Joint Measurability, Steering & Entropic Uncertainty

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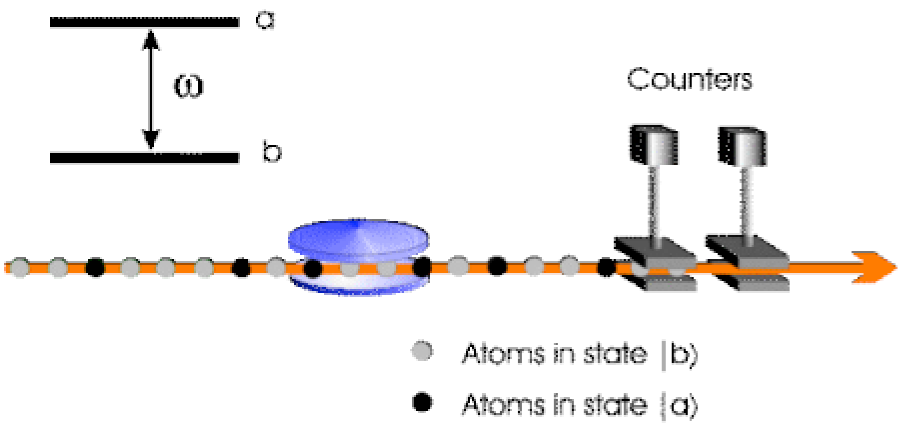
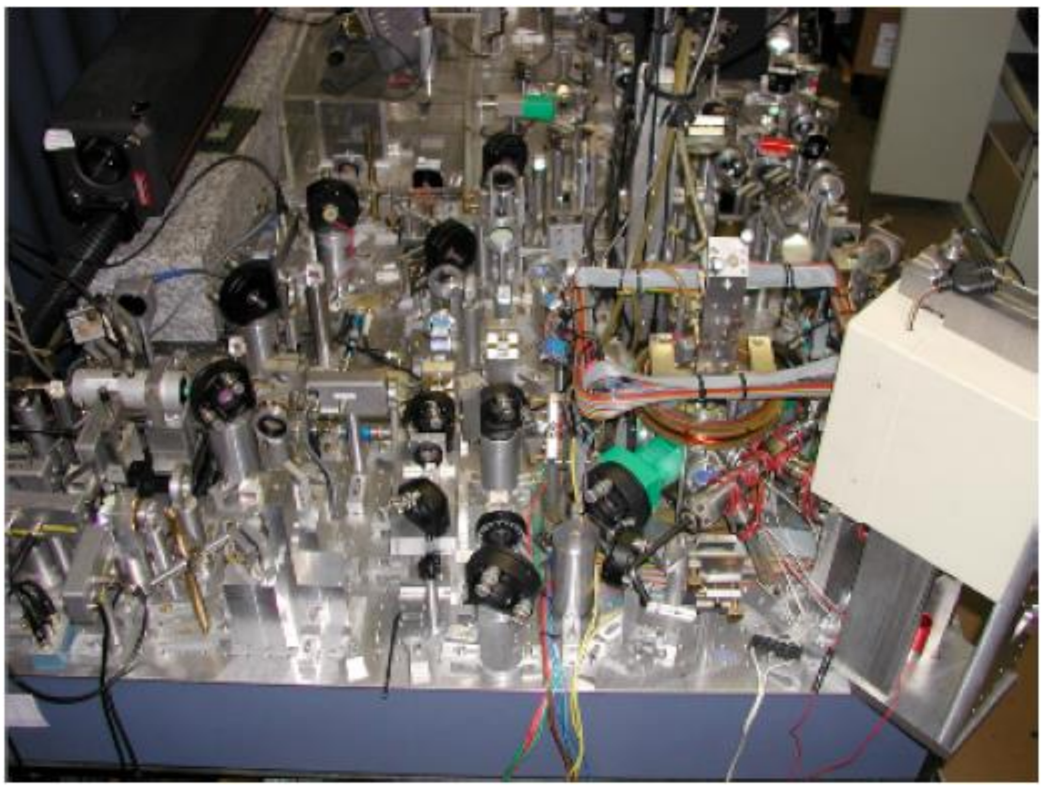
What's in Store?

- Quantum Information Science:

  An Ode to Einstein-Podolosky-Rosen Argument, Bell's Inequality, (Entropic) Uncertainty Relation(s)
- Quantum steering
- Notion of Joint Measurability
- Interconnections (i.e. Our work!!)



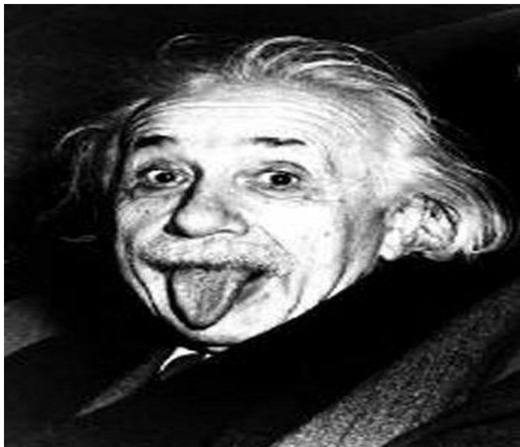
How theorists think about experiments!!



How experimenters think about experiments!!

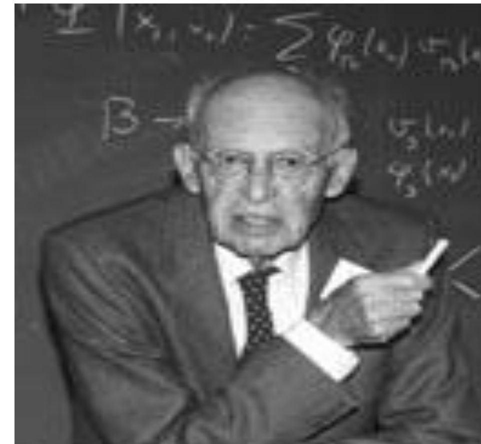
Einstein, Podolsky, Rosen (EPR) Paradox

Albert Einstein



Boris Podolsky

Nathan Rosen



Argument:

For the success of a physical theory, we must ask:

1)Is the theory correct?

