Possibility of adiabatic transport of a Majorana edge state through an extended gapless region

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Ref: A. Rajak, T. Nag, and A. Dutta, Phys. Rev. E **90**, 042107 (2014)

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Outline of talk

- Introduction to Majorana fermions
- 1-dimensional p-wave superconductor: phase diagram and topological phases
- Adiabatic dynamics and Kibble-Zurek scaling
- Violation of Kibble-Zurek scaling for edge Majorana state
- Application of complex hopping term
- Adiabatic quenching dynamics of periodic chain
- Slow quenching dynamics of edge Majorana and possibility of adiabatic transport
- Summary

Introduction to Majorana fermions

- A Majorana fermion is a fermion which is its own anti-particle $c^\dagger = c$
- Wave function of a Majorana fermion must be real
- The second quantized operators of Majorana fermions satisfy the relations
  $c_j^2 = 1$ and $\{c_j, c_l\} = 2\delta_{jl}$
- One ordinary fermion (with annihilation operator f) can be split into two Majorana fermions , so N ordinary fermions are equivalent to $2N$ Majorana fermions

  $f_1 = \frac{(c_1 + ic_2)}{2}, \; f_2 = \frac{(c_3 + ic_4)}{2}, \; \ldots$

Ref: E. Majorana, Nuovo Cimento 14, 171 (1937).

Majorana fermions in condensed matter systems

- Majorana fermions exist in spinless p-wave superconductors

  Ref: Kitaev A 2000 arXiv:cond-mat/0010440; Read and Green,PRB 61, 10267 (2000)

- An ordinary s-wave superconductor deposited on topological insulator hosts Majorana states at vortices

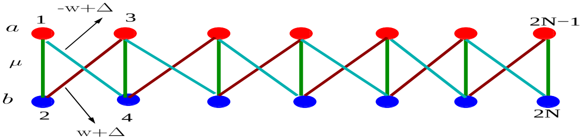
  Ref: L.Fu and C.L.Kane, PRL 100,096407 (2008)

- A semiconductor with strong spin-orbit coupling (e.g.,InAs) coupled to superconducting leads produce Majorana fermions at the ends of the wire

  Ref: R.M. Lutchyn et al., PRL 105, 077001 (2010); V. Mourik et al., Science 336, 1003 (2012); A. Das et al., Nat. Phys. 8, 887 (2012); A.D.K. Finck et al., PRL 110, 126406 (2013)

1-D p-wave superconducting chain

$$H = \sum_{n=1}^{N}[-w(f_n^\dagger f_{n+1} + f_{n+1}^\dagger f_n) + \Delta(f_n f_{n+1} + f_{n+1}^\dagger f_n^\dagger)] - \sum_{n=1}^{N}\mu(f_n^\dagger f_n - 1/2)$$



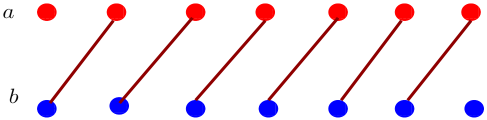
$a_n = f_n + f_n^\dagger, b_n = -i(f_n - f_n^\dagger)$

Majorana operator

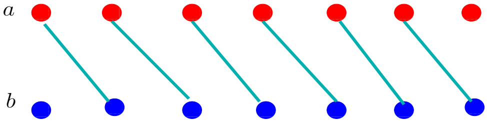
$$H = \frac{i}{2}\sum_{n=1}^{N-1}[(-w+\Delta)a_n b_{n+1} + (w+\Delta)b_n a_{n+1}] - \frac{i}{2}\sum_{n=1}^{N}\mu a_n b_n$$

Ref: Kitaev A 2000 arXiv:cond-mat/0010440.

