,\rangle$$

$$= -\langle\, \Psi_A\, \Psi_B\,(\, \xi_0\Phi_C\, \xi_0\Phi_D\,)\,\rangle + \langle\, \Psi_A\, \Psi_B\, \frac{b_0}{L_0}\,(\, X_0\Phi_C\, \xi_0\Phi_D\,)\,\rangle$$

$$= -\langle\, \Psi_A\, \Psi_B\,(\, \xi_0\Phi_C\, \xi_0\Phi_D\,)\,\rangle + \langle\!\langle\, \Psi_A\, \Psi_B\, \frac{b_0}{L_0}\,(\, X_0\Phi_C\, \Phi_D\,)\,\rangle\!\rangle$$

We will use the following relations:

$$X_0 = \{\, Q,\, \xi_0 \,\} \qquad \mathcal{X} = \{\, Q,\, \Xi \,\}$$

$$X_0 \Phi_a = \{\, Q,\, \xi_0 \,\}\, \Phi_a = Q \xi_0 \Phi_a \quad (\, a = A, B, C, D \,)$$

$$\langle\!\langle\, A,\ B \,\rangle\!\rangle = \langle\, \xi_0 A,\ B \,\rangle \qquad \langle\!\langle\, A,\ B \,\rangle\!\rangle = \langle\, \Xi A,\ B \,\rangle$$

$$\left\{\, Q,\ \frac{b_0}{L_0} \,\right\} = 1$$

$$Q(A * B) = QA * B + (-1)^{|A|} A * QB$$

$$\eta_0(A * B) = \eta_0 A * B + (-1)^{|A|} A * \eta_0 B$$

$$\{\, \eta_0,\, \xi_0 \,\} = 1 \qquad \{\, \eta_0,\, \Xi \,\} = 1$$

Similarly, for the t-channel contribution

$$\mathcal{A}^{(t)}_{FFBB} = \langle\!\langle\, \Psi_B\, \Phi_C\, \frac{b_0 \mathcal{X}}{L_0}\, \Phi_D\, \Psi_A \,\rangle\!\rangle \ ,$$

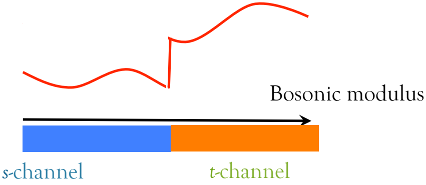
we rewrite it as

$$\begin{aligned}
&\langle\!\langle\, \Psi_B\, \Phi_C\, \frac{b_0 \mathcal{X}}{L_0}\, \Phi_D\, \Psi_A \,\rangle\!\rangle \\
&= -\langle\, \Psi_B\, \xi_0 \Phi_C\, \frac{b_0 \{\, Q,\, \Xi \,\}}{L_0}\, \Phi_D\, \Psi_A \,\rangle \quad \boxed{\text{Uplift to the large Hilbert space}} \\
&\underbrace{=}_{\boxed{\text{Integration by parts of } Q}} -\langle\, \Psi_B\, \xi_0 \Phi_C\, \Xi\, (\, \Phi_D\, \Psi_A \,)\, \rangle - \langle\, \Psi_B\, Q \xi_0 \Phi_C\, \frac{b_0 \Xi}{L_0}\, \Phi_D\, \Psi_A \,\rangle \\
&= -\langle\, \Psi_B\, \xi_0 \Phi_C\, \Xi\, (\, \Phi_D\, \Psi_A \,)\, \rangle + \langle\!\langle\, \Psi_B\, X_0 \Phi_C\, \frac{b_0}{L_0}\, \Phi_D\, \Psi_A \,\rangle\!\rangle
\end{aligned}$$

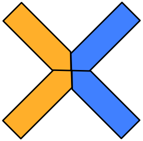
We found that

$$\Delta\mathcal{A}_{FFBB} \equiv \mathcal{A}_{FFBB} - \mathcal{A}^{\mathrm{WS}}_{FFBB}$$
$$= -\frac{1}{2}\langle\, \Psi_A\, \Psi_B\, (\, \xi_0\Phi_C\, \xi_0\Phi_D\,)\,\rangle - \langle\, \Psi_B\, \xi_0\Phi_C\, \Xi\, (\, \Phi_D\, \Psi_A\,)\,\rangle\ .$$

These contributions have appeared as **surface terms of bosonic moduli integrations**, so that they are localized at the boundary of the bosonic moduli regions.



Now let us include contributions from a Feynman diagram without propagators made from the quartic interaction S_{FFBB}.



$$\mathcal{A}^{(4)}_{FFBB} = -\frac{1}{2}\langle\, \xi_0 \Phi_D\, \Psi_A\, \Xi\,(\, \Psi_B\, \Phi_C\,)\,\rangle + \frac{1}{2}\langle\, \Psi_B\, \xi_0 \Phi_C\, \Xi\,(\, \Phi_D\, \Psi_A\,)\,\rangle\ .$$

(S_{FFBB} is required from the spacetime gauge invariance.)