2)Is the theory complete?

It is the second question that EPR tries to consider as applied to Quantum Mechanics.

EPR Contd....

Suppose $A|a\rangle = a|a\rangle$ and $B|a\rangle \neq b|a\rangle$

because A and B don't commute: i.e., both A and B do not represent realities for the system in state $|a>$.

This leaves us with two possibilities:

1)Quantum Theory is incomplete

2)Quantum Theory is fine: No simultaneous realities for two non-commuting operators.

EPR Contd....

Let's use Bohm's version of the paradox :

A pair of spin ½ particles ,A and B , with spin vectors $\boldsymbol{\sigma}_A$ and $\boldsymbol{\sigma}_B$ *are formed by decay in a spin singlet state, so that their spins are perfectly anti correlated.*

1)Measure $\boldsymbol{\sigma}_z$ at A's end.

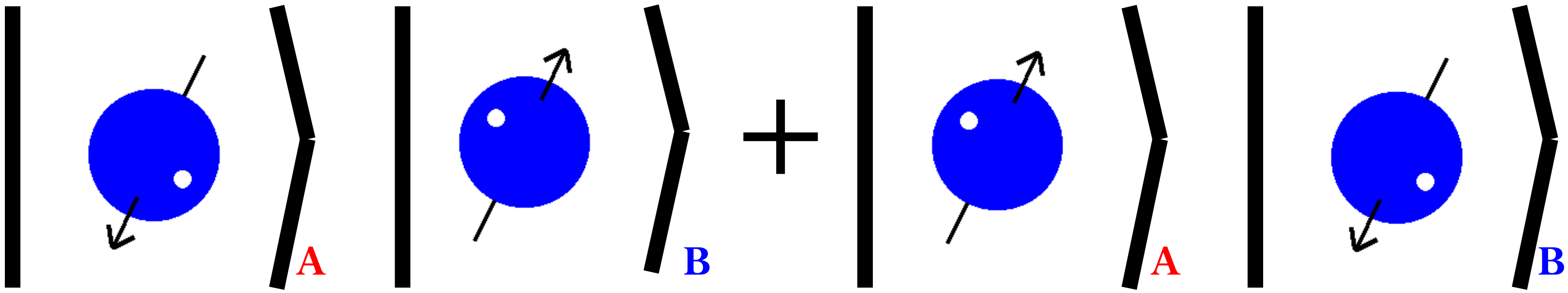
2)Next measure $\boldsymbol{\sigma}_x$ at A's end.

These measurements yield us the results at B's end though nothing was done at B.

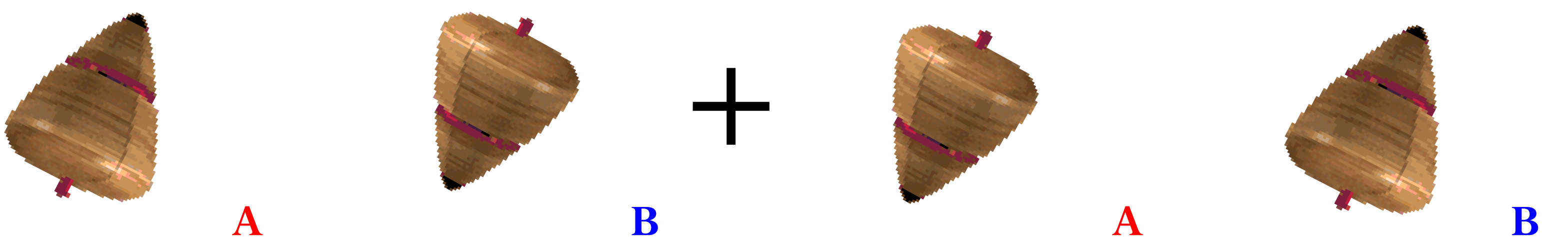
EPR Conclusion

- We have predicted the values of measurements at B's end though $[\sigma_{Bz}, \sigma_{Bx}] \neq 0$
- There are realities hidden at B's end which is NOT accounted by the wave function/state.
- Thus Quantum Mechanics is an incomplete description of reality that must be extended in some way to describe all these objective properties.

Hidden Variable Theory



Can the Spooky, Action-at-a-distance Predictions (Entanglement) of Quantum Mechanics…



…Be Replaced by Some Sort of Local, Statistical, Classical (Hidden Variable) Theory?

NO!—Bell's Inequality

The physical predictions of quantum theory disagree with those of any local (classical) hidden-variable theory!

Bell's Inequality

- *In 1964, **J S Bell** brought out a clear mathematical description of what is known as **local realism** and put forth a test to check whether quantum theory adheres to it.*
- *CHSH version of Bell's Inequality:*

Bell's Inequality(BI):

$$|C(A(a),B(b))-C(A(a),B(b'))| + |C(A(a'),B(b))+C(A(a'),B(b'))| \leq 2$$

Where C(A(a),B(b)) is the correlation among observables A,B measured on two spatially separated systems, which take values a, b respectively.

Quantum Entanglement

"Quantum entanglement is the characteristic trait of quantum mechanics, the one that enforces its entire departure from classical lines of thought."

— Erwin Schrödinger

Take Away from EPR

- Apparently provides the "values" of the non-commuting observables of both the particles
- Breaks uncertainty principle?

