Role of Detector and Defect in Quantum Random Walk

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Classical Random Walk (CRW) and Quantum Random Walk (QRW)

CRW	QRW
1. Particle can move either to the LEFT or to the RIGHT direction.	1. Particle moves in superposition of left and right; new degree of freedom chirality.
2. For diffusive CRW, $\langle x^2 \rangle \propto t^{\gamma_{cl}}$ with $\gamma_{\mathrm{cl}} = 1$. This scaling governs other dynamical behaviours.	2. Quantum walk propagates faster than the diffusive CRW; $\langle x^2 \rangle \propto t^{\gamma_q}$ where $\gamma_{\mathrm{q}} = 2$.

Quantum Random Walk

- Particle moves in superposition of left and right.
- State of the walker is expressed in $|x\rangle \otimes |d\rangle$ basis.
- $\mathcal{H}_P$ - the Hilbert space spanned by the positions of the particle. For a line with grid-length 1 this space is spanned by the basis states $\{|x\rangle : x \in Z\}$.
- The position Hilbert space $\mathcal{H}_P$ is augmented by a 'coin'-space $\mathcal{H}_C$ spanned by two basis states $\{|L\rangle, |R\rangle\}$, analogous to the role of the spin-1/2 space.
- Total unitary action U may be divided into two parts : translation T [on position eigen state] and rotation with Hadamard coin operator H [on chirality eigen state].
- Hadamard coin

$$H = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \Rightarrow \begin{cases} |L\rangle \to \frac{1}{\sqrt{2}}(|L\rangle + |R\rangle) \\ |R\rangle \to \frac{1}{\sqrt{2}}(|L\rangle - |R\rangle) \end{cases}$$

- Translation operator T then shifts the position of the particle according to the chirality state

$$T|x\rangle|L\rangle \to |x-1\rangle|L\rangle$$

$$T|x\rangle|R\rangle \to |x+1\rangle|R\rangle$$

[Aharonov et al PRA **48**, 1687(1993); J. Kempe, Contemporary Physics **44**, 307 (2003)]

QRW wave function :

- The two component wave function $\psi\ (x, t\ )$ describing the position of the particle at a particular time is written as :

$$\psi\ (x,t) = \begin{pmatrix} \psi_L\ (x,t) \\ \psi_R\ (x,t) \end{pmatrix}$$

- The occupation probability of site x at time t is given by

$$f\ (x,t) = \left|\psi_L\right|^2 + \left|\psi_R\right|^2$$

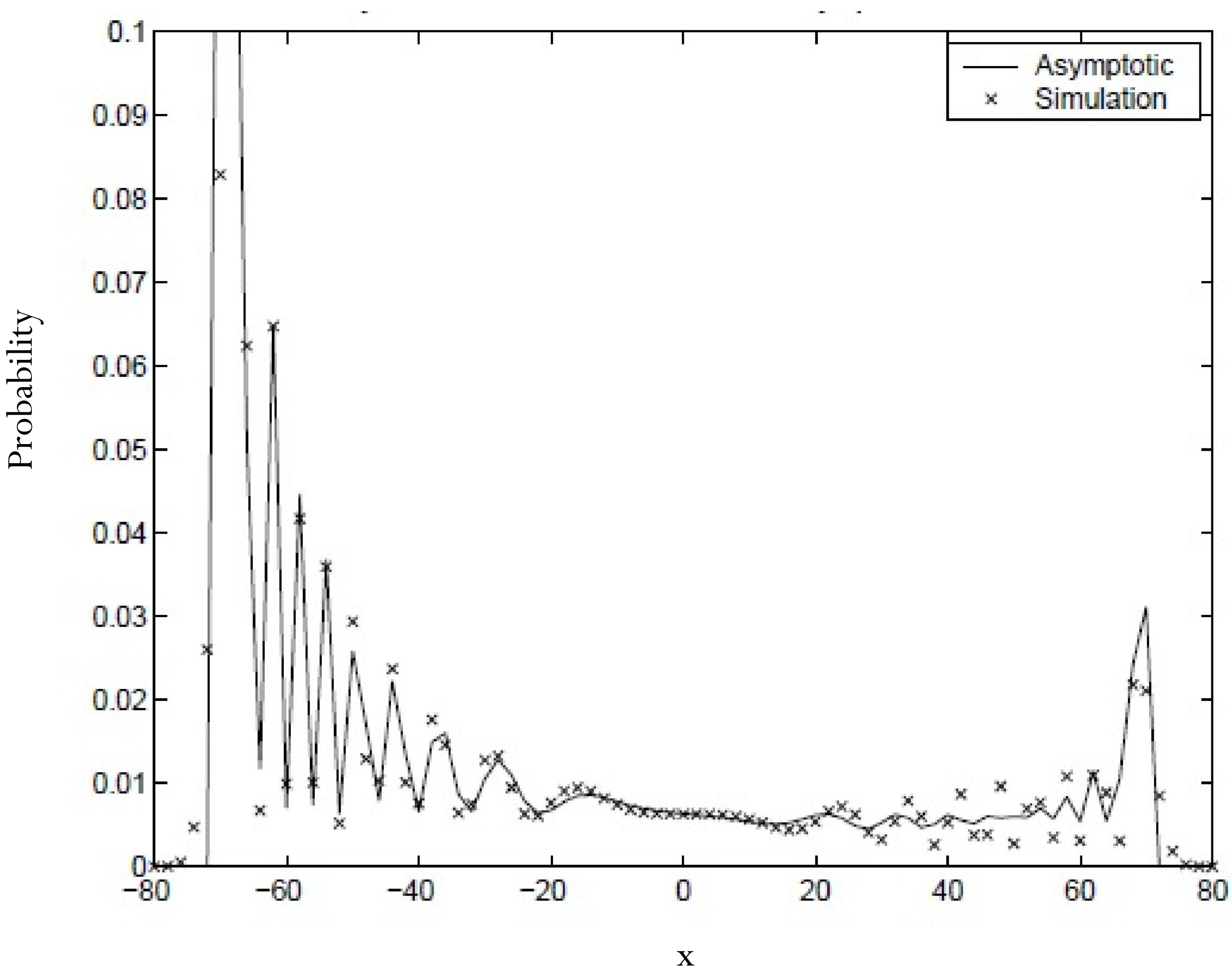
- The walk is initialized at the origin :

$\psi_L\ (0,0) = a_0$, $\psi_R\ (0,0) = b_{0;}$ $a_0^2 + b_0^2 = 1$.

