Young quantum - 2015

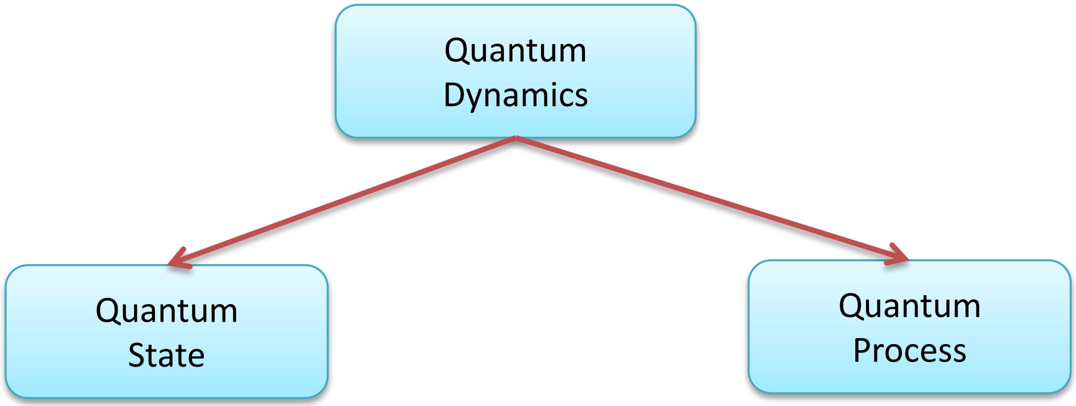
Tracking a quantum system

by

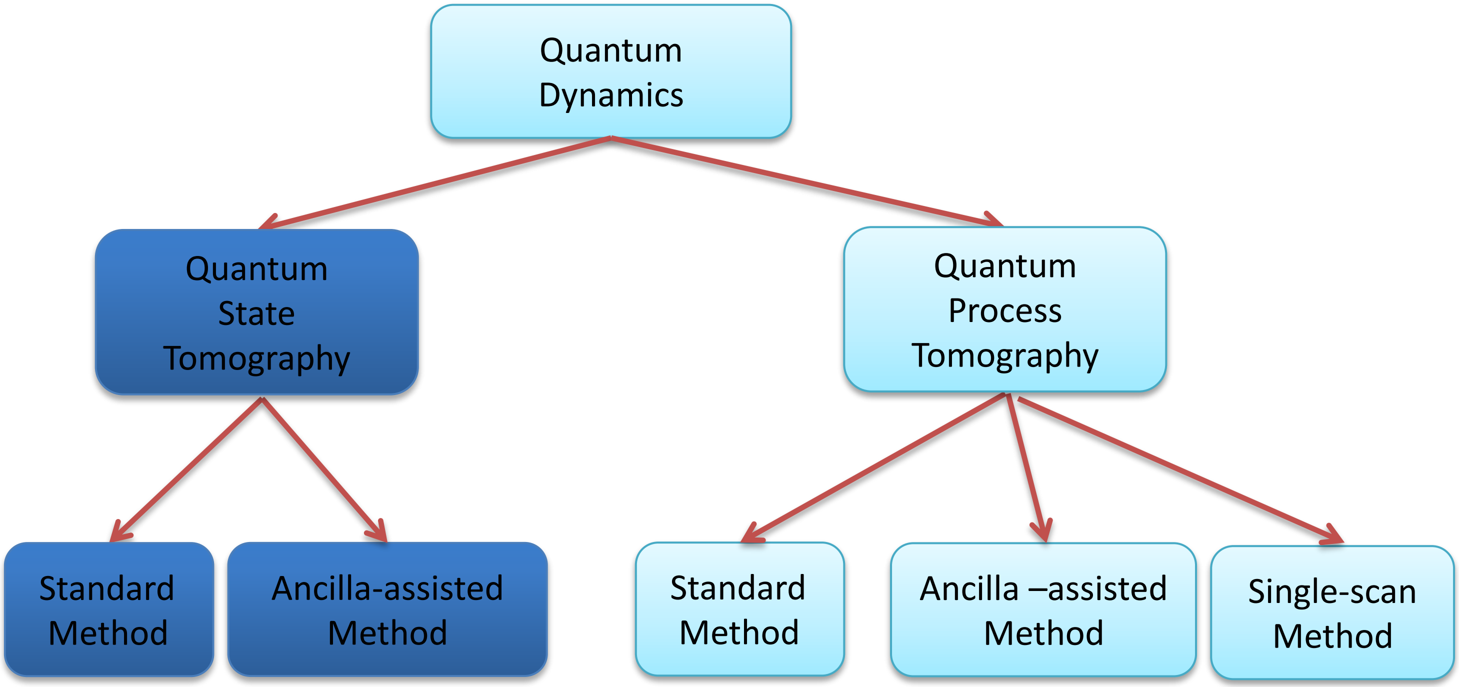
Abhishek Shukla

IISER PUNE

Plan of the talk



Plan of the talk



Quantum State Tomography

Quantum State Tomography: Process of complete characterization of an unknown state.

Why tomography?

To confirm efficiency of state preparation.

To see the effect of imperfect control fields.

To understand the effect of decoherence.

General Procedure:

1. Read the density matrix elements which are directly observable
2. Convert other elements into readable elements via unitary transformation
3. Repeat until all the elements are quantified

Quantum state : 1-qubit

The most general 1-qubit quantum state:

Constant
background-population
- least interesting

purity-factor
depends on state preparation,
temperature, energy gap etc.

Incompatible observables

$$\rho = \frac{1}{2}\left[\, \mathbb{1} + \varepsilon_{purity}\,(n_x\sigma_x + n_y\sigma_y + n_z\sigma_z)\,\right]$$

Transverse components of precessing magnetization

can be measured by time-separated ensemble detections

- quadrature detection

Needs a separate

measurement

ρ needs to be prepared at least two times

Quantum State tomography: 1-qubit

standard method :

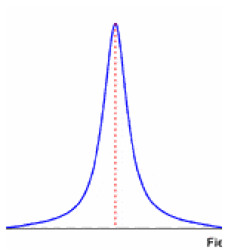
3 unknowns , 2 Direct observables : σ_x , σ_y (single quantum)

1st Experiment

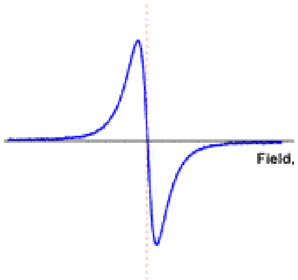
	$\|0\rangle$	$\|1\rangle$
$\langle 0\|$	½+P	X+iY
$\langle 1\|$		½−P

→

Quadrature signal from transverse magnetization



Real part $\langle\sigma_x\rangle \propto$ X



Imag part $\langle\sigma_y\rangle \propto$ Y

NMR

2nd Experiment

½+P	X+iY
	½−P

G_z →

½+P	
	½−P

90_y →

½	P
	½

→ Real part $\propto$ P

Fidelity: Measure of overlap b/w experimental & theoretical (expected) density matricies

$$F = \frac{tr(\rho_{exp}\,\rho_{th})}{[tr(\rho_{exp}^2)\,tr(\rho_{th}^2)]^{1/2}}$$

Quantum State Tomography: 2-qubits

$\|00\rangle$	$\|01\rangle$	$\|10\rangle$	$\|11\rangle$	
13	**1,2**	**3,4**	**9,10**	$\langle 00\|$
	14	**11,12**	**5,6**	$\langle 01\|$
		15	**7,8**	$\langle 10\|$
				$\langle 11\|$

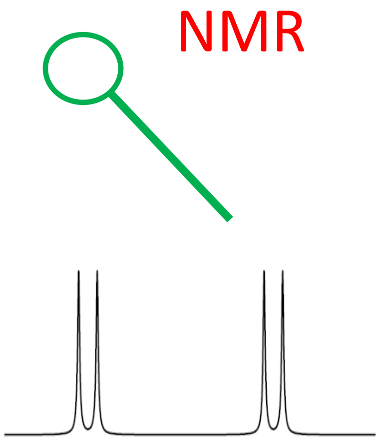
Direct measurement →

$\sigma_{x/y} \otimes |0\rangle\langle 0|$

$\sigma_{x/y} \otimes |1\rangle\langle 1|$

$|0\rangle\langle 0| \otimes \sigma_{x/y}$

$|1\rangle\langle 1| \otimes \sigma_{x/y}$



Fixes 8 unknowns
(4 real + 4 imag)