- Ref: *"Is quantum mechanical description of reality complete?" (EPR,* Physical review **47** , 777 (1935*)).*

Former paradoxes of quantum foundations are now resources of quantum information science!

Entropic Uncertainty Relation(EUR)

- From an information-theoretic perspective, it is natural to capture this "ignorance" in terms of Shannon entropies rather than variances.
- Entropic Uncertainty Relations (EUR) have broadened and strengthened the original notion of Heisenberg's UP.

MAASSEN & UFFINK EUR:

$$H_\rho(X) + H_\rho(Z) \geq -2\log_2 C(X,Z)$$

where $C(X,Z) = \max_{x,z} |\langle x|z\rangle|$.

- The lower bound limiting the sum of entropies is independent of the state **ρ**.

H. Maassen & J. B. M. Uffink, Phys. Rev. Lett. **60**, 1103(1988).

EUR contd…

- The term **C(X,Z)** can assume a maximum value $\mathbf{1/\sqrt{d}}$ resulting in the maximum entropic bound of $\mathbf{\log_2 d}$**,** where **d** denotes the dimension of the system.

EXAMPLE: *consider a qubit prepared in a completely random mixture given by* $\boldsymbol{\rho = I/2}$ *(I denotes 2 × 2 identity matrix). Measurements of the observables* $X = \sigma_x$ *and* $Z = \sigma_z$ *in this state leads to Shannon entropies of measurement* $H_\rho(X) = H_\rho(X) = 1$; $C(X,Z) = 1/\sqrt{2}$ *and the uncertainty bound is* $-2\log_2 C(X,Z) = 1$; *the Massen-Uffink relation is satisfied.*

Enter Entanglement!

- After a long debate and a series of laboratory experiments, we understand today that, rather than constituting refutations of uncertainty relations, *these apparent violations are a signature of entanglement.*

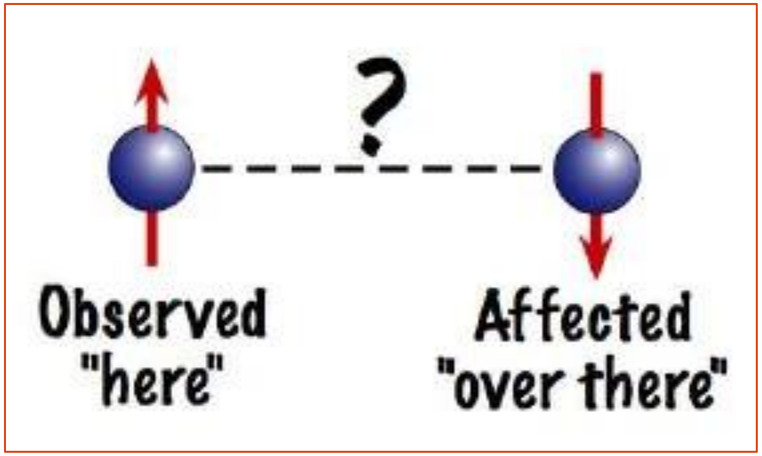
- This is the point of departure for the new entropic uncertainty relation of Berta *et al.*, *(M. Berta, M. Christandl, R. Colbeck, J. M. Renes, and R. Renner, Nature Physics 6, 659(2010))* which had been conjectured previously *(J. M. Renes, J. C. Boileau, Phys. Rev. Lett.* ***103****, 020402 (2009))*.

But…….

What is ENTANGLEMENT?

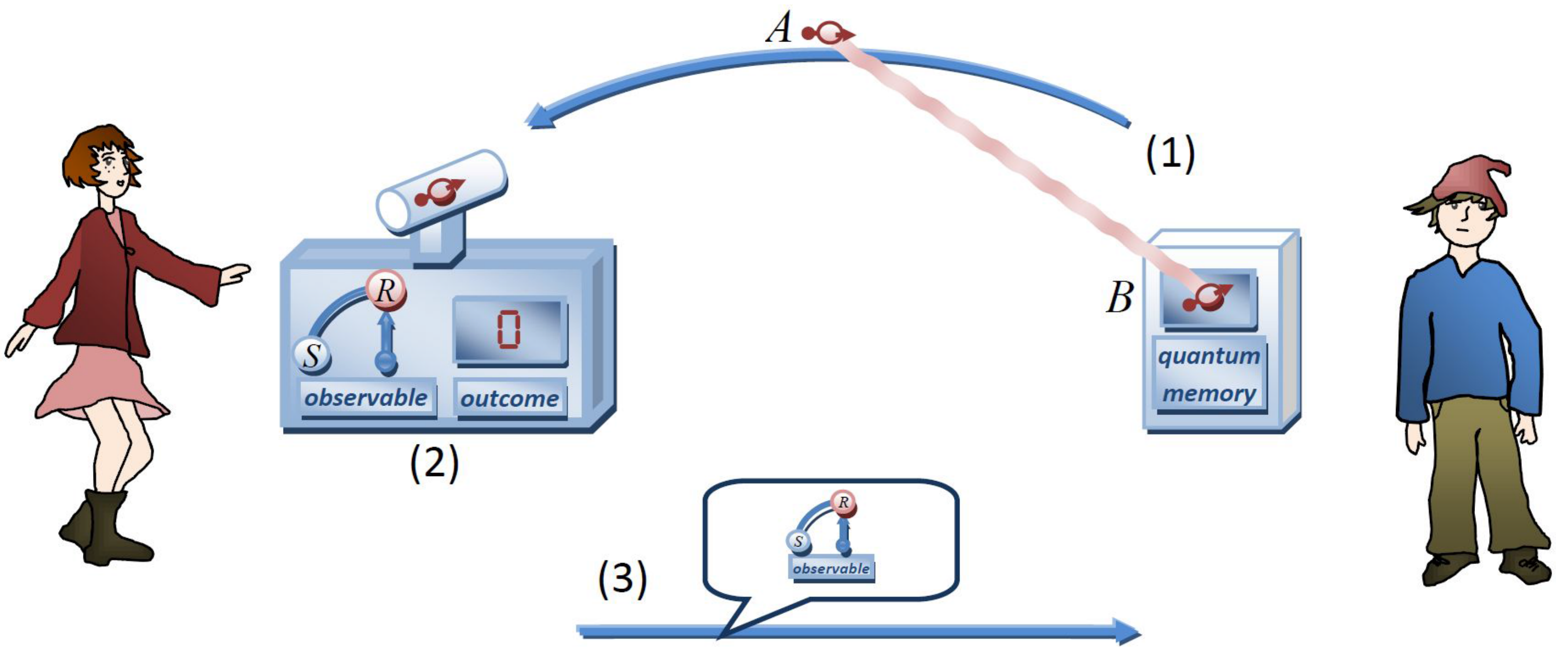
- *"…a correlation that is stronger than any classical correlation"---------John S Bell*
- *"…a trick that quantum magicians use to produce phenomena that cannot be imitated by classical magicians."---------Asher Peres*