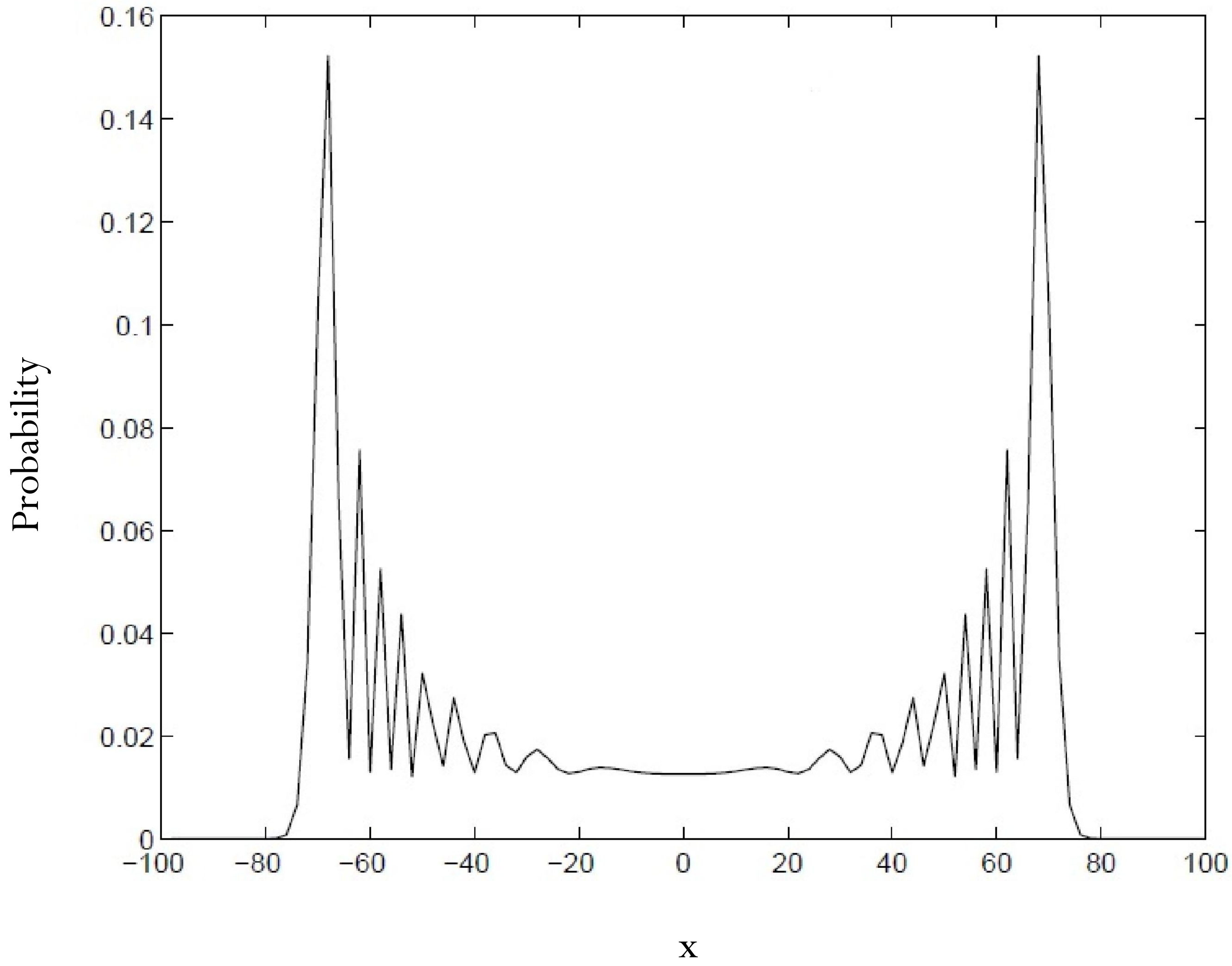
- Initial state with left chirality, i.e., $a_0 = 1$, $b_0 = 0$.

$$\psi_L(x,t) = \frac{1+(-1)^{x+t}}{2} \int \frac{dk}{2\pi} \left(1 + \frac{\cos k}{\sqrt{1+\cos^2 k}} \right) \exp\left[-i\left(\omega_k t + kx\right)\right]$$

$$\psi_R(x,t) = \frac{1+(-1)^{x+t}}{2} \int \frac{dk}{2\pi} \left(\frac{\exp(ik)}{\sqrt{1+\cos^2 k}} \right) \exp\left[-i\left(\omega_k t + kx\right)\right]$$



Asymptotic
Simulation
Probability
x
0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.09
0.1
-80
-60
-40
-20
0
20
40
60
80