U_k, followed by measurement ↓

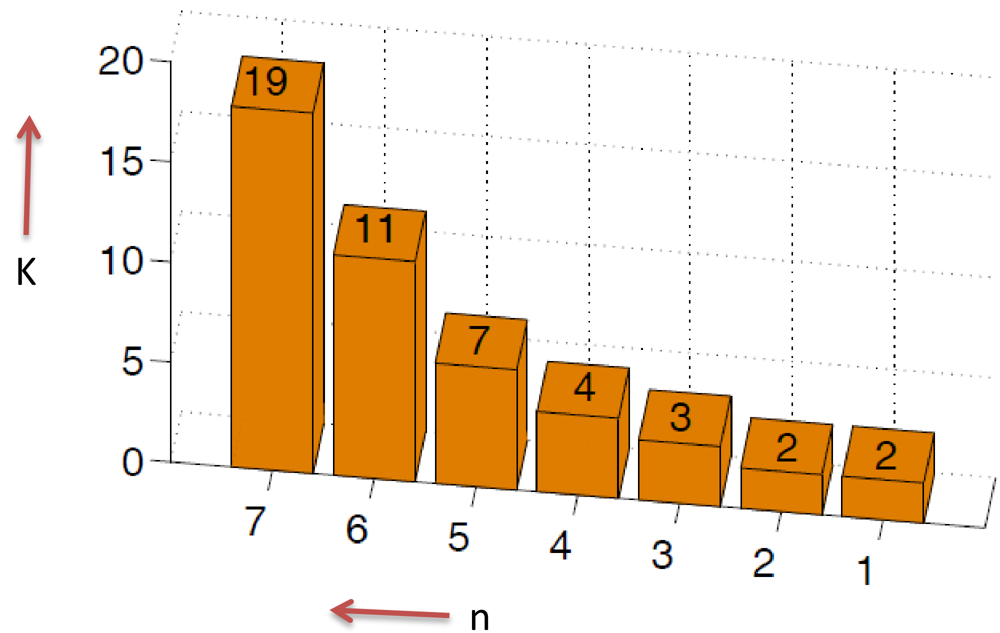
8 Linear equations

Again, at least two independent experiments are needed !!

Quantum State Tomography: n-qubits

Minimum number of experiments

$$K = \frac{\text{total number of unknowns}}{\text{total number of direct observables}} = \left\lceil \frac{N^2 - 1}{nN} \right\rceil \sim 2^n / n$$



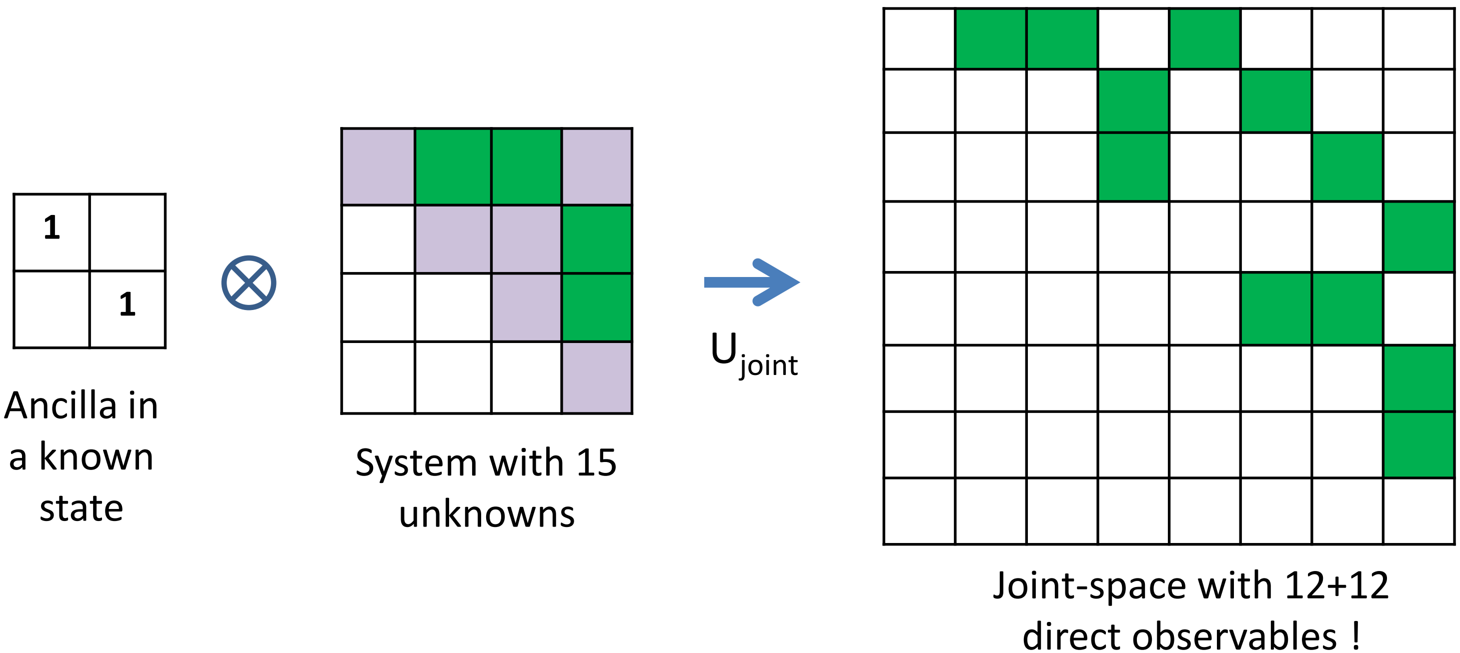
Grows Exponentially with n

Is it possible to reduce number of experimernts : ?

Ancilla-Assisted Quantum State Tomography (AAQST)

E. Allahverdyan et. al.
PRL 2004

Ancilla qubits lead to a larger number of direct observables



Minimum number of experiments

$$K = \frac{\text{total number of unknowns}}{\text{total number of direct observables}} = \lceil 15/24 \rfloor = 1$$

Complete characterization of a quantum state by a SINGLE-joint measurement !!

Advantage: repeated state preparations avoided

QST Vs AAQST

Minimum number of experiments:

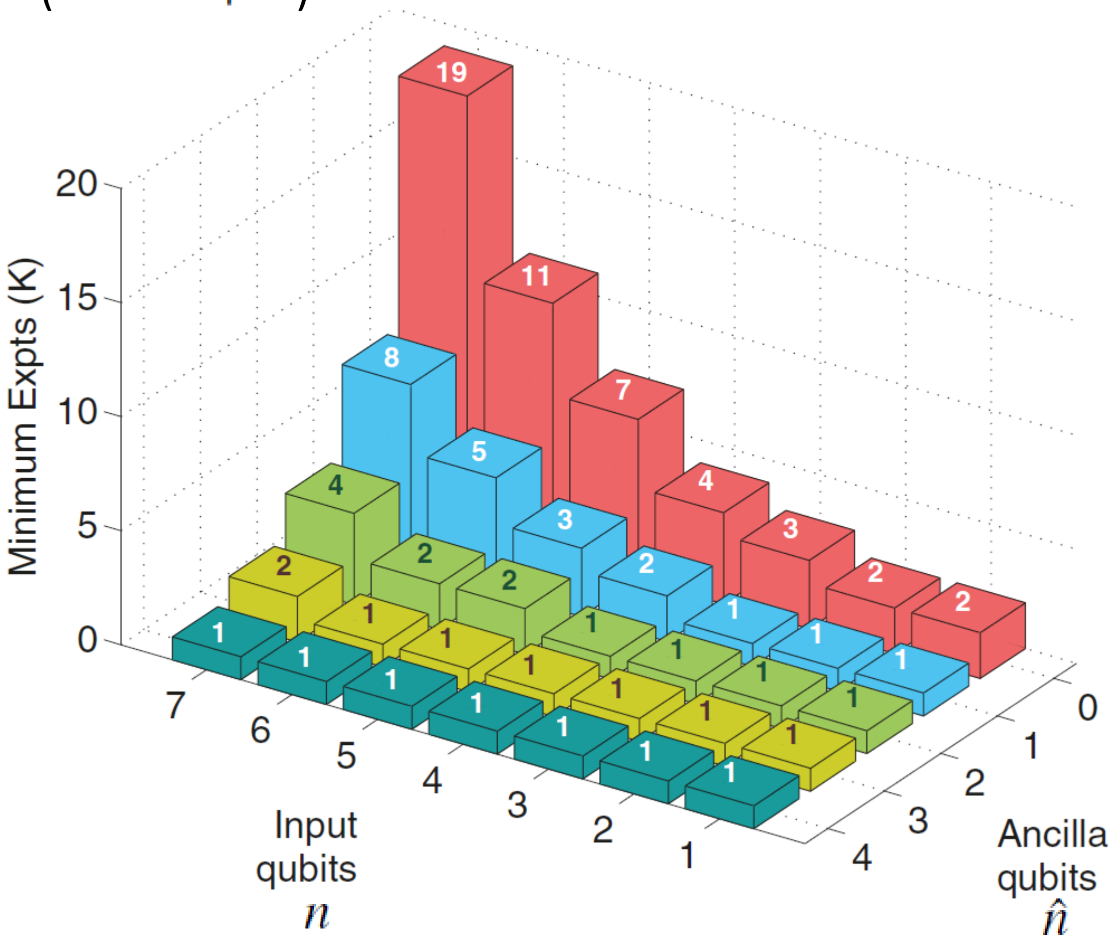
AAQST $K = \left\lceil \frac{N^2 - 1}{\tilde{n}\tilde{N}} \right\rceil \leq K = \left\lceil \frac{N^2 - 1}{nN} \right\rceil$ QST

$(\tilde{n} = n + \hat{n})$

with sufficient number of ancilla qubits, i.e.,

$\tilde{n}\tilde{N} \geqslant (N^2 - 1)$

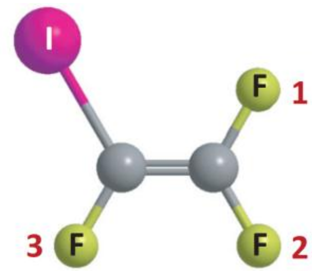
K=1 !!



Abhishek Shukla et al, PRA 2013

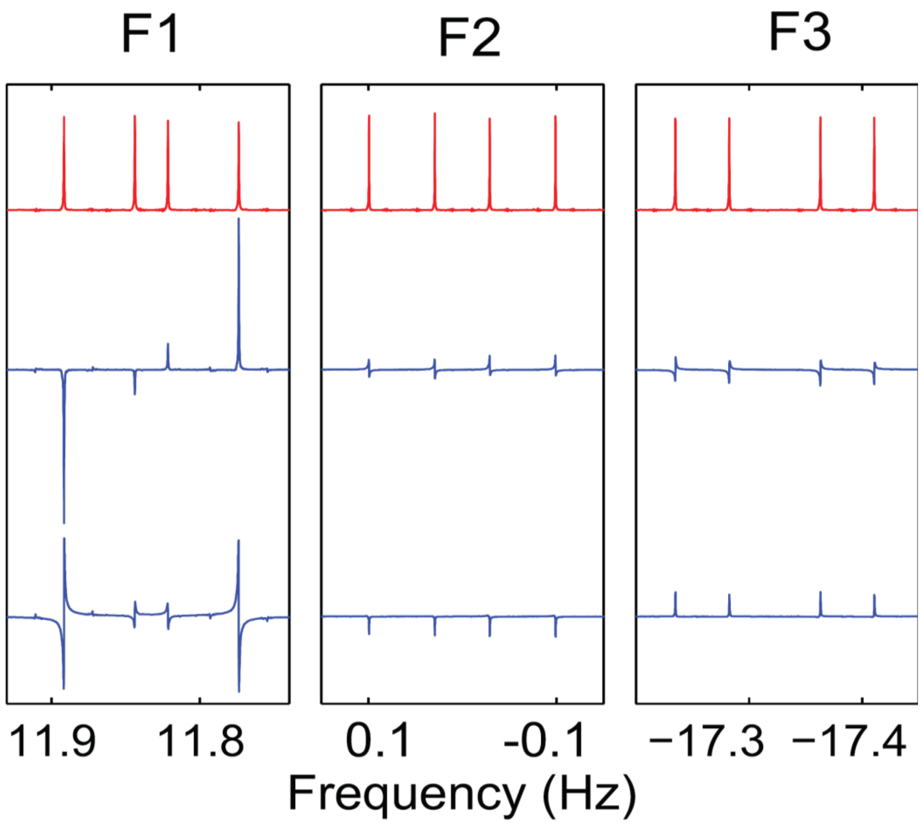
1-spin ancilla and 2-spin system

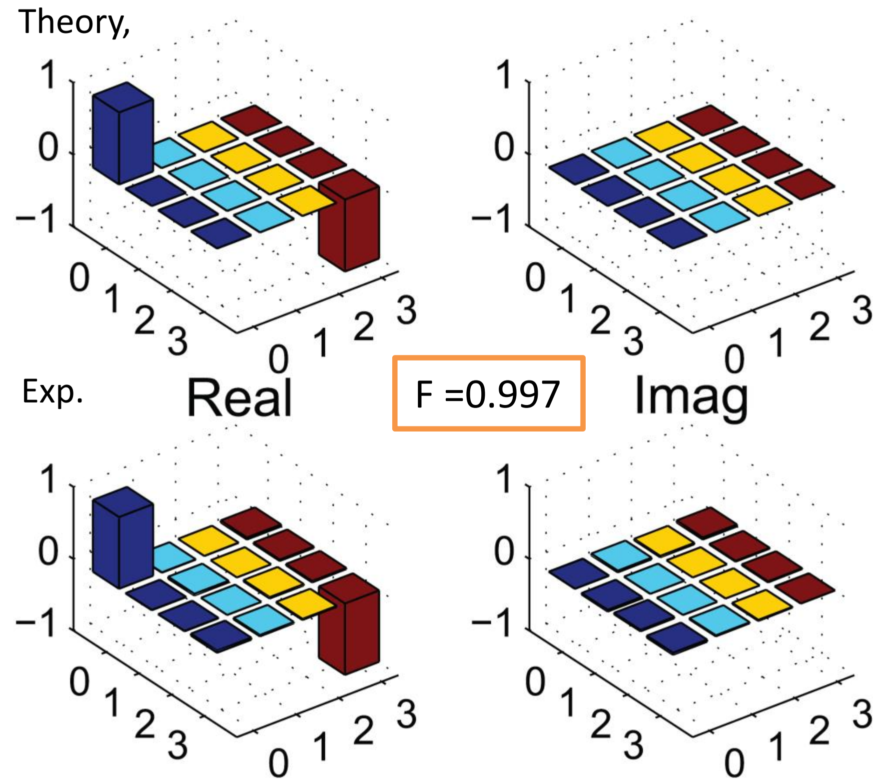
<u>State</u> $\rho_1 = \sigma^2_z + \sigma^3_z$