Spooky!!!!



A Quantum Game

(M. Berta, M. Christandl, R. Colbeck, J. M. Renes, and R. Renner, Nature Physics 6, 659(2010))



(1) Bob sends a particle to Alice, which may, in general, be entangled with his quantum memory.
(2) Alice measures either R or S and notes her outcome.
(3) Alice announces her measurement choice to Bob.

The entropic uncertainty relation, when Bob possesses a quantum memory (Berta et. al., Nature Physics 6, 659 (2010)) is given by

$$S(\mathbb{X}|B) + S(\mathbb{Z}|B) \geq -2\log_2 \mathcal{C}(\mathbb{X},\mathbb{Z}) + S(A|B)$$

$$S(\mathbb{X}|B) = S\left(\rho_{AB}^{(\mathbb{X})}\right) - S(\rho_B),$$

$$S(\mathbb{Z}|B) = S\left(\rho_{AB}^{(\mathbb{Z})}\right) - S(\rho_B)$$

are the conditional von Neumann entropies of the post measured states

$$\rho_{AB}^{(\mathbb{X})} = \sum_x |x\rangle\langle x| \otimes \rho_B^{(x)}$$

$$\rho_B^{(x)} = \mathrm{Tr}_A[\rho_{AB}(E_{\mathbb{X}}(x) \otimes \mathbb{1}_B)]$$

and

$$\rho_{AB}^{(\mathbb{Z})} = \sum_z |z\rangle\langle z| \otimes \rho_B^{(z)}$$

$$\rho_B^{(z)} = \mathrm{Tr}_A[\rho_{AB}(E_{\mathbb{Z}}(z) \otimes \mathbb{1}_B)]$$

$S(A|B)$ can be negative if the state ρ_{AB} is entangled

A Quantum Game Cont….

- Bob, when he prepares an entangled state ρ_{AB} and sends one part of it to Alice, can indeed beat the uncertainty bound and can predict Alice's outcomes with certainty-----------but this was done with ***Projective Valued (PV) measurements***----- i.e ***Incompatible set of Measurements***!!

Berta et. al EUR

- When Alice's system is in a maximally entangled state with Bob's quantum memory, $\mathbf{S(A|B) = -\log_2 d}$ and as $\mathbf{-2\log_2 C(X,Z) \leq \log_2 d}$ one can achieve a trivial lower bound of **zero**. Thus, with the help of a quantum memory maximally entangled with Alice's state, Bob can beat the uncertainty bound and can predict the outcomes of incompatible observables X, Z precisely.

(Caveat here is----Bob should perform Incompatible measurements on his part of the state to predict Alice's outcomes)

Quantum Steering

- Steering is yet another facet of the *mystical* world of Quantum Mechanics
- Consider non-separable states of a bi-partite system:

  $|\psi> = \sum_{n=1}^{n=\infty} c_n|\psi_n>|u_n> = \sum_{n=1}^{n=\infty} d_n|\varphi_n>|v_n>$