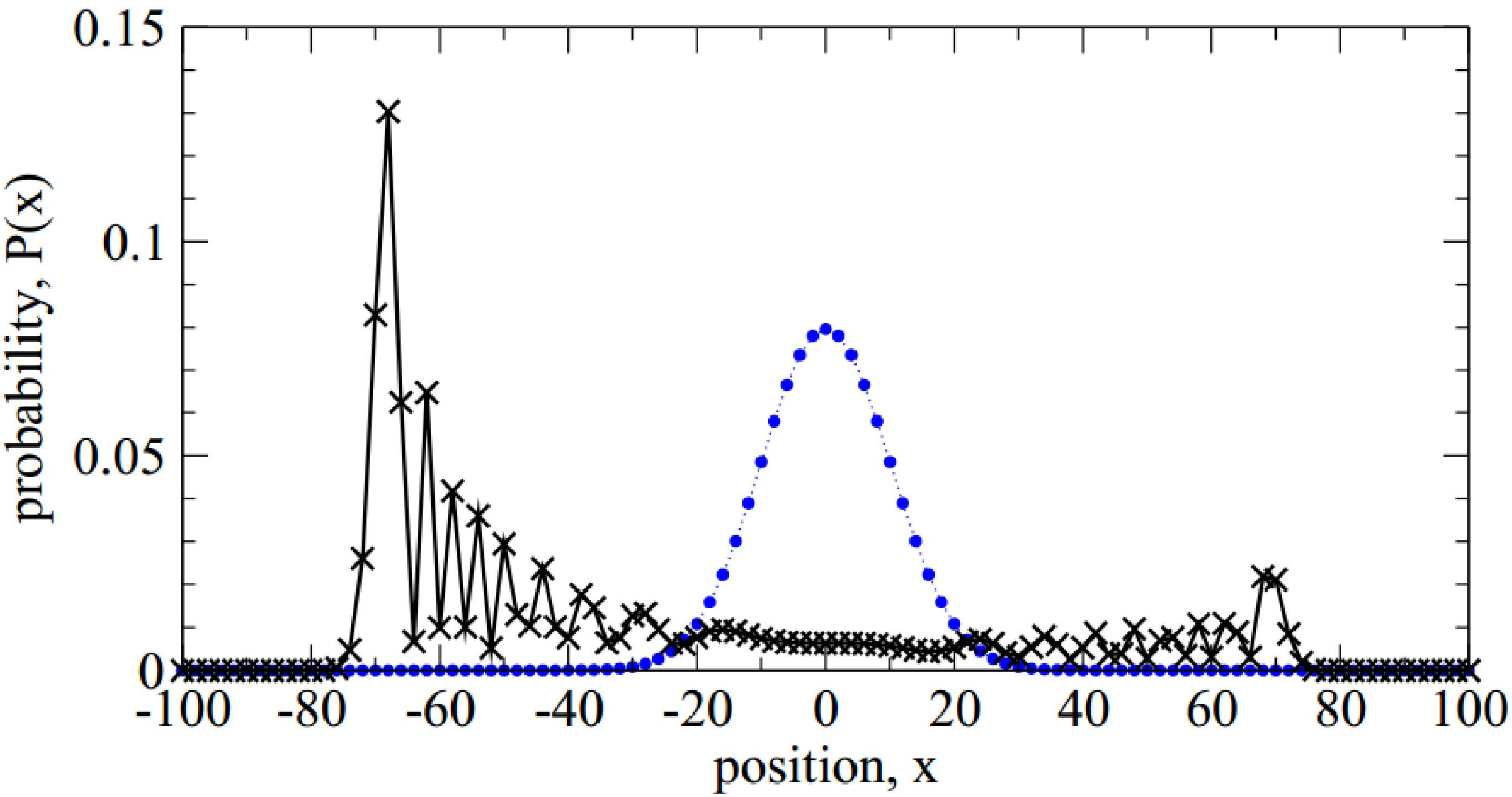

Probability
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
−100
−80
−60
−40
−20
0
20
40
60
80
100
x

Classical Random Walk

T \ i	-5	-4	-3	-2	-1	0	1	2	3	4	5
0						1					
1					1/2		1/2				
2				1/4		1/2		1/4			
3			1/8		3/8		3/8		1/8		
4		1/16		1/4		3/8		1/4		1/16	
5	1/32		5/32		5/16		5/16		5/32		1/32

Quantum Random Walk

T \ i	-5	-4	-3	-2	-1	0	1	2	3	4	5
0						1					
1					1/2		1/2				
2				1/4		1/2		1/4			
3			1/8		3/8		3/8		1/8		
4		1/16		3/8		1/8		3/8		1/16	
5	1/32		11/32		1/8		1/8		11/32		1/32



0.15
0.1
0.05
0
-100
-80
-60
-40
-20
0
20
40
60
80
100
probability, P(x)
position, x

Quantum computation:

- One of the earliest work by Feynman (1985): A hamiltonian can be constructed to implement any quantum circuit . The dynamics of any time-independent Hamiltonian can be viewed as a quantum walk on a weighted graph. Hence a quantum circuit can in principle be implemented by a quantum walk!
- More recently it was proved that continuous-time quantum walks are universal for quantum computation, using unweighted graphs of low degree.

  Hamiltonian matrix is the adjacency matrix of a low degree graph.

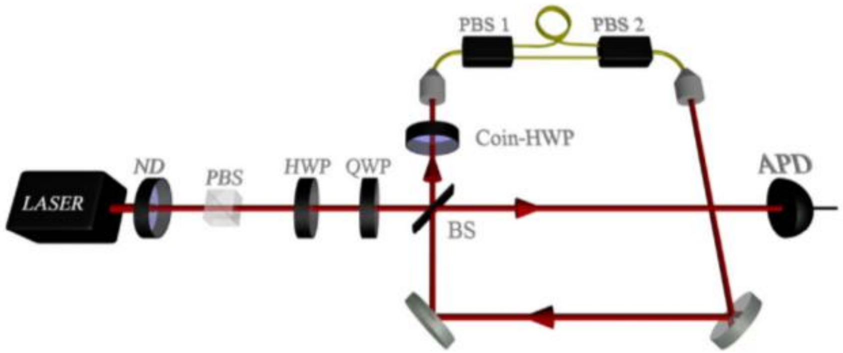
  Childs, Phys. Rev. Lett. **102**, 180501 (2009)
- Correspondence of the two types of walks - - the continuous walk can be obtained as a limit of the discrete walk.

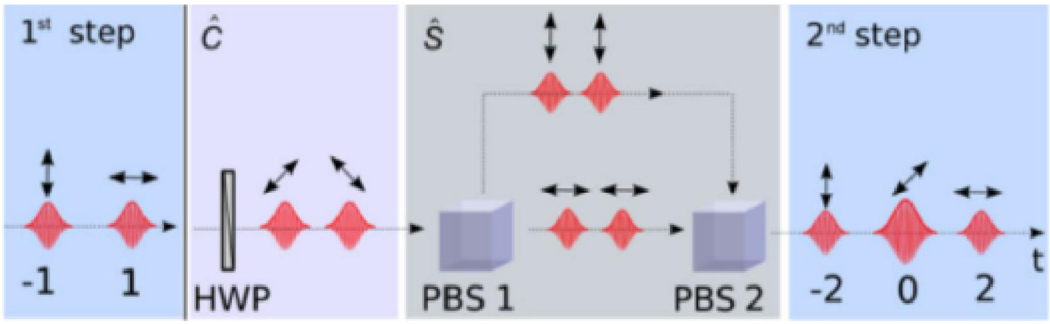
  Childs, Commun. Math. Phys. **294**, 581603 (2010)
- Universal computation by discrete time walks also!