Number of unknown parameters - 15
Number of direct observables : = 24

TriFluoroIodoethylene





One experiment is sufficient to characterize a 2-qubit density matrix

Abhishek Shukla et al, PRA 2013

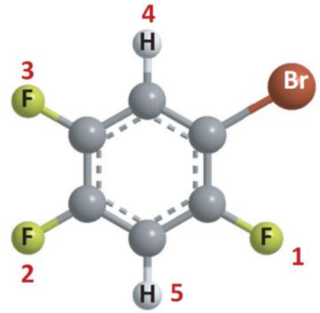
2-spin ancilla and 3-spin system

Abhishek Shukla et al, PRA 2013

State

$\rho_1 = \sigma^1_z + \sigma^2_z + \sigma^3_z$

Bromotrifluorobenzene



Number of unknown parameters - 63

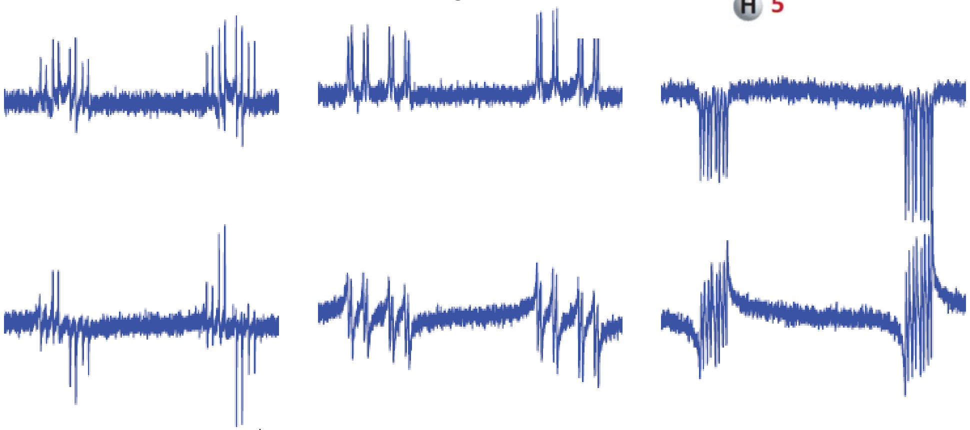
Number of direct observables : $\tilde{n}\tilde{N}$ = 80

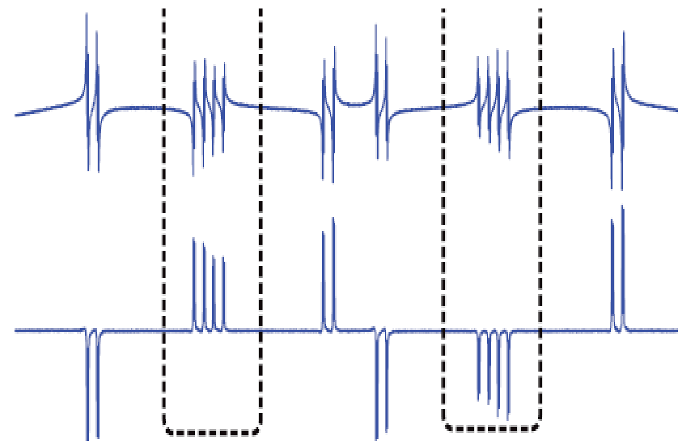
System: F_1, F_2 & F_3

ancilla: H_1 & H_2

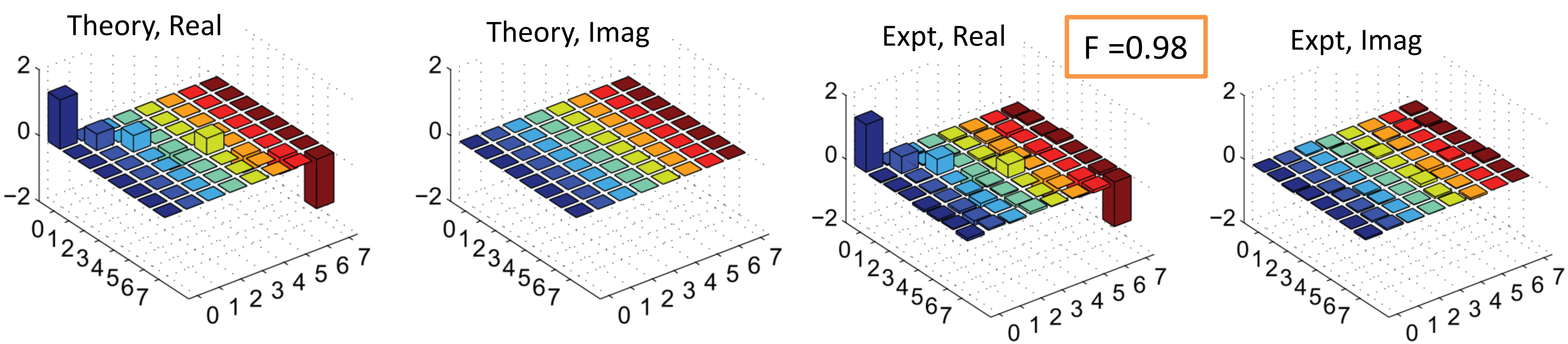
Real

Imag



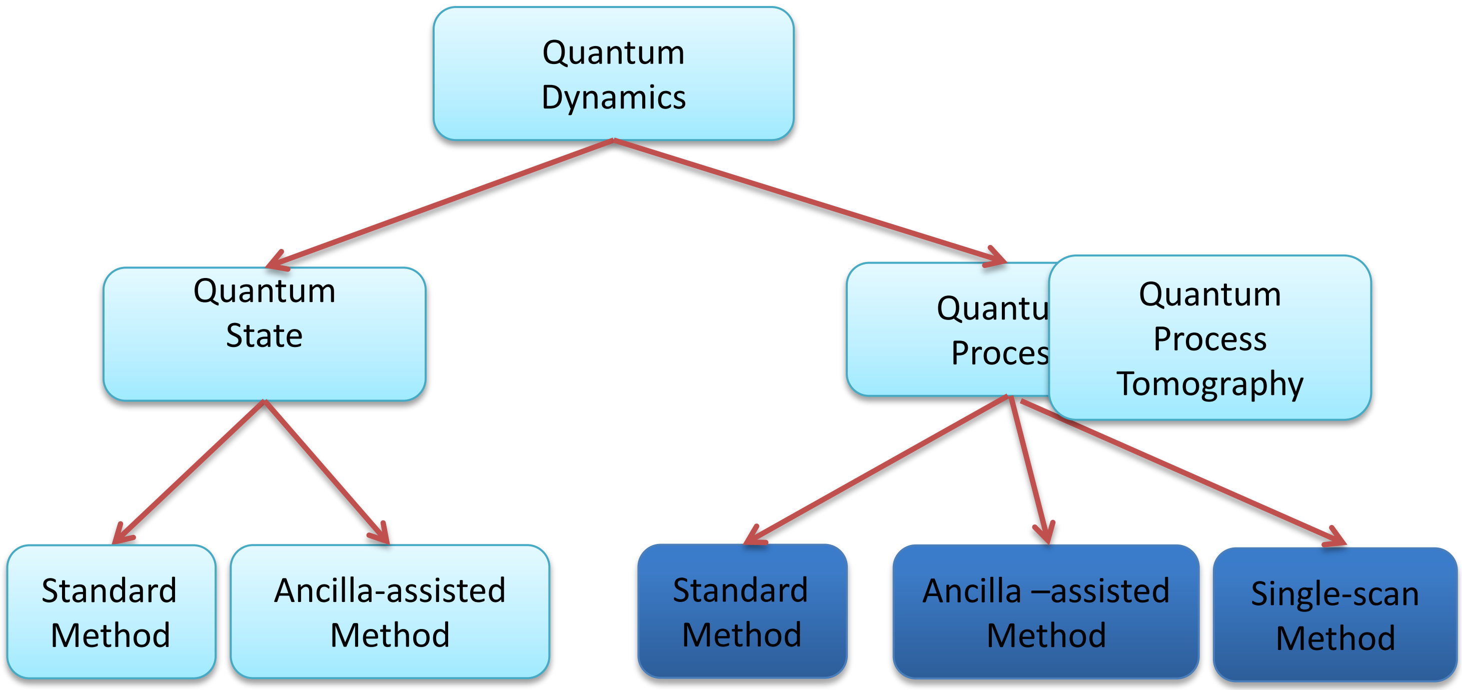


F =0.98



One experiment is sufficient to characterize a 3-qubit density matrix

Plan of the talk



Quantum Process Tomography

A most general quantum process maps a quantum state to a quantum state

Superoperator

$$\rho \xrightarrow{\;\mathcal{E}\;} \rho'$$

Linear map

χ matrix representation

$$\varepsilon(\rho) = \sum_{mn} \tilde{E}_m \rho \tilde{E}_n^\dagger \chi_{mn}$$

fixed set of operators ($\tilde{E}_m$)

completely characterizes the process (χ_{mn})

<u>Why Quantum Process Tomography?</u>

To characterize unknown processes.