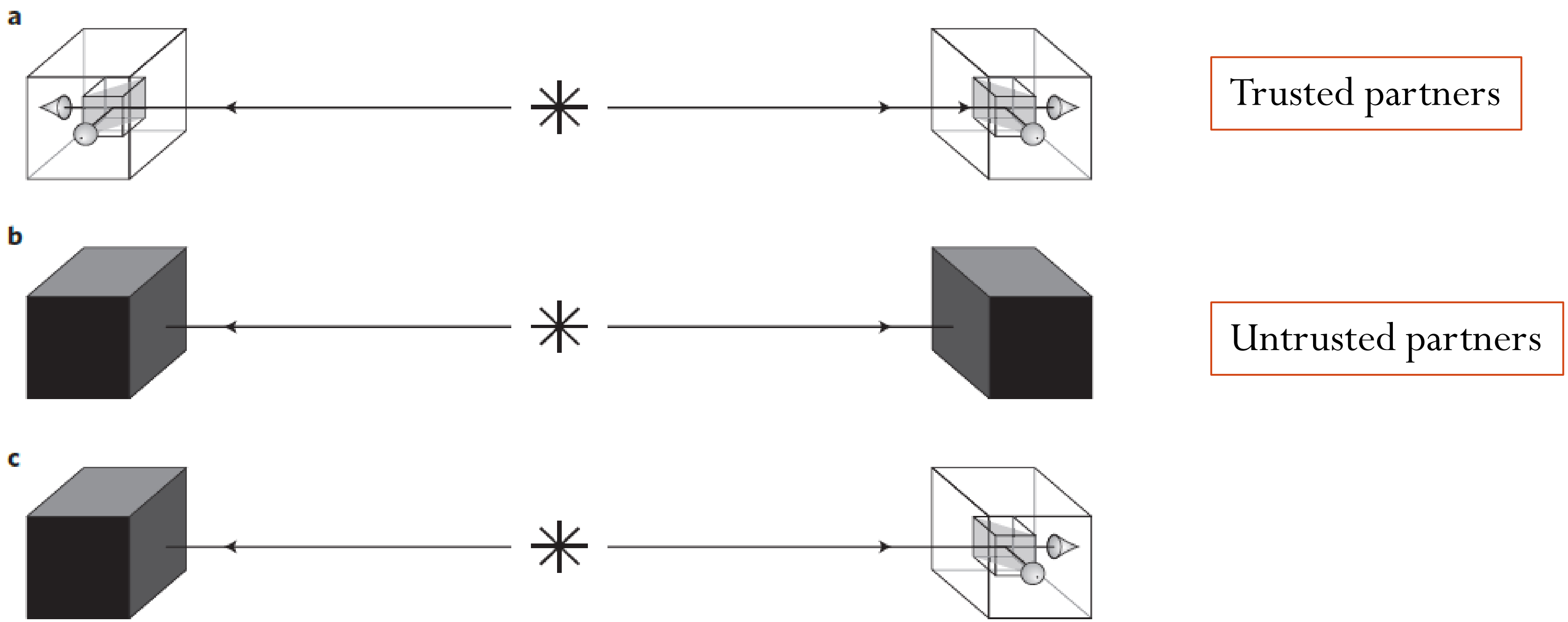
- Bob can ***steer*** Alice's state into either $|\psi_n>$s or $|\varphi_n>$s depending upon his choice of measurement

- *"It is rather discomforting that the theory should allow a system to be steered …… into one or the other type of state at the experimenter's mercy in spite of having no access to it."* ----------***Erwin Schrodinger***.

  Proc. Cam. Phil. Soc. **32**, 446-452(1936)

Steering as an Information theoretic task

- Task: Alice , Bob and Charlie are 3 experimenters. Suppose Charlie (always the third party!) wants to know whether Alice & Bob are entangled (E Cavalcanti et. al, PRA **80**, 032112 (2009))



Pic Credit: N. Brunner Nature Physics 6, 842–843 (2010)

A Steering Game!!

- Bob, who holds a source of photons tries to convince a skeptical Alice that they share an entangled state
- To convince Alice, he claims that after having sent the photon, he can steer it's state from a distance
- If the photons are actually entangled, Bob can remotely prepare (***steer***) different states for Alice's photon---of course dependent on his (random) measurement outcomes
- But how can Alice make sure that he is not cheating? He can send an ***uncorrelated*** photon and pretend to have made a measurement!

Entropic Steering Inequality

- Alice asks Bob to perform different measurements in each experimental run and evaluates ***steering inequality*** ---- which is ***always*** obeyed when Bob is not trustworthy i.e. he cannot steer Alice's state by his measurements ------N. Brunner , Nature Physics 6, 842–843 (2010)
- Steering inequality:

Consider pairs of binary measurements: $X, X', Z, Z' \in \{1,-1\}$

$$\mathbf{H(X|X') + H(Z|Z') \geq -2\,Log_2 C(X,Z)}$$

where $C(X,Z) = \max_{x,z} |\langle x|z\rangle|$

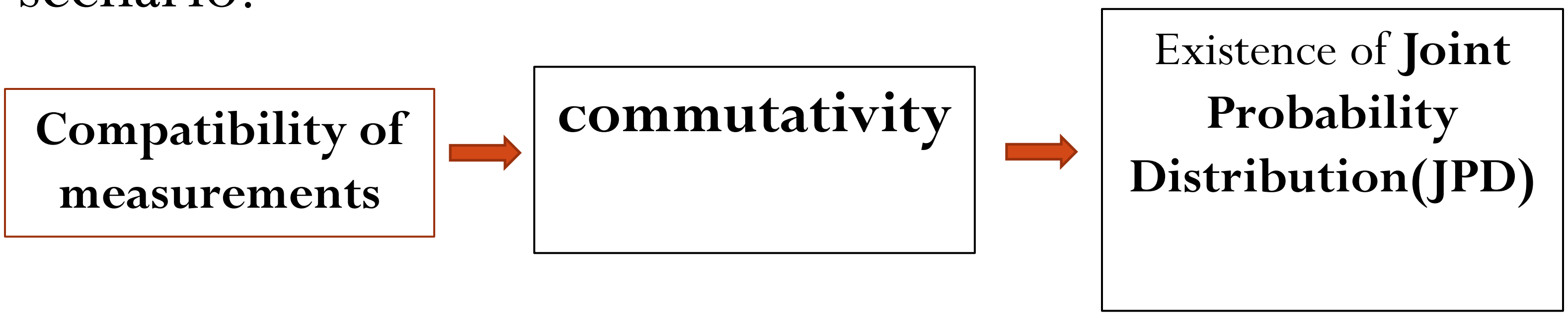
Our Work

We investigate the entropic uncertainty relation(Berta Et. Al. Nature Physics 6, 659 (2010)) for a pair of discrete observables (of Alice's system) when an entangled quantum memory of Bob is **restricted to record outcomes of jointly measurable POVMs** only. Within the joint measurability regime, the sum of entropies associated with Alice's measurement outcomes – conditioned by the results registered at Bob's end – are constrained to obey an entropic steering inequality. In this case, Bob's non-steerability reflects itself as his inability in predicting the outcomes of Alice's pair of non-commuting observables with better precision, even when they share an entangled state.