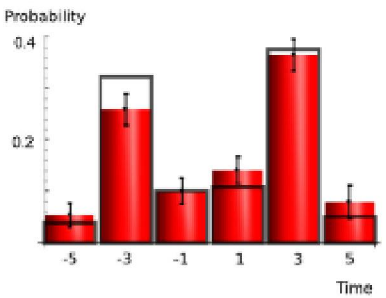
  Lovett et al, Phys Rev A **81**, 042330 (2010)

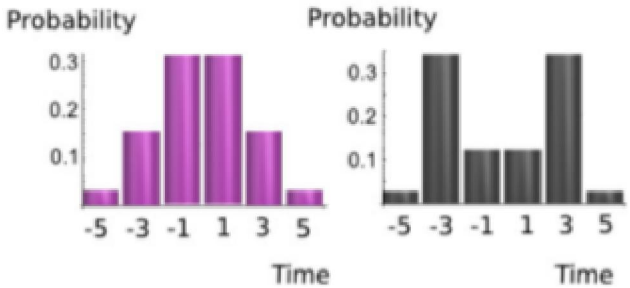
Both the continuous and the discrete-time quantum walks can be regarded as computational primitives.

Outline of a typical experiment









➢Schreiber et al, PRL **104**, 050502 (2010); PRL **106**, 180403 (2011) : 28 steps

Semi-Infinite and Infinite Walks

- **Semi-Infinite Walk (SIW) :**
- One absorbing boundary [measurement wise : detector at a particular position].
- The walk starts from some given site at $t = 0$, and a detector is placed at some other given site $\bar{x}$. The detector detects the particle with probability unity if it reaches there and the corresponding evolution is stopped.
- **Infinite Walk (IW) :**
- No boundary [measurement wise : no detector upto the time of observation].
- The system is allowed to evolve unitarily from a given initial state at $t_i = 0$, up to a terminating time $t' = \Delta t$, when finally a measurement is done and evolution is stopped. The entire process can be repeated by increasing terminating times by Δt (which is 1 for our case).

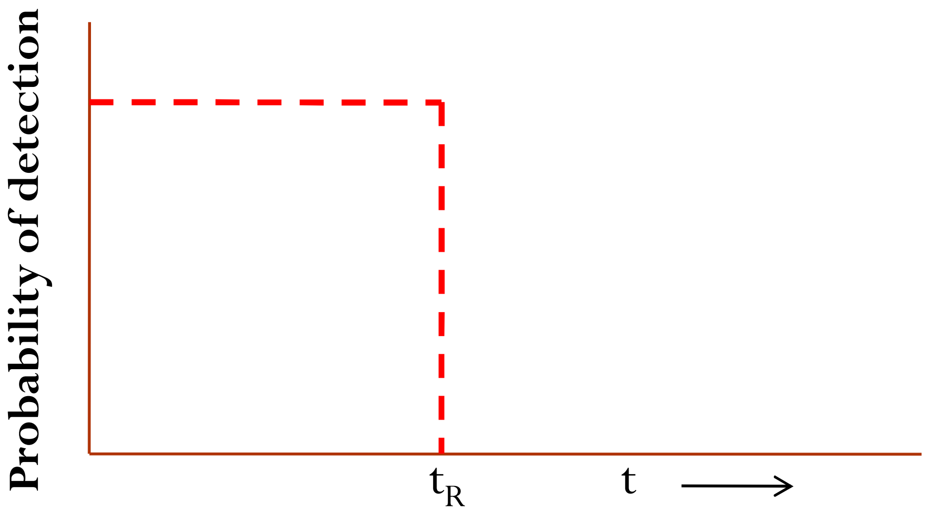
*[SG, P. Sen and A. Das, Phys. Rev. E **81**, 021121(2010)]*

Quenching in different systems :