To see the effect of imperfect control fields

To understand the effect of decoherence

Experimental QPT: Single qubit

Chuang et al, J. Mod. Optics, 1997

Total: 6 QST (each needs 2 expt)

↓

$\mathcal{E}$ needs to be applied 12 times !
typically takes an hour

$\rho_{3(2)} = |+\rangle\langle+| \pm i\,|-\rangle\langle-| - (1 \pm i)1/2$

$$\rho_1 = \begin{pmatrix} \mathbf{1} & 0 \\ 0 & 0 \end{pmatrix} \xrightarrow{\mathcal{E}^{(S)}} \begin{pmatrix} \lambda_{11} & \lambda_{12} \\ \lambda_{13} & \lambda_{14} \end{pmatrix} \xrightarrow[\text{(2 scans)}]{\text{QST}} \{\lambda_{1k}\}$$

$$\rho_2 = \begin{pmatrix} 0 & \mathbf{1} \\ 0 & 0 \end{pmatrix} \xrightarrow{\mathcal{E}^{(S)}} \begin{pmatrix} \lambda_{21} & \lambda_{22} \\ \lambda_{23} & \lambda_{24} \end{pmatrix} \xrightarrow[\text{(2 scans)}]{\text{QST}} \{\lambda_{2k}\}$$

$$\rho_3 = \begin{pmatrix} 0 & 0 \\ \mathbf{1} & 0 \end{pmatrix} \xrightarrow{\mathcal{E}^{(S)}} \begin{pmatrix} \lambda_{31} & \lambda_{32} \\ \lambda_{33} & \lambda_{34} \end{pmatrix} \xrightarrow[\text{(2 scans)}]{\text{QST}} \{\lambda_{3k}\}$$

$$\rho_4 = \begin{pmatrix} 0 & 0 \\ 0 & \mathbf{1} \end{pmatrix} \xrightarrow{\mathcal{E}^{(S)}} \begin{pmatrix} \lambda_{41} & \lambda_{42} \\ \lambda_{43} & \lambda_{44} \end{pmatrix} \xrightarrow[\text{(2 scans)}]{\text{QST}} \{\lambda_{4k}\}$$

$$\varepsilon(\rho) = \sum_{mn} \tilde{E}_m \rho \tilde{E}_n^\dagger \chi_{mn} = \sum_k \lambda_{jk} \rho_k$$

, where $\tilde{E}_m \rho_j \tilde{E}_n^\dagger = \sum_k \beta_{jk}^{mn} \rho_k$

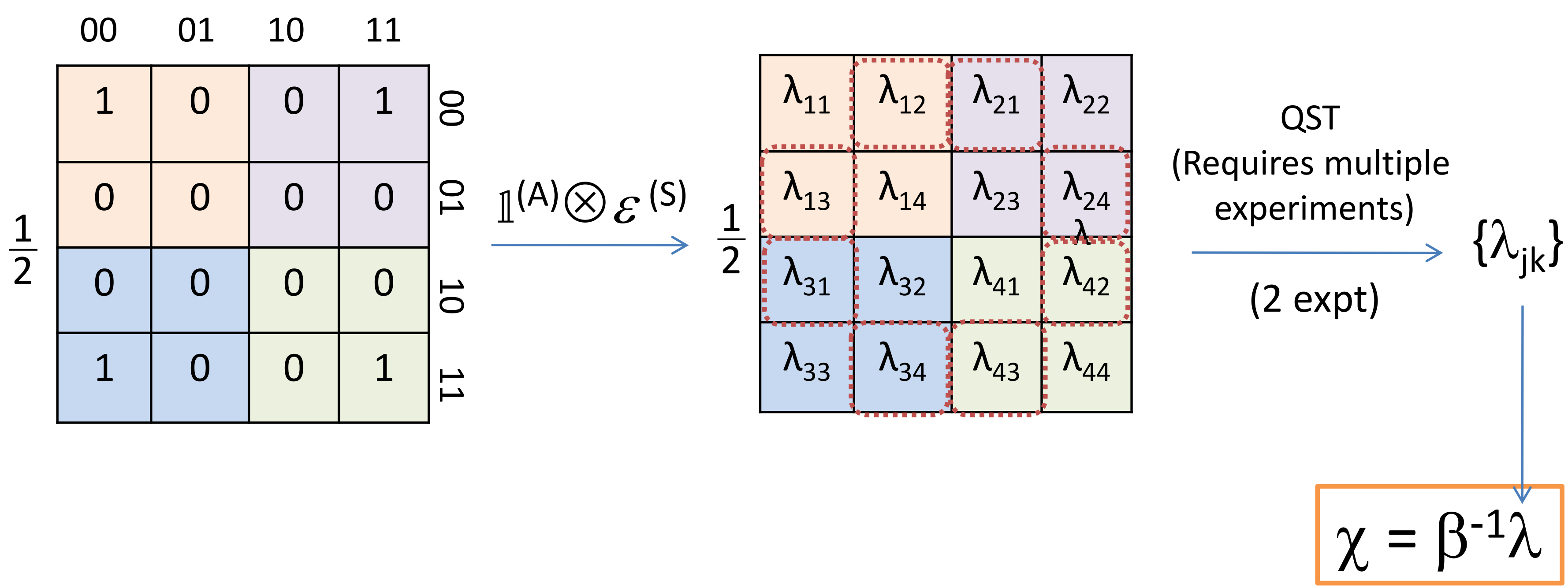
$\chi = \beta^{-1}\lambda$

computed from $\{\tilde{E}_m\}$ and $\{\rho_j\}$

Ancilla Assisted Process Tomography (AAPT):

Altepeter et al, PRL 2003

Ancilla-system joint space



Still process need to be applied 2 times

Single-Scan Process Tomography (SSPT):

Abhishek Shukla et al, PRA 2014

AAPT + AAQST → SSPT

Additional ancilla

1	
	1

$\otimes$

Ancilla-System joint space

1	0	0	1
0	0	0	0
0	0	0	0
1	0	0	1

$\mathbb{I}^{A1} \otimes \varepsilon^{S}$ → $\frac{1}{4}$

λ_{11}	λ_{12}	λ_{21}	λ_{22}	0	0	0	0
λ_{13}	λ_{14}	λ_{23}	λ_{24}	0	0	0	0
λ_{31}	λ_{32}	λ_{41}	λ_{42}	0	0	0	0
λ_{33}	λ_{34}	λ_{43}	λ_{44}	0	0	0	0
0	0	0	0	λ_{11}	λ_{12}	λ_{21}	λ_{22}
0	0	0	0	λ_{13}	λ_{14}	λ_{23}	λ_{24}
0	0	0	0	λ_{31}	λ_{32}	λ_{41}	λ_{42}
0	0	0	0	λ_{33}	λ_{34}	λ_{43}	λ_{44}