H S Karthik, A R Usha Devi, A K Rajagopal, Joint Measurability, Steering and Entropic Uncertainty, **Phys. Rev. A 91, 012115 (2015)**

But, What is Joint Measurability?

- Measurements of observables which do ***not commute*** are declared to be ***incompatible*** in the quantum scenario.



- But Quantum Mechanics places restriction on how ***sharply*** two non-commuting observables can be measured jointly.

Are Joint un-sharp measurements possible?

Joint Measurements of Positive Operator Valued (POV) Observables

- The orthodox notion of ***sharp*** Projective Valued(PV) measurements of self adjoint observables gets broadened to include ***un-sharp*** measurements of POV observables

- Do classical features emerge when one merely confines to measurements ***compatible*** un-sharp observables?

- Is it possible to classify physical theories based on the ***fuzziness*** required for Joint measurability?

Extended framework: Joint measurements of Positive Operator Valued (POV) observables

1. P. Busch, Phys. Rev. D **33**, 2253 (1986).
2. T. Heinosaari, D. Reitzner, and P. Stano, Found. Phys. **38**, 1133 (2008).
3. P. Busch, P. Lahti, and P. Mittelstaedt, *The Quantum Theory of Measurement*, No. v. 2 in Environmental Engineering, Springer, 1996.
4. M. M. Wolf, D. Perez-Garcia, and C. Fernandez, Phys. Rev. Lett. **103**, 230402 (2009).
5. M. Banik, Md. R. Gazi, S. Ghosh, and G. Kar, Phys. Rev. A **87**, 052125 (2013).
6. N. Stevens and P. Busch, Phys. Rev. A **89**, 022123 (2014)
7. M. T. Quintino, T. Vertesi, and N. Brunner, arXiv:1406.6976; Phys. Rev. Lett. **113**, 160402 (2014)
8. R. Uola, T. Moroder, and O. Gühne, arXiv:1407.2224; Phys. Rev. Lett. **113**, 160403 (2014).

Example of un-sharp POV observables

- It is well known that both $\boldsymbol{\sigma}_\mathbf{x}$ and $\boldsymbol{\sigma}_\mathbf{z}$ don't commute and hence cannot be measured jointly
- But the un-sharp observables $\mathbf{S}_\mathbf{x}$ and $\mathbf{S}_\mathbf{z}$ defined by the POVM's $S_x(\pm) = (½)(I+(1/\sqrt{2})\sigma_x)$ and $S_z(\pm) = (½)(I+(1/\sqrt{2})\sigma_z)$ are shown to be jointly measurable.

Consider $$G(i,j) = \frac{1}{4}(\mathbb{1} + \frac{i}{\sqrt{2}}\sigma_x + \frac{j}{\sqrt{2}}\sigma_z),\ i,j \in \{-1,+1\}$$

- Here, $S_x(\pm) = \sum_j G(\pm,j)$ and $S_z(\pm) = \sum_i G(i,\pm)$. One can jointly determine the probabilities of the POVM's $\mathbf{S}_\mathbf{x}$ and $\mathbf{S}_\mathbf{z}$ by measuring **G**

- *More formally, a POVM set $\{\mathcal{E}_k\}$ is said to be jointly measurable iff there exists a grand POVM G from which the probabilities of the POVM set $\{\mathcal{E}_k\}$ can be calculated.*
- i.e., a pair of POVM measurements $\{E_1(x_1)\}$ and $\{E_2(x_2)\}$ are jointly measurable iff there exists $\{G(\lambda), 0 \leq G(\lambda) \leq I, \sum_\lambda G(\lambda) = I\}$ from which $\{E_1(x_1)\}$ and $\{E_2(x_2)\}$ can be constructed as follows:

  1) Measure G on a state ρ

  2) From this, $P(\lambda) = Tr[\rho G(\lambda)]$ is calculated.

- If the elements $\{E_1(x_1)\}$ and $\{E_2(x_2)\}$ can be constructed as marginals of $\{G(\lambda), \lambda = x_1, x_2, \ldots.\}$ such that $E_1(x_1) = \sum_{x2} G(x_1, x_2)$ and $E_2(x_2) = \sum_{x1} G(x_1, x_2)$, then they are said to be jointly measurable.

- In general, if the effects can be constructed in terms of $G(\lambda)$ as $E_i(x_i) = \sum_\lambda p(x_i|i,\lambda)G(\lambda) \;\; \forall i$, where $0 \le p(x_i|i,\lambda) \le 1$, $\sum_\lambda p(x_i|i,\lambda) = 1$, then $\{E_i\}$ are jointly measurable.

- For all the jointly measurable POVM's $\{E_i\}$, the probability $p(x_i|i)$ of the outcome x_i in the measurement of $\{E_i\}$ can be post processed based on the results of the measurement of the grand POVM G

$$p(x_i|i) = \mathrm{Tr}[\rho E_i(x_i)] = \sum_\lambda p(x_i|i,\lambda)p(\lambda)$$

Neat Examples

- **Example: 1) Trine spin Axes**
- Consider noisy spin observables which form triplets of POVM's: $k \in \{1,2,3\}$

  $E_k(\pm) = (½)(I+(\eta/2)\sigma.n_k)$, $0 \leq \eta \leq I$.

  Here $\boldsymbol{\sigma}$ is the Pauli spin operators. $\mathbf{n_k}$ is the direction along which the spin is measured.