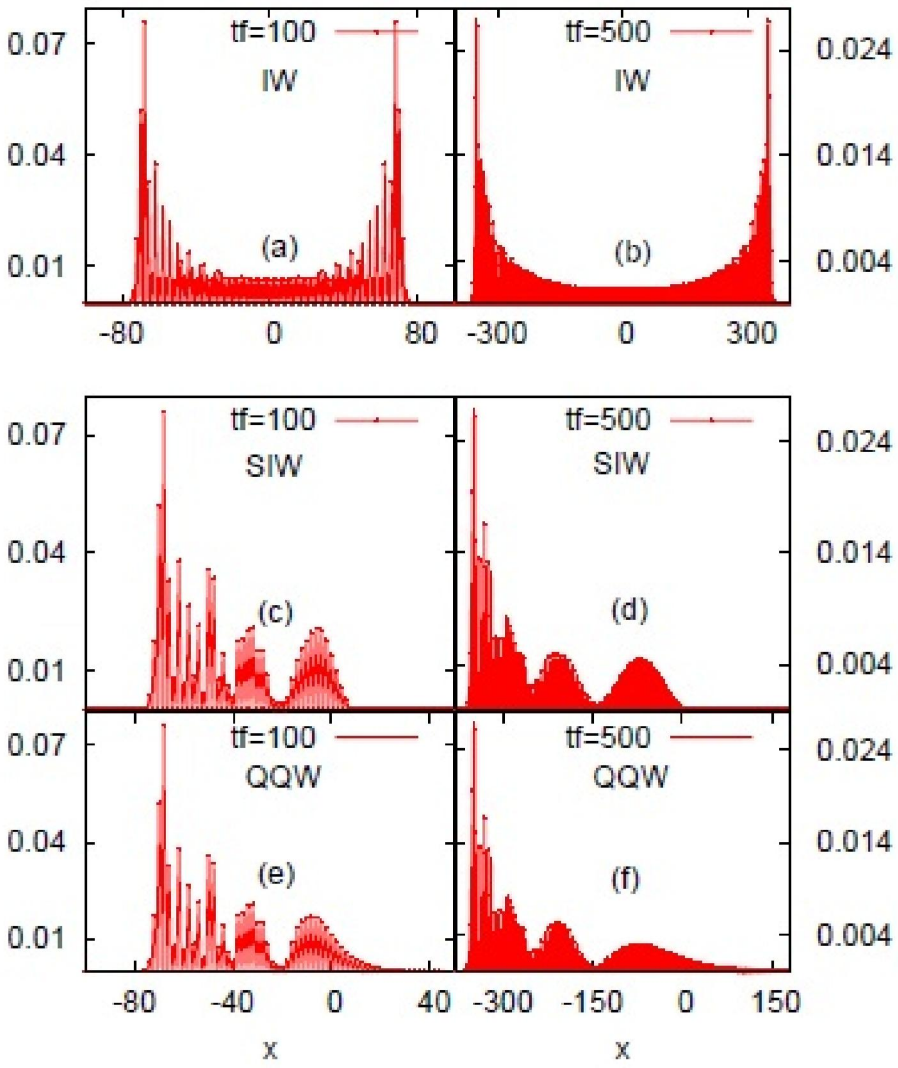
- For *spin glass* system, slow quenching is applied to obtain the classical ground states of the system where there are many minima in the energy landscape, separated by barriers which may be overcome by quantum tunnelling.
- In *ultracold atoms in an optical lattice* fast quenching is applied by shifting the position of the trap potential and studying its response. In case of the *transverse Ising* or *XY model*, there is a deviation from the equilibrium state under quenching as the quantum critical point is crossed and the quantity of interest is the 'defect'.

Quenched Quantum Walk (QQW)

- Effect of quenching on a discrete quantum random walk : a detector placed at a position x_D is removed abruptly at time t_R from its path ➔QQW.
- As x_D and t_R are the parameters of the system, we further modify our notation: $f(x, t, x_D, t_R)$ is the occupation probability of site x at time t respectively for some given x_D and t_R.
- if the particle is at x_D with probability α and the detector detects the particle with probability β, then the total absorption probability at x_D will be $\alpha\beta$. In our case we have chosen $\beta = 1$.



Snapshots of probabilities



$x_D=10$

$x_D=10$, $t_R=50$

IW :

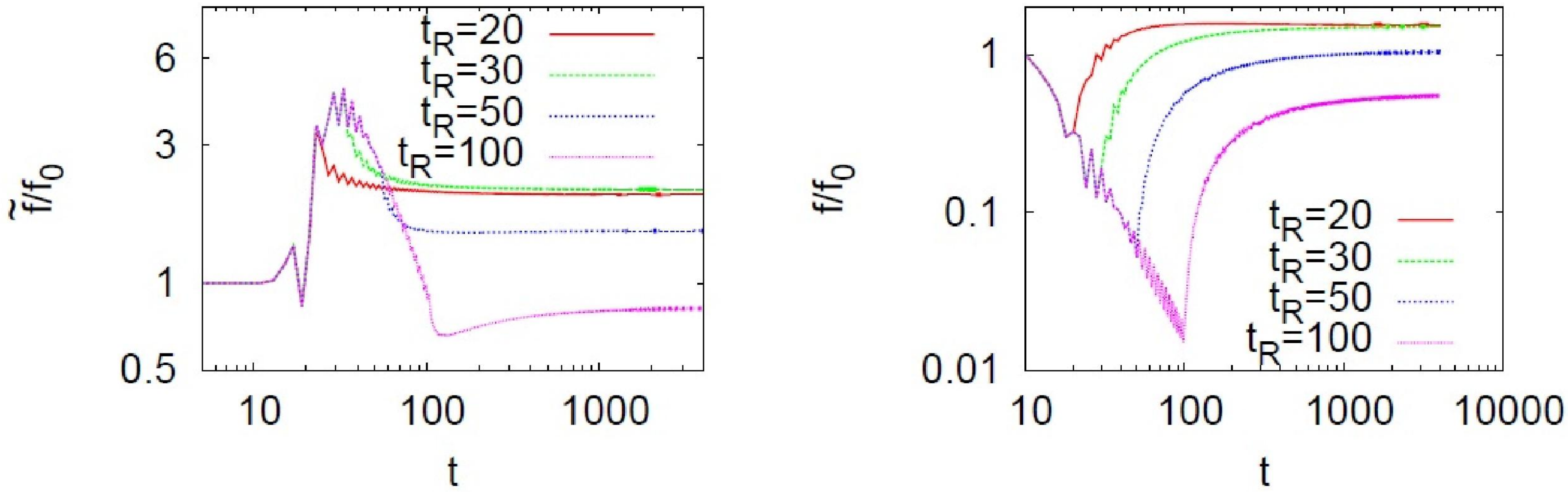
- Displacement in time t proportional to t.
- Maximum of the occupation probability at $x \sim t/\sqrt{2}$.

SIW and **QQW** :