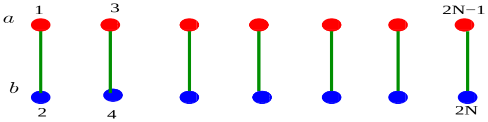
end modes



Case A:
$w = \Delta$ and $\mu = 0$

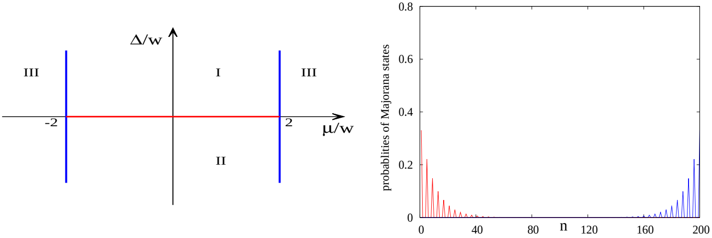


Case B:
$w = -\Delta$ and $\mu = 0$



Case C:
$w = \Delta = 0$ and $\mu \neq 0$

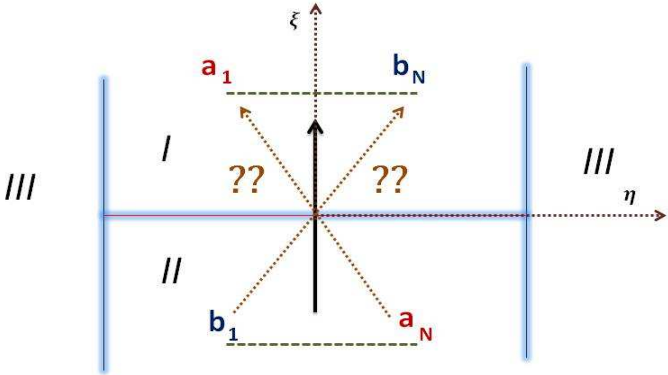
Phase diagram and end modes



- Left panel:I and II → topologically non-trivial phases, *III* → topologically trivial phase
- Number of zero-energy Majorana modes is the topological invariant of topological phases
- Right panel: Two isolated Majorana states are in two edges of a 100-sites open Majorana chain in phase I ($\mu = 0.0$, $\Delta = 0.1$ and $w = 1.0$). a_n (b_n) is non-zero for the left (right) end of the chain respectively

Ref: M. Thakurathi, A.A. Patel, D. Sen, and A. Dutta, Phys. Rev. B **88**, 155133 (2013); A. Rajak and A. Dutta, Phys. Rev. E **89**, 042125 (2014)

Non-equilibrium dynamics of edge Majorana and adiabatic transport ...



Quenching of the Majorana chain

- Quenching scheme: $\Delta(t) = -1 + 2t/\tau$ with rate $\tau \gg 1$, $\mu = 0$ and $t \in [0, \tau]$, crosses QCP $\Delta = 0$ at $t = \tau/2$
- Initial state $|\Psi(0)\rangle$ can be a bulk state or an edge Majorana for $\xi(0) = -1$ within the phase II.
- Probability of Majorana getting excited to the positive energy band P_{def} (negative energy band P_{neg}) in phase I

$$P_{\mathrm{def(neg)}} = \sum_{\epsilon^+ > 0 (\epsilon^- < 0)} |\langle \epsilon^{+(-)} | \Psi(\tau) \rangle|^2$$

  where $|\Psi(\tau)\rangle$ is the time-evolved initial state.
- The Majorana excitation probability $P_{\mathrm{m}} = |\langle \epsilon_0 | \Psi(\tau) \rangle|^2$ where $|\epsilon_0\rangle$ is the final equilibrium edge Majorana in the Phase I.

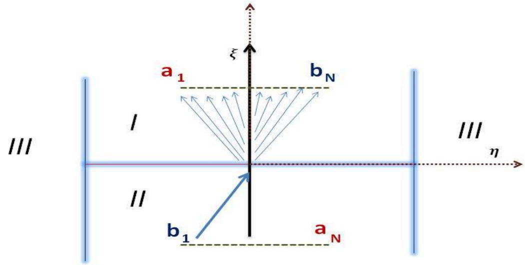
Ref: A. Bermudez, et al.,New J.Phys. **12**, 055104 (2010)

Violation of Kibble-Zurek scaling and no adiabatic transport

- Periodic chain or initial interior bulk state of an open chain $\Rightarrow$ expected KZ scaling ($n \sim \tau^{\frac{-\nu d}{\nu z+1}} \sim \tau^{-1/2}, \ \nu = z = 1)$
- Edge state $\Rightarrow$ deviates from the usual KZ scaling and is non-universal
- $P_{\mathrm{def}} = P_{\mathrm{neg}} = 0.5$ and $P_{\mathrm{m}} = 0$
- Completely fuses in the positive and negative energy bands, adiabatic transport of Majorana $(b_1 \rightarrow b_N$ or $a_N \rightarrow a_1)$ is impossible

Ref: A. Bermudez, et al.,New J.Phys. **12**,055104 (2010)

Complete delocalization of edge Majorana throughout the chain



- Equilibrium left edge MF at QCP:
  $|L\rangle \sim (b_1 + b_3 + b_5 + \cdots, \ b_N)|\Omega\rangle \sim \frac{1}{\sqrt{2}}(|\frac{\pi}{2}\rangle_+ + |\frac{\pi}{2}\rangle_-)$
- Could the edge Majorana be transported...