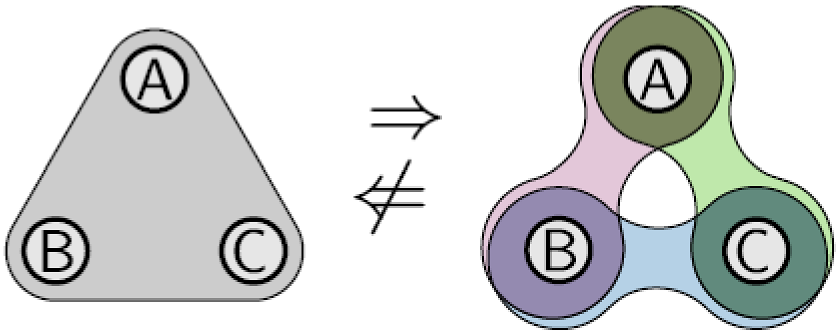
$$\hat{n}_1 = (0,0,1),\quad \hat{n}_2 = (\frac{\sqrt{3}}{2},0,-\frac{1}{2}),\quad \hat{n}_3 = (-\frac{\sqrt{3}}{2},0,-\frac{1}{2}).$$

The triple of measurements defined by the noisy spin observables along three equally spaced axes in a plane is jointly measurable if $\boldsymbol{\eta \leq 2/3}$.

- **2) Orthogonal spin Axes**

$$\hat{n}_1 = (0,0,1),\quad \hat{n}_2 = (1,0,0),\quad \hat{n}_3 = (0,1,0).$$

The triple of measurements defined by the noisy spin observables along three orthogonal spin axes is jointly measurable if $\boldsymbol{\eta \leq 1/\sqrt{3}}$.



Existence of a global observable for three observables A, B & C implies that there exists joint observables for each of the possible pairs {A,B}, {A,C}, {B,C} but the converse need not be true for un-sharp observables.

See: P. Busch, Phys. Rev. D 33, 2253 (1986)

T Heinosari, D Reitzner and P Stano, Found. Phys. 38, 1133 (2008)

Steering and Joint Measurability

- As measurement of a single *grand* POVM can be used to construct results of measurements of all compatible POVMs, joint measurability entails a joint probability distribution for all compatible observables.
- Existence of joint probabilities in turn implies that the set of all Bell inequalities are satisfied (A. Fine, Phys. Rev. Lett. 48, 291 (1982)) when only compatible measurements are employed.
- Wolf et. al., (Phys. Rev. Lett. 103, 230402 (2009)) have shown that incompatible measurements of a pair of POVMs with dichotomic outcomes are necessary and sufficient for the violation of Clauser-Horne-Shimony-Holt (CHSH) Bell inequality.
- Equivalence between steering and joint measurability: A set of POVMs are not compatible iff they can be employed for the task of non-local quantum steering.

  (See: M. T. Quintino, T. Vértesi, and N. Brunner, arXiv:1406.6976; Phys. Rev. Lett. 113, 160402 (2014)

  R. Uola, T. Moroder, and O. Gühne, arXiv:1407.2224; Phys. Rev. Lett. 113, 160403 (2014))

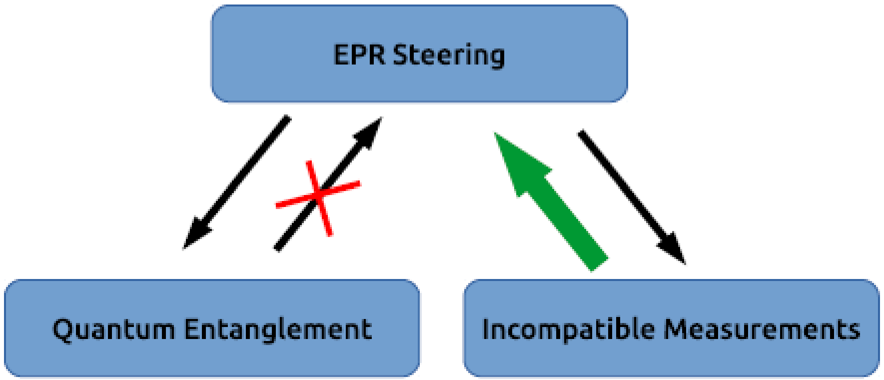
If Alice's state – after Bob's measurement of the POVM $\mathbb{E}_k$ with an outcome x – has the Local Hidden State (LHS) structure

$$\rho_{x|k} = \sum_{\lambda} p(\lambda)\, p(x|k,\lambda)\rho_\lambda$$

then it is *not steerable* and it satisfies every steering inequality.

If Bob can only perform measurements of compatible POVMs at his end, Alice's state would take the LHS form even when they share an entangled state. For steering to take place, it is necessary and sufficient that Bob measures incompatible POVMs.

Steering and Joint Measurability



Steering implies both entanglement and incompatible measurements at Bob's end

M. T. Quintino, T. Vértesi, and N. Brunner, arXiv:1406.6976; Phys. Rev. Lett. 113, 160402 (2014)

R. Uola, T. Moroder, and O. Gühne, arXiv:1407.2224; Phys. Rev. Lett. 113, 160403 (2014)

Uncertainty & Joint Measurability

- What kind of limitations are imposed by restricting to joint measurability of un-sharp observables?
- We explore the implications of joint measurability on entropic uncertainty relation in the presence of a quantum memory *(M. Berta, M. Christandl, R. Colbeck, J. M. Renes, and R. Renner, Nature Physics 6, 659(2010))*

Uncertainty & Joint Measurability

- When Bob has only classical information about Alice's state, the uncertainties in the outcomes of the observables X, Z obey the EUR (H. Maassen & J. B. M. Uffink, Phys. Rev. Lett. **60**, 1103(1988), M. Krishna & K. R. Parthasarathy, Sankhya, 64, 842, (2002))

$$\mathbf{H_\rho(X) + H_\rho(Z) \geq -2\log_2 C(X,Z)}$$

Projective Measurements

$\mathbf{X}$-----> $|x\rangle\langle x|$

$\mathbf{Z}$-----> $|z\rangle\langle z|$

$C(\mathbf{X},\mathbf{Z}) = \max_{x,z} |\langle x|z\rangle|$.