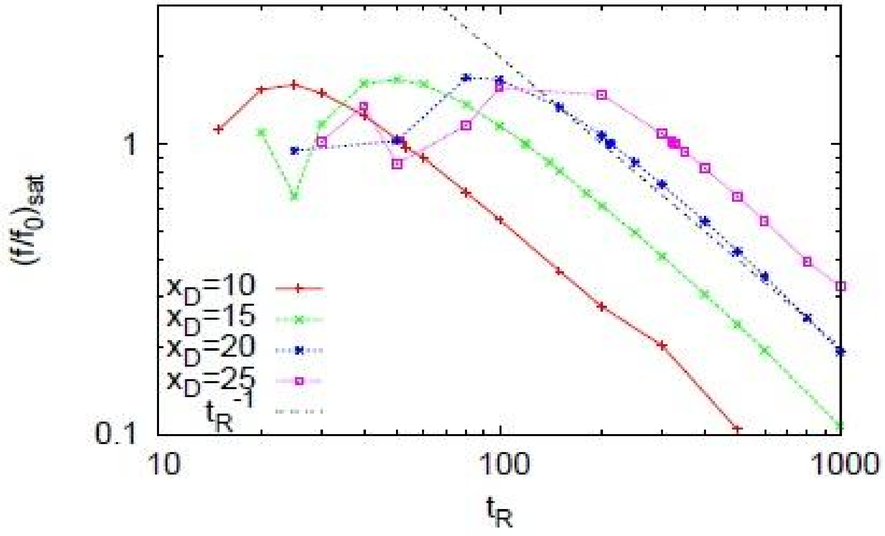
- Up to $t = t_R$, QQW and SIW are equivalent.
- For $t > t_R$, the probability “spills out” beyond x_D for QQW.
- Far away from x_D, there is little difference.
- Away from the boundary, we find that maximum probability is again at a value of $|x| \sim t/\sqrt{2}$.

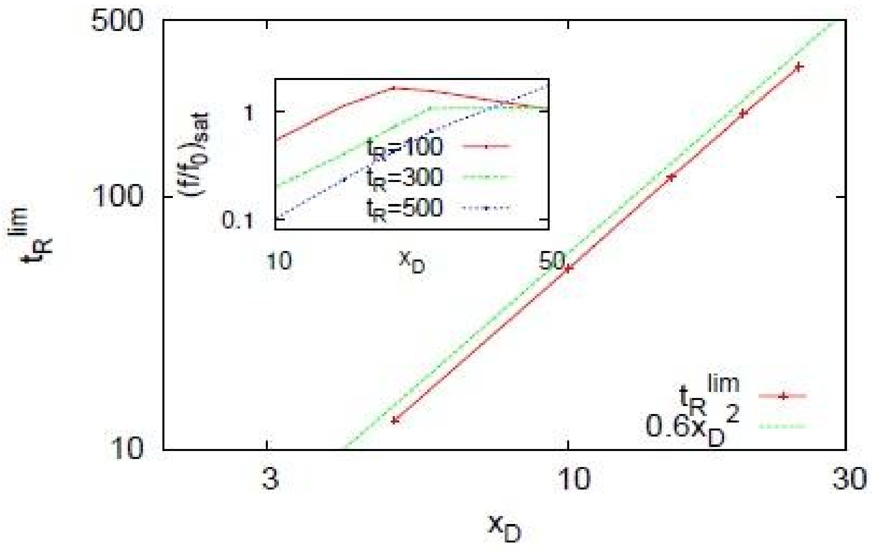
The study for quench must be for a time $t \geq t_R \geq x_D$

$x = x_D = 10$



$\tilde{f}$= normalized occupation probability at x at time t; in ensemble format, fraction of copies that survived the measurement up to time t_R, $\sum_x \tilde{f}(x,t) = 1$,
f= occupation probability at x at time t taking absorption probability into account, $\sum_x f(x,t) = 1 - d$; d= probability that it was absorbed earlier,
f_0= occupation probability at x at time t for IW.





➢The saturation values of the ratio, $(f/f_0)_{sat}$ plotted against t_R shows an initial non-monotonic behaviour. For large t_R, $(f/f_0)_{sat}$ clearly scales as $1/t_R$ for all values of x_D .

➢For fixed and large t_R, $(f/f_0)_{sat}$ varies as $x_D{}^2$

=======➔

For large t_R, $(f/f_0)_{sat} = kx_D{}^2/t_R$

➢For classical walker as $t_R \rightarrow \infty$, f_c/f_{c0} scales as $x_D/\sqrt{t_R}$ → persistence probability at time t_R

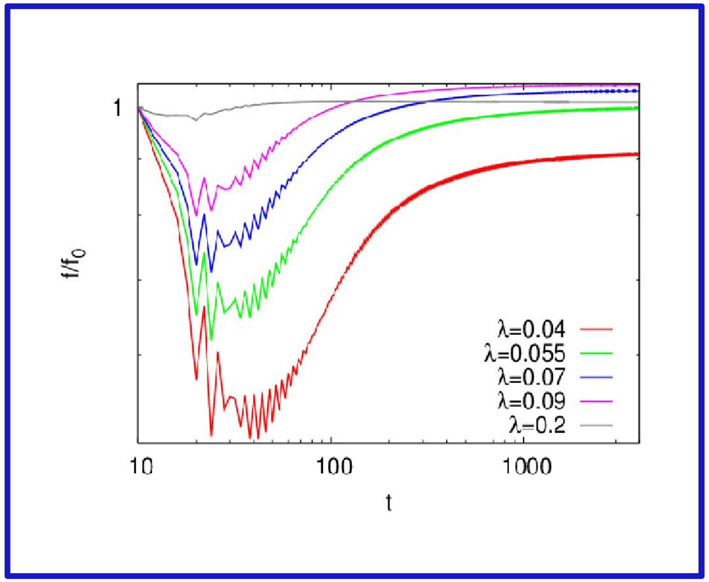
Varying Detection Probability

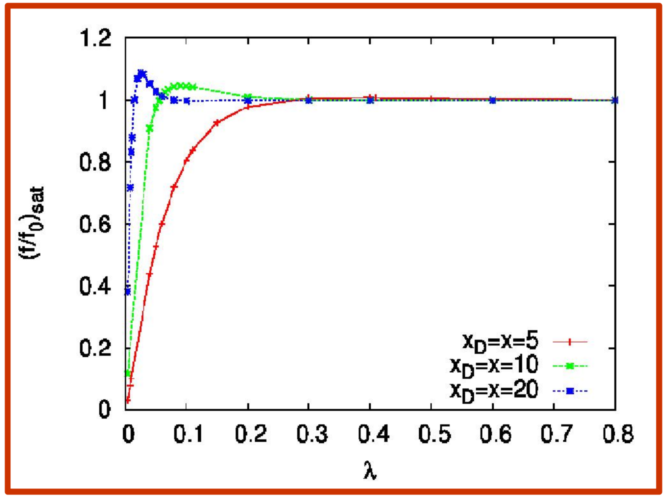
- The particle is at x_D with probability α and the detector detects the particle with probability β : Total absorption probability at x_D will be $\alpha\beta$.