We will use the following relations:

$$X_0 = \{\, Q,\, \xi_0 \,\} \qquad \mathcal{X} = \{\, Q,\, \Xi \,\}$$

$$X_0 \Phi_a = \{\, Q,\, \xi_0 \,\} \, \Phi_a = Q \xi_0 \Phi_a \quad (\, a = A, B, C, D \,)$$

$$\langle\!\langle\, A,\, B \,\rangle\!\rangle = \langle\, \xi_0 A,\, B \,\rangle \qquad \langle\!\langle\, A,\, B \,\rangle\!\rangle = \langle\, \Xi A,\, B \,\rangle$$

$$\left\{\, Q,\, \frac{b_0}{L_0} \,\right\} = 1$$

$$Q(A * B) = QA * B + (-1)^{|A|} A * QB$$

$$\eta_0(A * B) = \eta_0 A * B + (-1)^{|A|} A * \eta_0 B$$

$$\{\, \eta_0,\, \xi_0 \,\} = 1 \qquad \{\, \eta_0,\, \Xi \,\} = 1$$

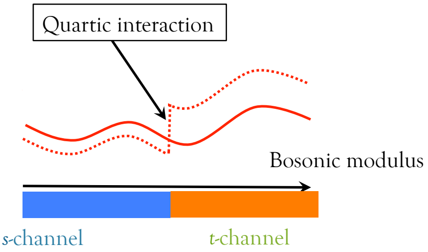
We make ξ_0 assigned at Φ_C (the target state) and eventually find

$$\mathcal{A}^{(4)}_{FFBB} = -\Delta\mathcal{A}_{FFBB} \ ,$$

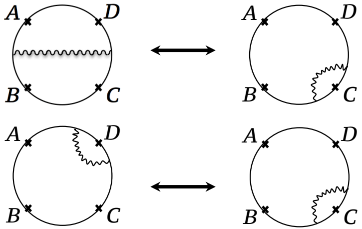
so that

$$\mathcal{A}^{\mathrm{WS}}_{FFBB} = \mathcal{A}^{(s)}_{FFBB} + \mathcal{A}^{(t)}_{FFBB} + \mathcal{A}^{(4)}_{FFBB} \ .$$

Summary



The quartic interaction implements a *vertical integration* (Cf. [Sen-Witten (2015)]) .



Comments

Generalization to Sen's formulation

We can regard our calculation as one for the gauge-invariant formulation with constraints on the Ramond string field,

$$S_R^{(0)} = -\frac{1}{2}\langle\!\langle\, \Psi,\, \mathcal{Y} Q \Psi \,\rangle\!\rangle$$

or one for a gauge-invariant formulation with a spurious free field developed by Sen,

$$S_R^{(0)} = \frac{1}{2}\langle\!\langle\, \tilde{\Psi},\, Q X_0 \tilde{\Psi} \,\rangle\!\rangle - \langle\!\langle\, \tilde{\Psi},\, Q \Psi \,\rangle\!\rangle \ ,$$

from which the propagators are calculated to give

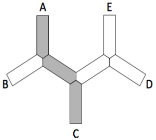
$$\begin{aligned} |\overbracket{\Psi\rangle\!\rangle\langle\!\langle\Psi}| &= \frac{b_0 X_0}{L_0} \ , \\ |\overbracket{\Psi\rangle\!\rangle\langle\!\langle\tilde{\Psi}}| &= \frac{b_0}{L_0} \ . \end{aligned}$$

Five-point amplitudes

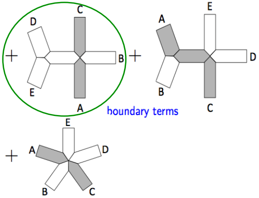
In an alternative approach for constructing open superstring field theory, a constraint was imposed on Ramond string field after deriving Feynman rules from equations of motion, just as in type IIB supergravity. [Michishita (2003)]

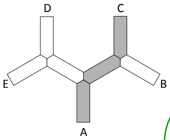
It was reported that the four-point amplitudes were correctly reproduced, but the five-point amplitudes were not.

A refined set of Feynman rules was later proposed and it was confirmed that it reproduces correct five-point amplitudes. [Kunitomo (2014)]

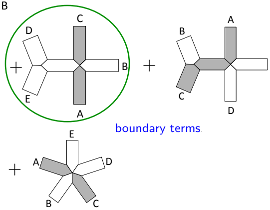


$$= \langle\!\langle\, \Psi_A \textcolor{red}{X} \Phi_B \frac{b_0}{L_0} \Psi_C \frac{b_0}{L_0} \textcolor{red}{X} \Phi_D \Phi_E \,\rangle\!\rangle$$





$$= \langle\!\langle\, \textcolor{red}{X} \Phi_D \Phi_E \frac{b_0}{L_0} \Psi_A \frac{b_0}{L_0} \textcolor{red}{X} \Phi_B \Psi_C \,\rangle\!\rangle$$



Conclusions and Future prospects

We have decoded the relation between the **supermoduli space of disks** and the **spacetime gauge symmetry** in open superstring field theory.

Issues of covering the supermoduli space at the **one-loop level** are closely related to the question whether open superstring field theory is **consistent without closed-string degrees of freedom** or not.

We hope that our analysis provide important data for developing open superstring field theory at the quantum level.

Thank you.