POVM measurement

$\mathbf{X} \equiv \{E_{\mathbf{X}}(x)\}$

$\mathbf{Z} \equiv \{E_{\mathbf{Z}}(z)\}$

$C(\mathbf{X},\mathbf{Z}) = \max_{x,z} \left\|\sqrt{E_{\mathbf{X}}(x)}\,\sqrt{E_{\mathbf{Z}}(z)}\right\|$

- **Denote Bob's POVMs as $\mathbb{X}'$ or $\mathbb{Z}'$ accordingly when Alice announces her choice of measurements of the observables $\mathbb{X}$ or $\mathbb{Z}$.**
- **The entropic uncertainty relation can be recast in terms of the conditional entropies $H(\mathbb{X}|\mathbb{X}')$, $H(\mathbb{Z}|\mathbb{Z}')$ of Alice's measurement outcomes of $\mathbb{X}$, $\mathbb{Z}$, conditioned by Bob's measurements of $\mathbb{X}'$, $\mathbb{Z}'$:**

$$H(\mathbb{X}|\mathbb{X}') + H(\mathbb{Z}|\mathbb{Z}') \geq -2\log_2 \mathcal{C}(\mathbb{X},\mathbb{Z}) + S(A|B).$$

(Above entropic uncertainty relation follows by using the reuslt that measurement always increases entropy i.e., $H(\mathbb{X}|\mathbb{X}') \geq S(\mathbb{X}|B)$, and $H(\mathbb{Z}|\mathbb{Z}') \geq S(\mathbb{Z}|B)$)

- **The conditional entropies $H(\mathbb{X}|\mathbb{X}')$, $H(\mathbb{Z}|\mathbb{Z}')$ are constrained to obey an *entropic steering inequality* (J. Schneeloch, C. J. Broadbent, S. P. Walborn, E. G. Cavalcanti, and J. C. Howell, Phys. Rev. A 87, 062103 (2013)),**

$$H(\mathbb{X}|\mathbb{X}') + H(\mathbb{Z}|\mathbb{Z}') \geq -2\log_2 \mathcal{C}(\mathbb{X}, \mathbb{Z})$$

if Bob is unable to remotely steer Alice's state by his local measurements.

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  if Bob is unable to remotely steer Alice's state by his local measurements.

- **But incompatibility of measurements and steering imply one another ⇒ the entropic steering inequality can never be violated if Bob's measurements $\mathbb{X}'$, $\mathbb{Z}'$ are compatible.**

Beating the uncertainty bound

- **The conditional entropies $H(\mathbb{X}|\mathbb{X}')$, $H(\mathbb{Z}|\mathbb{Z}')$ are constrained to obey an *entropic steering inequality* (J. Schneeloch, C. J. Broadbent, S. P. Walborn, E. G. Cavalcanti, and J. C. Howell, Phys. Rev. A 87, 062103 (2013)),**

$$H(\mathbb{X}|\mathbb{X}') + H(\mathbb{Z}|\mathbb{Z}') \geq -2\log_2 \mathcal{C}(\mathbb{X},\mathbb{Z})$$

if Bob is unable to remotely steer Alice's state by his local measurements.

- **But incompatibility of measurements and steering imply one another $\Rightarrow$ the entropic steering inequality can never be violated if Bob's measurements $\mathbb{X}'$, $\mathbb{Z}'$ are compatible.**

- **Violation of the entropic steering inequality would in turn correspond to a reduced bound in the entropic uncertainty relation in the presence of quantum memory**

Beating the uncertainty bound

If Bob is constrained to perform only compatible measurements on his system, he cannot beat the uncertainty bound and win the *uncertainty game* by predicting the outcomes as precisely as possible, even when he shares an entangled state with Alice.

An example

1. **Qubit observables σ_x, σ_z are measured randomly by Alice.**
2. **To predict Alice's outcomes, Bob measures noisy dichotomic outcome POVMs $\mathbb{E}_x$ or $\mathbb{E}_z$.**
3. **When Alice and Bob share a maximally entangled state, the right hand side of Berta et al inequality is zero binding the entropic uncertainties in a trivial manner. But can Bob's noisy measurements help in attaining this trivial bound?**
4. **The conditional entropies $H(\mathbb{X}|\mathbb{X}') = H(\mathbb{Z}|\mathbb{Z}') = H[(1+\eta)/2]$ (where $H(p) = -p\,\log_2 p - (1-p)\,\log_2(1-p)$ is a binary entropy function; η is the unsharpness parameter associated with Bob's noisy measurements.**
5. **The sum of entropies obey $H(\mathbb{X}|\mathbb{X}') + H(\mathbb{Z}|\mathbb{Z}') \geq 1$ (satisfy entropic steering inequality) when the unsharpness parameter lies in the joint measurability range for Bob's POVMs i.e., $0 \leq \eta \leq \frac{1}{\sqrt{2}}$. Hence, the entropic uncertainty bound cannot be beaten even when they share a maximally entangled state.**

Conclusions

- Entropic Uncertainty Relations(EUR) constrain the sum of entropies associated with the probabilities of outcomes of a pair of Non-Commuting(NC) observables.
- An extended EUR of Berta et. Al brought out that it's possible to beat the lower bound on uncertainties, when the system is entangled with a quantum memory.
- We investigate the situation where Bob's quantum memory is restricted to record outcomes of Jointly Measurable POVM's.
- In such a scenario, it's realized that the l.h.s of EUR is constrained to obey an Entropic Steering Relation.
- Finally, we see that a quantum memory cannot assist in beating the entropic uncertainty bound -----when it is confined to register results of compatible POVM's only.

Thank You for your attention!!