Application of complex hopping term

$$H = \sum_{n=1}^{N} \left[-w_0 e^{i\phi} f_n^\dagger f_{n+1} - w_0 e^{-i\phi} f_{n+1}^\dagger f_n + \Delta(f_n f_{n+1} + f_{n+1}^\dagger f_n^\dagger) \right] - \sum_{n=1}^{N} \mu (f_n^\dagger f_n - 1/2)$$

- The Hamiltonian in the momentum space takes the form

$$h_k = (2w_0 \sin\phi \sin k)\ I \ - \ (2w_0 \cos\phi \cos k \ + \ \mu)\ \sigma^z + (2\Delta \sin k)\ \sigma^y$$

The Hamiltonian in terms of Majorana fermions:

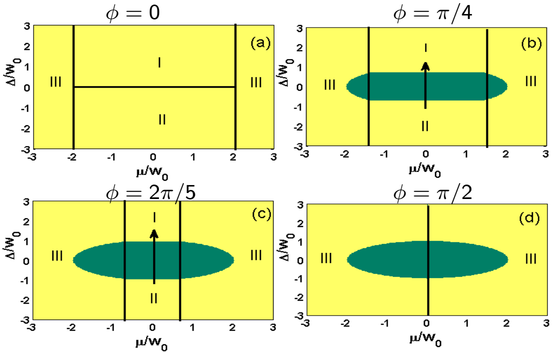
$$H \ = \ -\frac{i}{2} \sum_{n=1}^{N-1} \Big[w_0 \cos\phi \ (a_n b_{n+1} \ + \ a_n b_{n-1}) \ - \ \Delta(a_n b_{n+1} - a_n b_{n-1})$$

$$+ \quad w_0 \sin\phi \ (a_n a_{n+1} \ + \ b_n b_{n+1}) \Big] \ + \ \sum_{n=1}^{N} \mu \ a_n b_n$$

Under time-reversal $a_n \to a_n$ and $b_n \to -b_n$, operators like $i a_m a_n$ or $i b_m b_n$ break time-reversal symmetry

Ref: W. DeGottardi et al., Phys. Rev. B **88** 165111 (2013)

Phase diagram



- One can locate the QCPs given by the relation

$$(2w_0 \cos\phi \cos k_c + \mu)^2 + 4\Delta^2 \sin^2 k_c = 4w_0^2 \sin^2\phi \sin^2 k_c$$

- Two vertical lines (horizontal lines) given by $\mu/w_0 = \pm 2\cos\phi$ $(\Delta/w_0 = \pm\sin\phi)$

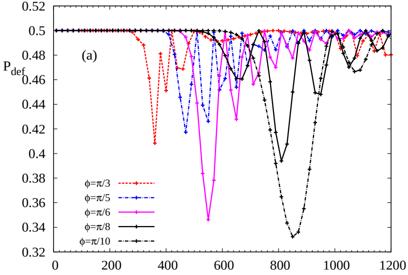
Quenching dynamics of an initial edge Majorana

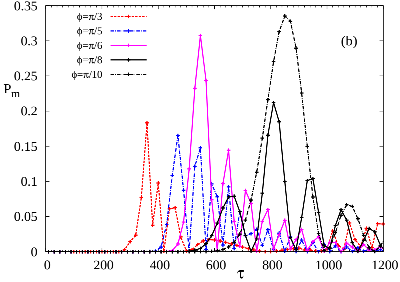
- Address:
  Is adiabatic transport of edge Majorana from one topological phase to the other across the extended quantum critical region (from $\Delta = -\sin\phi$ to $\Delta = \sin\phi$) possible ?