↓ AAQST

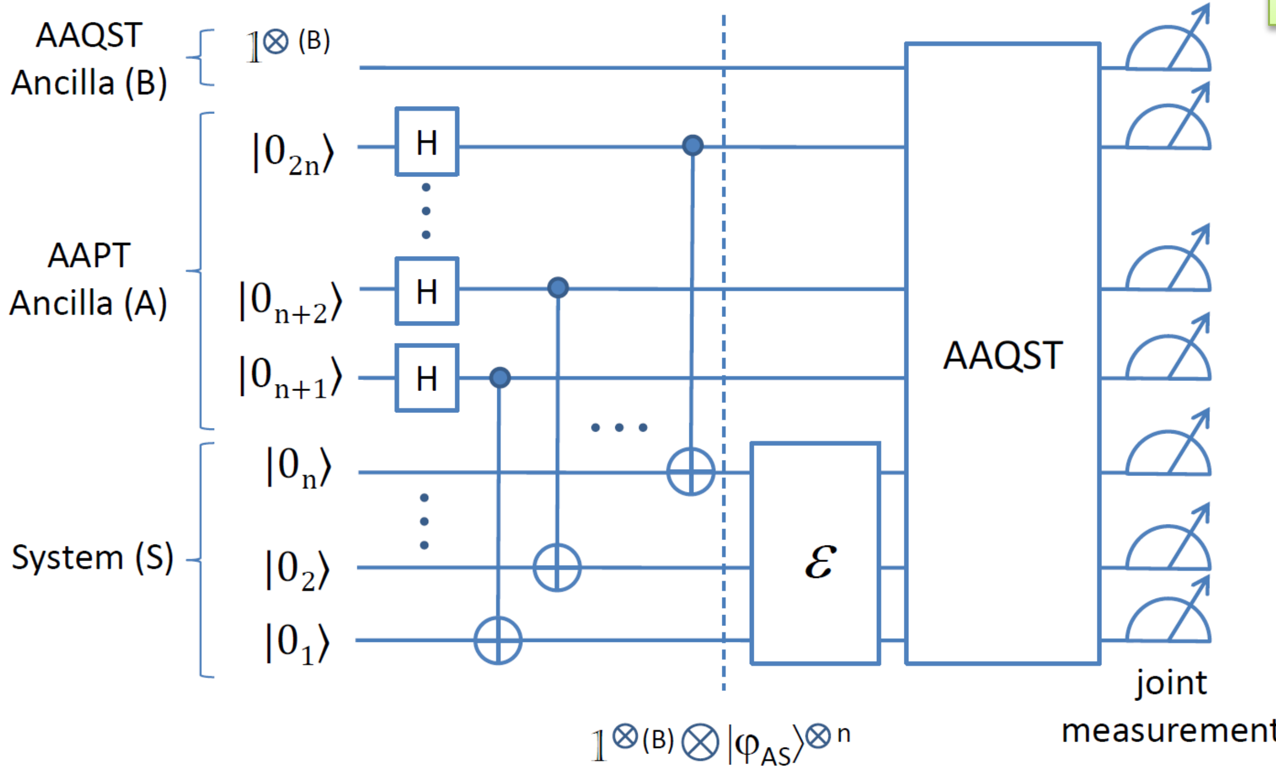
Single expt!!

$$\chi = \beta^{-1}\lambda$$

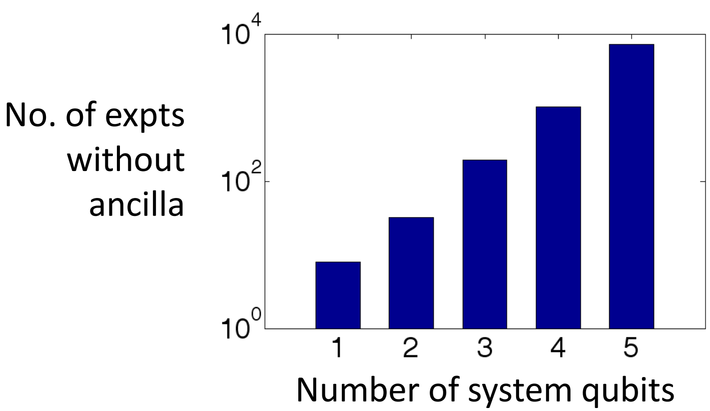
- Process tomography in seconds !
- Single application of process !
- Useful for characterizing dynamic / random processes

SSPT of n-qubits: circuit

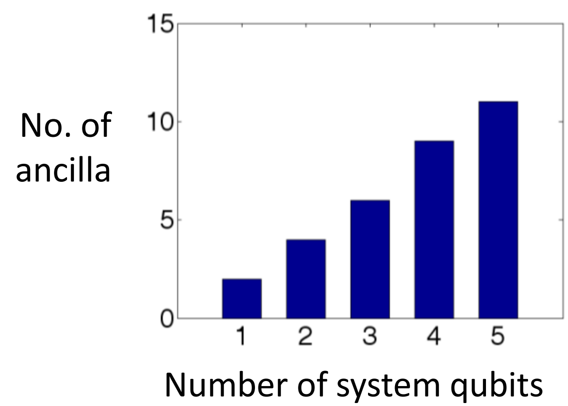
Abhishek Shukla et al, PRA 2014



Standard method:

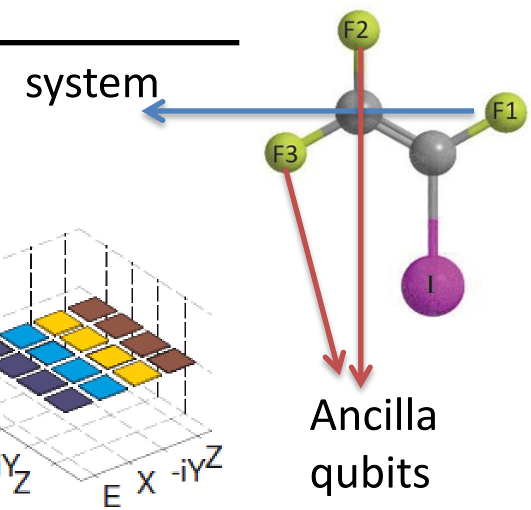


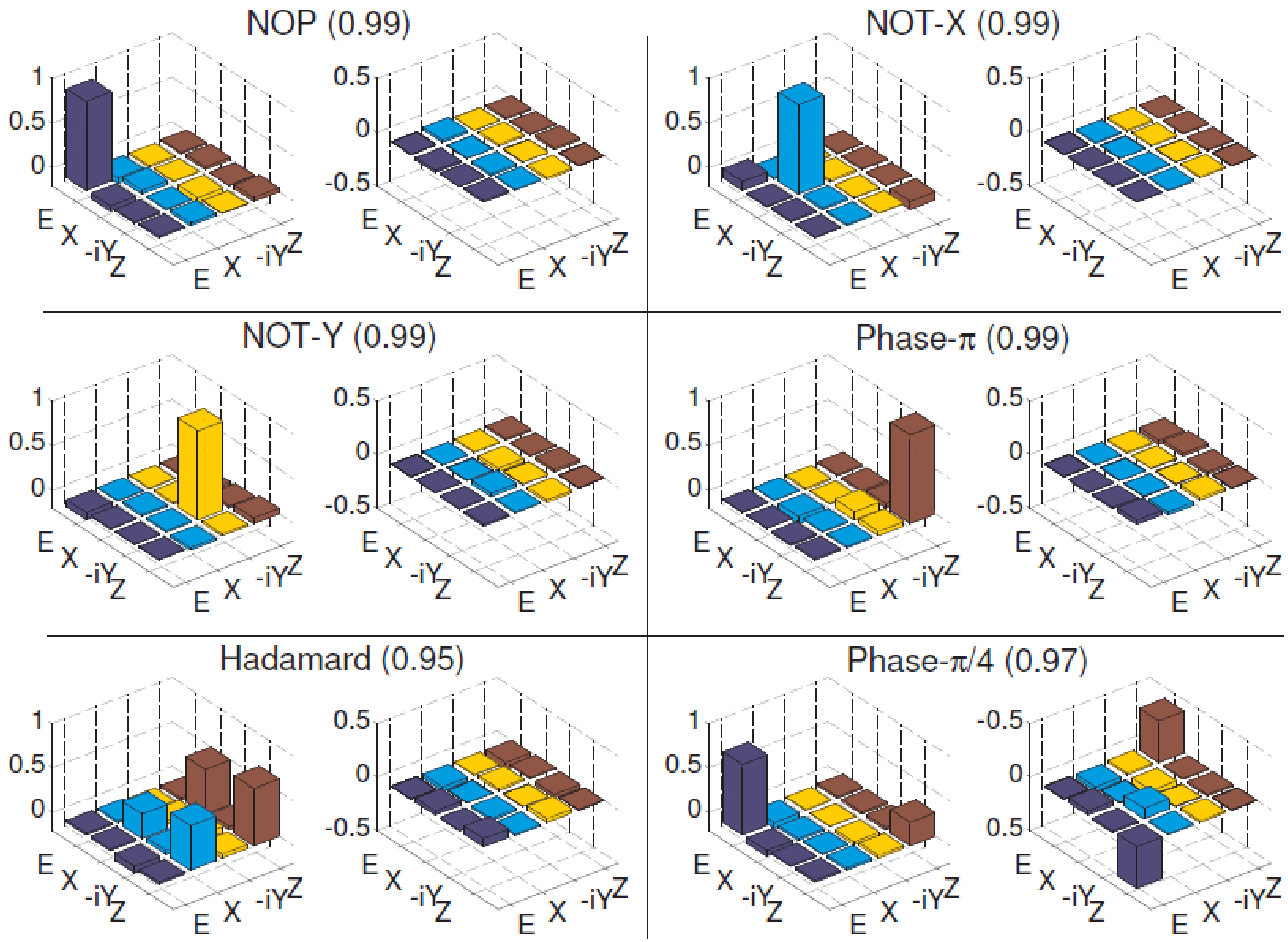
SSPT: (Requires SINGLE experiment)