- β is different from 1, varying exponentially with parameter λ , i,e, $\exp(-\lambda t)$ or like a power law $t^{-\lambda}$.

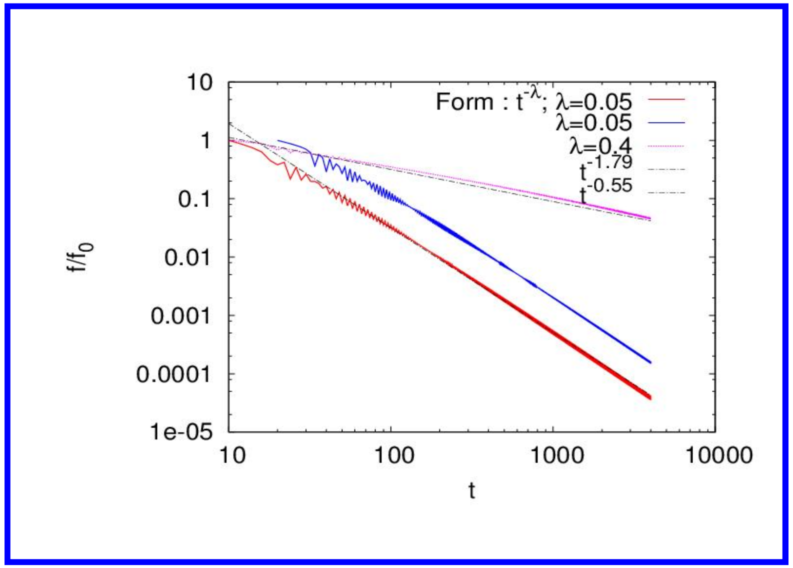
Ratio of probabilities

Exponential :





Power law :



Classical

- Exponential decay :

$$\frac{f_c}{f_{c0}} = 1 - \left[\int_0^T F_c(x_D, t)\,dt + \int_T^\infty F_c(x_D, t)\exp(-\lambda t)\,dt \right]$$

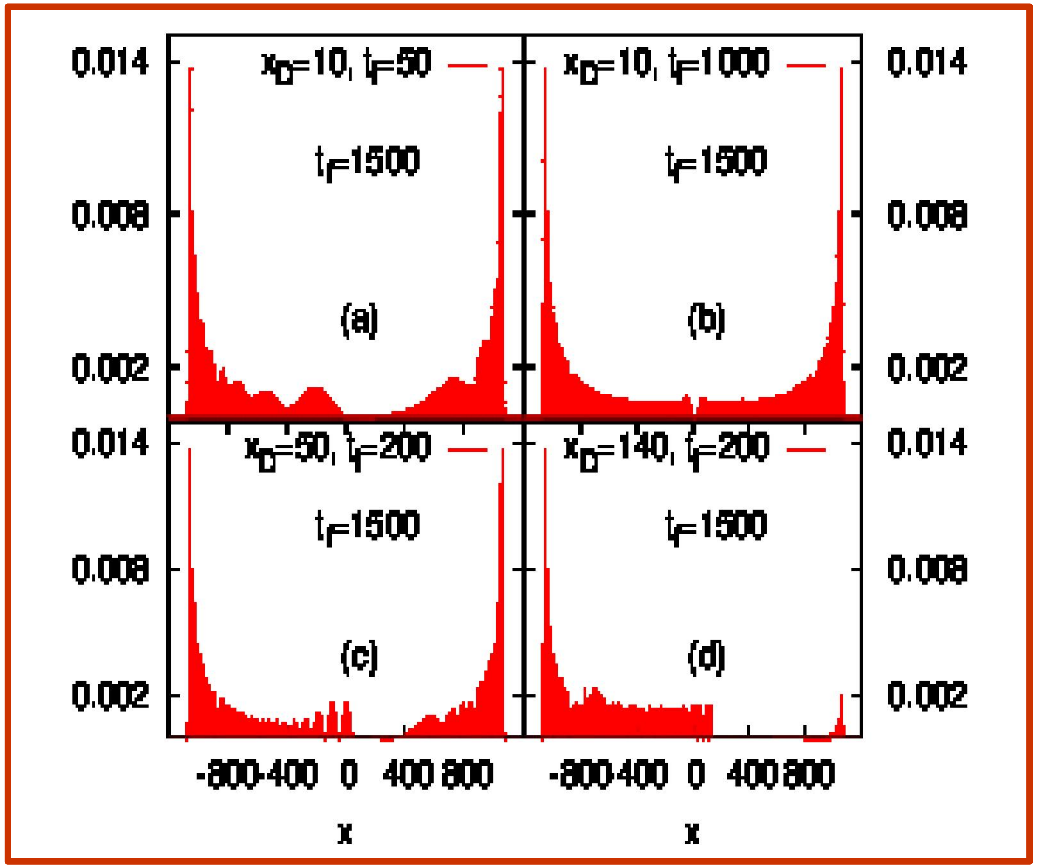
$$\sim erf\left(\frac{x_D}{\sqrt{4DT}}\right) - erf\left(\frac{x_D}{\sqrt{4DT}}\right)\exp(-\lambda T) - \frac{\lambda x_D\, erf\left(\sqrt{\lambda T}\right)}{\sqrt{\lambda D}}$$

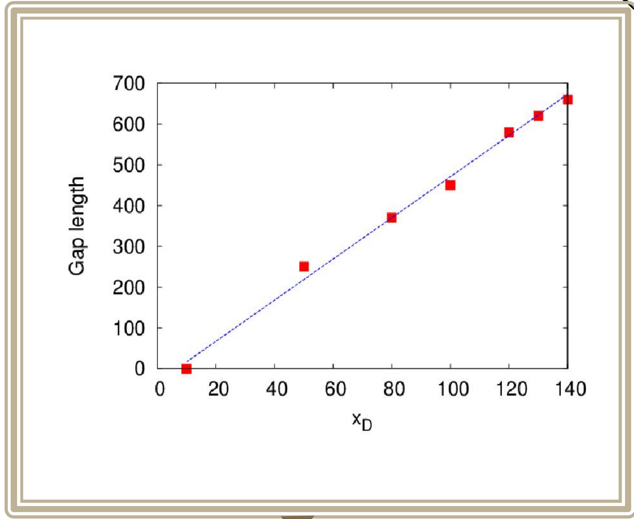
- Power law decay :