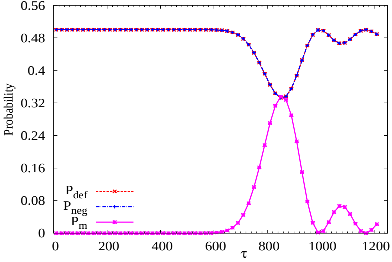
- Observations:
  1. The $P_{\rm m}$ becomes finite for some values of τ

  2. The adiabatic transport happens for set of values above a threshold value of transit time $(\Delta t_{\rm th})$ through the gapless region

Probability of Majorana







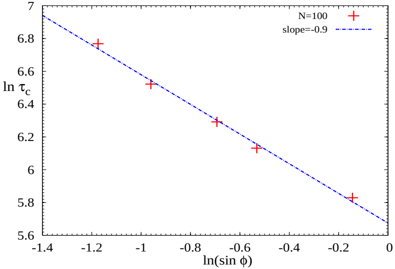
Exists a characteristic τ (denoted by τ_c) for which there is the first significant dip (peak) in $P_{\rm def}$ (P_m) followed by a few drops (peaks)

$N = 100$

Optimum transit time

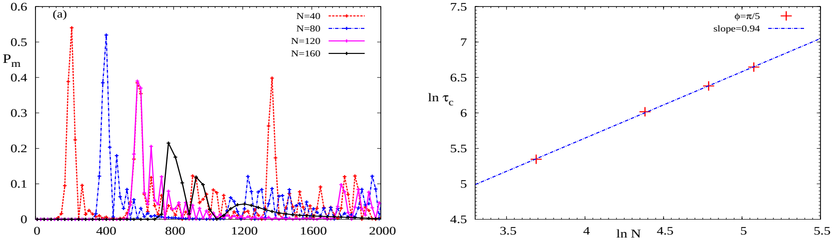
- Define two instants of time when the system enters and leaves the gapless phase as t_e and t_o, respectively and the transit time $\Delta t = t_0 - t_e = \tau \sin\phi$
- Exists an optimum value of transit time which is independent of ϕ

$$\Delta t_{\rm op} = \tau_c \sin\phi = \frac{\tau_c \; W_d}{2}$$



Clearly, τ_c depends on ϕ and decreases as it increases from 0 to $\pi/2$; $\tau_c \to \infty$; when $\phi \to 0$, satisfy Bermudez et al's result

Optimum transit time and system size



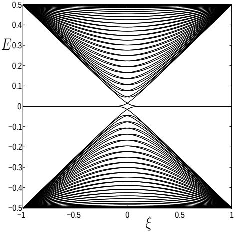
- $\Delta t_{\text{op}} \propto N$
- Energy difference between two consecutive levels decreases with N, as a result relaxation time of the system increases and then τ_c increases linearly with system size N
- For few peaks following the first peak of P_m exists $\Delta t_{\text{op1}},\ \Delta t_{\text{op2}},\ \cdots,\ \Delta t_{\text{opn}}\ (\Delta t_{\text{op(n}-1)} < \Delta t_{\text{opn}})$
- τ_c depends on the quenching length, but Δt_{op} remains fixed..it is a universal constant which depends only on N

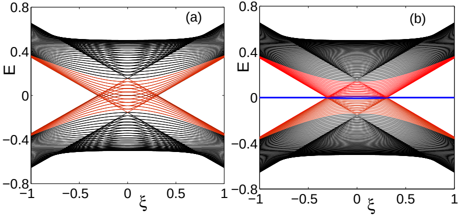
P_m for $t > \tau$

Does Majorana state persist after the quenching is stopped?

- For $t > \tau$, $|\Psi(\tau)\rangle$ evolves as as $\exp(-iH(\Delta = 1)t)|\Psi(\tau)\rangle$
- $P_m(t > \tau) = |\langle\epsilon_0|\exp(-iH(\Delta = 1)t)|\Psi(\tau)\rangle|^2$ does not change with time t
- A finite probability of the edge Majorana in phase I exists even for $t > \tau$

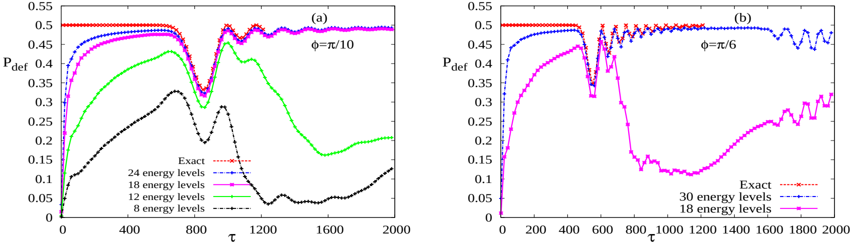
Energy levels: Band Inversion





- Band inversion occurs inside the zone bounded by $\Delta/w_0 = \pm \sin\phi$ (gaplees region under PBC)
- Number of inverted energy levels increases with N and ϕ
- Is it going give any new feature in the dynamics of edge Majorana?