Experimental 1-qubit SSPT Results

Abhishek Shukla et al, PRA 2014

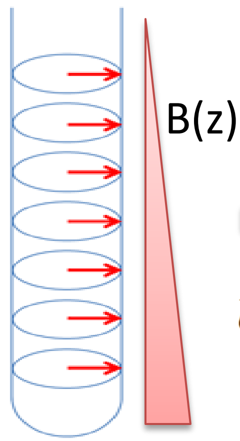




Twirling process

$$\rho_{\text{in}} = \begin{pmatrix} \rho_{00} & \rho_{01} \\ & \rho_{11} \end{pmatrix} \xrightarrow{\;\varepsilon_{\text{twirl}}\;} \begin{pmatrix} \rho_{00} & \\ & \rho_{11} \end{pmatrix} = \rho_{\text{out}}$$

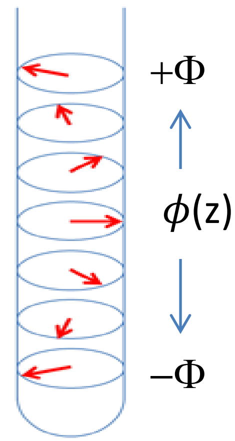
off-diagonal elements depahsed
diagonal elements conserved



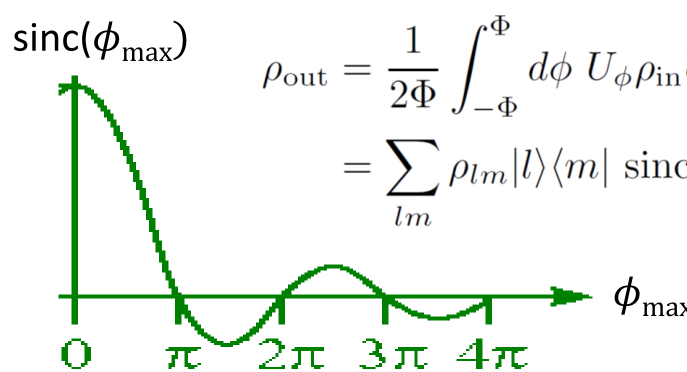
All in phase

Gradient Pulse (G_z)

$$U_\phi = \exp\left(-i\frac{\phi}{2}\sum_{j=1}^{n}\sigma_{jz}\right)$$



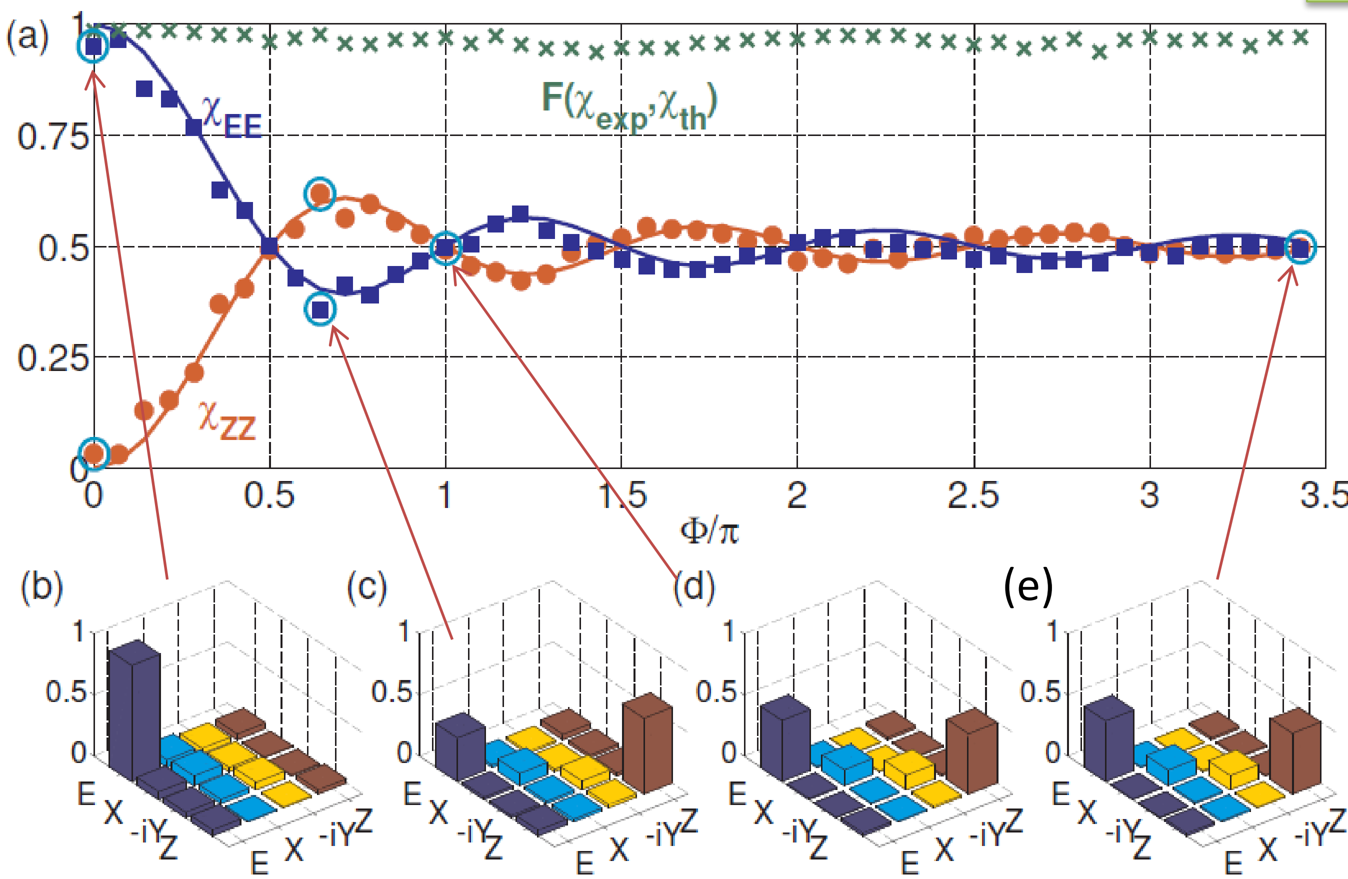
Dephased (twirling)
due to gradient



$$\rho_{\text{out}} = \frac{1}{2\Phi}\int_{-\Phi}^{\Phi} d\phi\; U_\phi \rho_{\text{in}} U_\phi^\dagger$$
$$= \sum_{lm} \rho_{lm} |l\rangle\langle m| \;\text{sinc}(q_{lm}\Phi)$$

Tracking a twirl via SSPT: Experimental results

Abhishek Shukla et al, PRA 2014



Main advantages of SSPT:

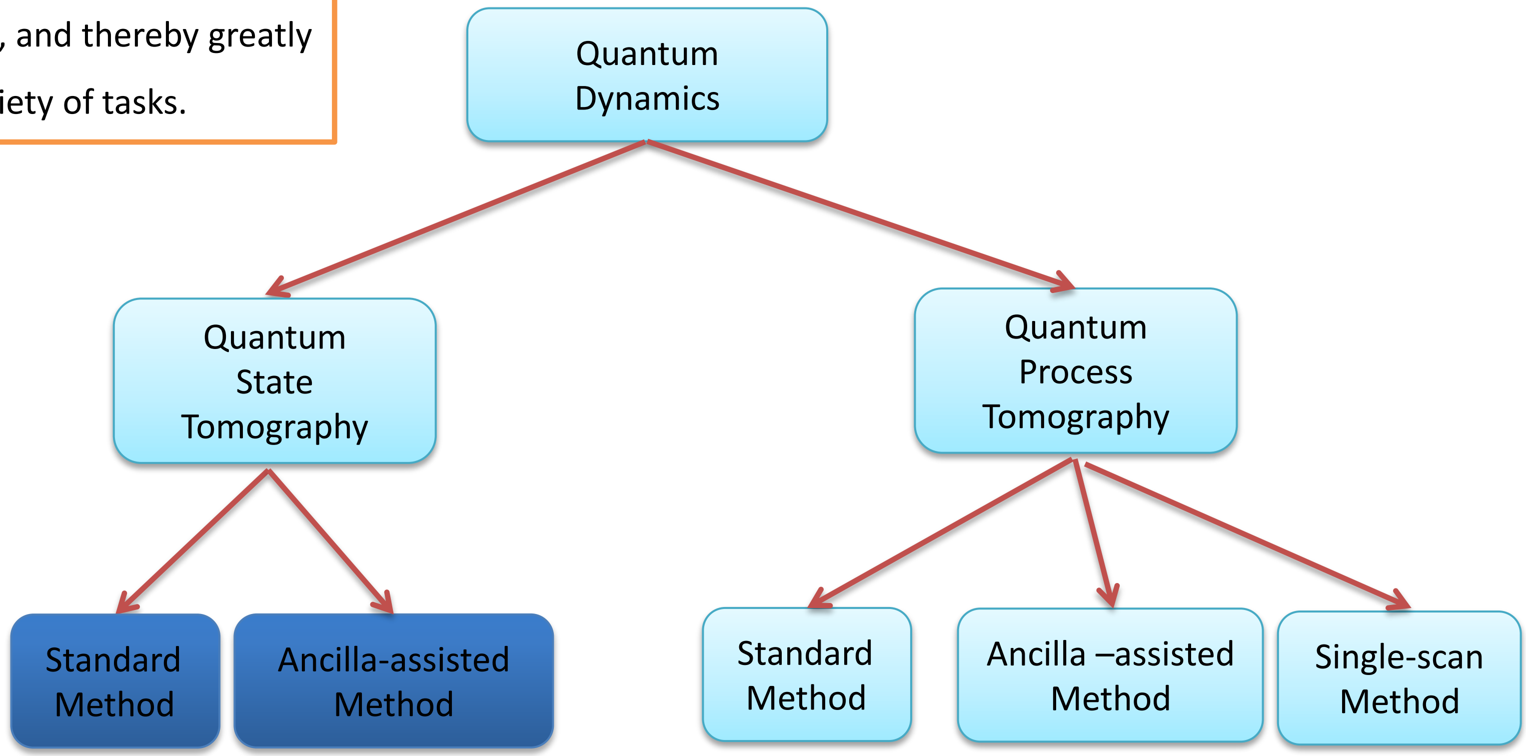
1. Ultrafast: Time taken:

   In the above case, SSPT ~ 4 minutes (QPT ~ An hour)

2. SSPT is the only way for characterizing a dynamical (or random or irreproducible) process

Plan of the talk

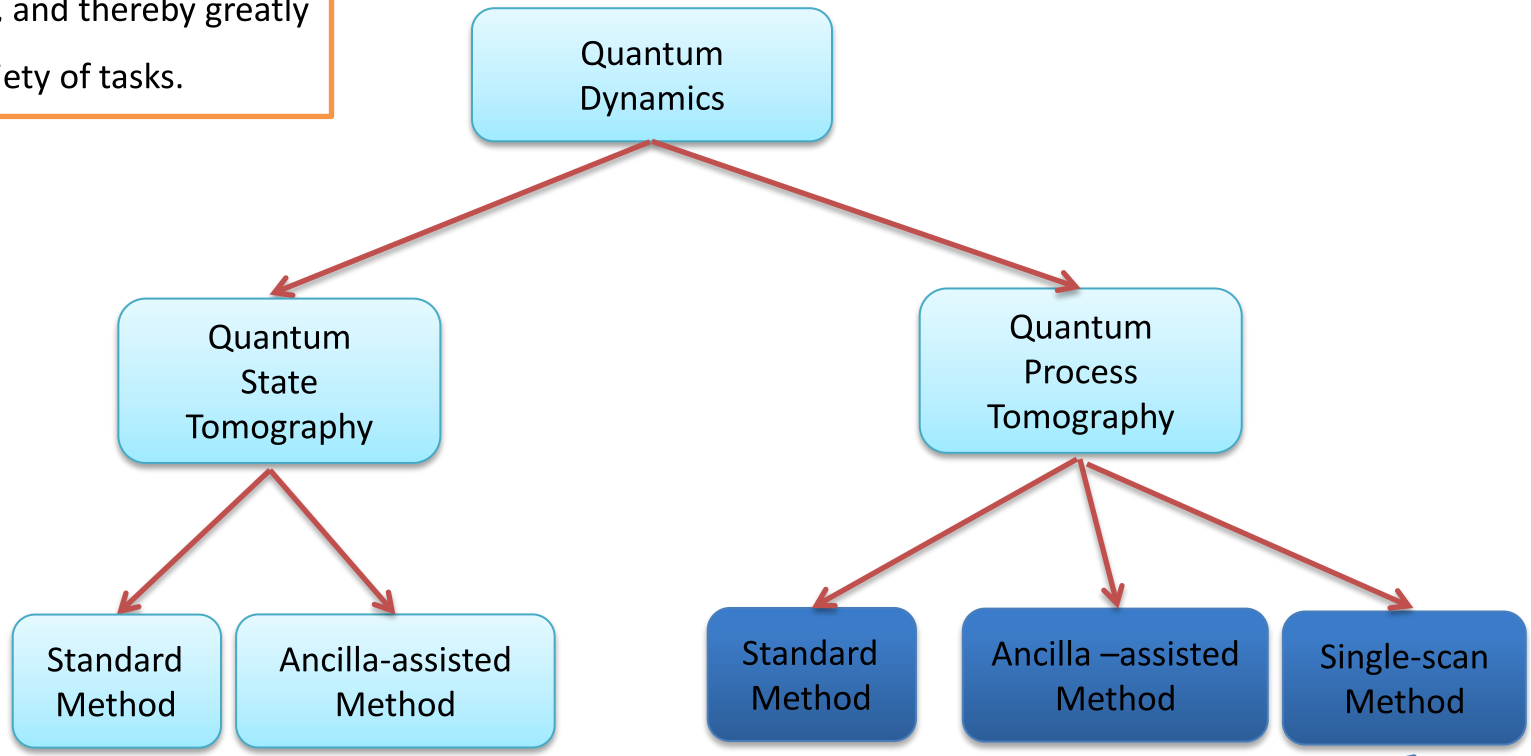
In QIP, ancilla qubits help in extending the computational space, and thereby greatly benefit a variety of tasks.



- Ancilla qubits allow efficient state characterization
- A single collective measurement in the extended space suffices for complete characterization of the 'system'-state.

Plan of the talk

In QIP, ancilla qubits help in extending the computational space, and thereby greatly benefit a variety of tasks.



- Anclla qubits allow complete characterization of a quantum process in a single joint measurement

– method of choice for probing a time-dependent (random/irreproducible) process

Acknowledgement:

- I acknowledge , Dr. T. S. Mahesh for his guidance and support.
- I acknowledge , Director IISER-Pune for the experimental facilities he has provided.
- I also thanks to my lab mats and physics department friends for their discussions and support.
- I also thanks my friends in Chemistry department to provide me support via various chemicals.

To all of u for your attention and time.