Plausible conjecture



- Exists a threshold value of $\Delta t_{\rm th} = \Delta t_{\rm op1}$, above which Majorana starts delocalizing maximally within the inverted band
- This phenomena leads to adiabatic tunneling of edge Majorana
- For $\Delta t < \Delta t_{\rm th}$ edge Majorana diffuses into the interior bulk, no possibility of recombination
- First significant peak height of P_m decreases with increasing N and ϕ, interaction with more number of inverted energy levels

Summary

- A phase factor in the hopping term results an extended gapless region and breaks time-reversal symmetry of the Hamiltonian that leads to give zero-energy Majoranas involving both a and b
- The extended gapless region is insensitive to quenching of a periodic chain, but show interesting behavior for edge state quenching
- Exists inverted energy levels near zero-energy for non-zero ϕ and for $\Delta t \geq \Delta t_{\rm th}$ edge Majorana can be transported adiabatically from one topological phase to the other
- Adibatic transport of edge Majorana becomes more probable with a less number of inverted bands within the gapless region

Thank You

Adiabatic dynamics and Kibble-Zurek scaling

- A driven quantum many body system has two time scales
  (I) Relaxation time : Inverse of minimum gap
  (II) Time scale for driving the Hamiltonian
- Adiabatic dynamics must follow the relation :
  Relaxation time $<<$ Time scale of driving

  Relaxation time diverges close to the critical point: Dynamics becomes diabatic. defect generation
- Density of defect scales as

$$n \sim \tau^{\frac{-\nu d}{\nu z+1}} \qquad \text{(Kibble-Zurek (KZ) scaling)}$$

where, $d \rightarrow$ dimension of the system, $\nu \rightarrow$ correlation length exponent and $z \rightarrow$ dynamical exponent

Ref: W. H. Zurek, U. Dorner, and P. Zoller, Phys. Rev. Lett. **95**, 105701 (2005); A. Polkovnikov, Phys. Rev. B **72**, 161201(R) (2005)

Quenching dynamics of the periodic chain

- The Hamiltonian h_k with $w_0 = 1$ following an appropriate unitary transformation takes the form

$$h_k(t) \; = \; (2\sin\phi\sin k)\; I \; + \; (2\Delta(t)\sin k)\; \sigma^z \; + \; (2\cos\phi\cos k)\; \sigma^y$$

- $\Delta(t) = -1 + 2t/\tau$ with quench time $\tau \gg 1$ and $t \in [0, \tau]$ along the path $\mu = 0$
- General state for a given $k-$ mode $|\psi_k(t)\rangle = u_k(t)|1_k\rangle + v_k(t)|2_k\rangle$
- Identity part of the Hamiltonian is irrelavant for dynamics; time evolution is dictated by LZ term
- Probability of excitation for each mode $p_k = e^{-2\pi\gamma_k}$
  where, $\gamma_k \; = \; \delta_k^2 / \left|\frac{d}{dt}(E_1 - E_2)\right|$, $\delta_k = 2\cos\phi\cos k$ and $E_{1,2} = \pm 2\Delta(t)\sin k$

Ref: C. Zener, Proc. R. Soc. London, Ser. A **137**, 696 (1932); L.D. Landau and E. M. Lifshitz, *Quantum Mechanics: Non-relativistic Theory*

Quenching dynamics of the periodic chain

Defect density in the final state is given by

$$n = \frac{2\pi}{N} \int_{-\pi}^{\pi} p_k \, dk \sim \frac{1}{\pi} \frac{1}{\cos\phi \sqrt{\tau}}$$

- The maximum contribution in the above integration comes from $k_c = \pi/2$ where the energy gap of LZ part vanishes for $\Delta = 0$
- Dynamics is completely insensitive to gapless region
- KZ scaling is satisfied analytically as well as numerically, note that τ gets renormalized to $\tau_{\text{eff}} = \tau \cos^2\phi$