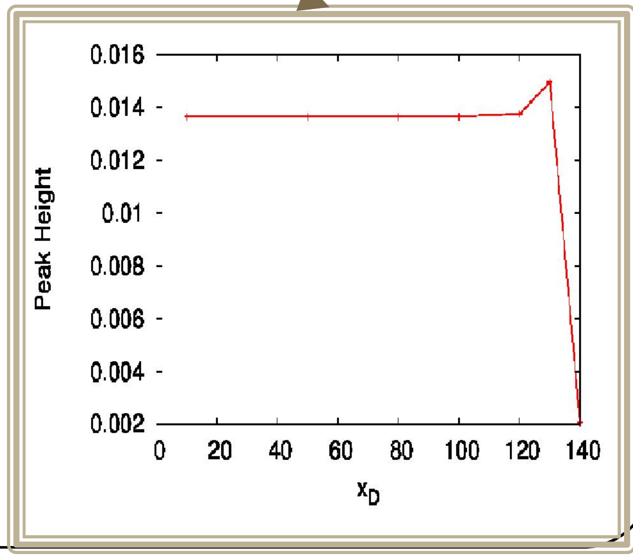
$$\frac{f_c}{f_{c0}} = 1 - \left[\int_0^T F_c(x_D, t)\,dt + \int_T^\infty F_c(x_D, t)\, t^{-\lambda}\,dt \right]$$

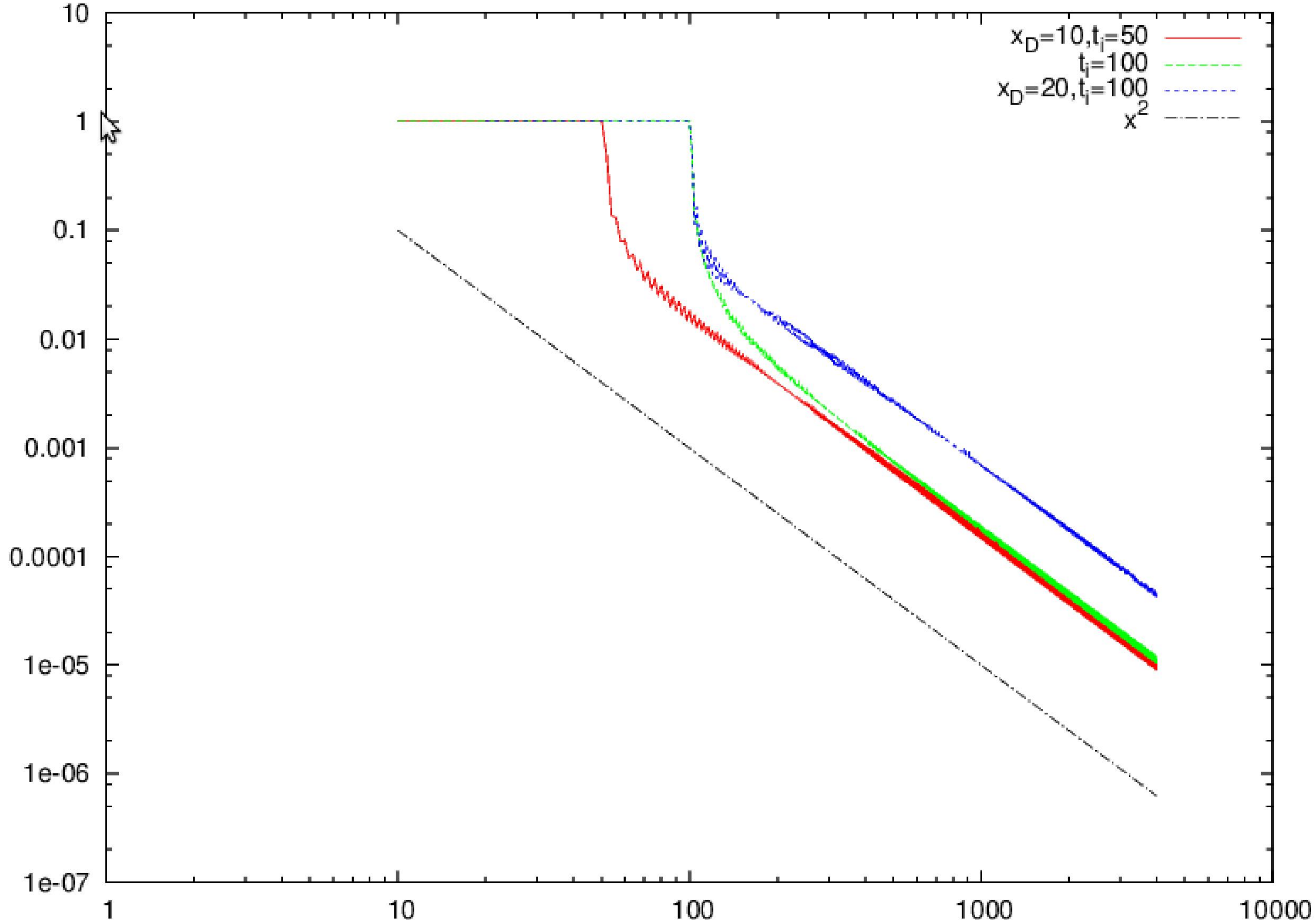
$$\sim erf\left(\frac{x_D}{\sqrt{4DT}}\right) - \frac{x_D^{2(-\lambda-1/2)+1}\, D^{\lambda}\, 4^{\lambda+1/2}\,\Gamma\left(\lambda + \frac{1}{2}, \frac{{x_D}^2}{4DT}\right)}{2\sqrt{\pi}}$$

Detector introduced at time t_i :



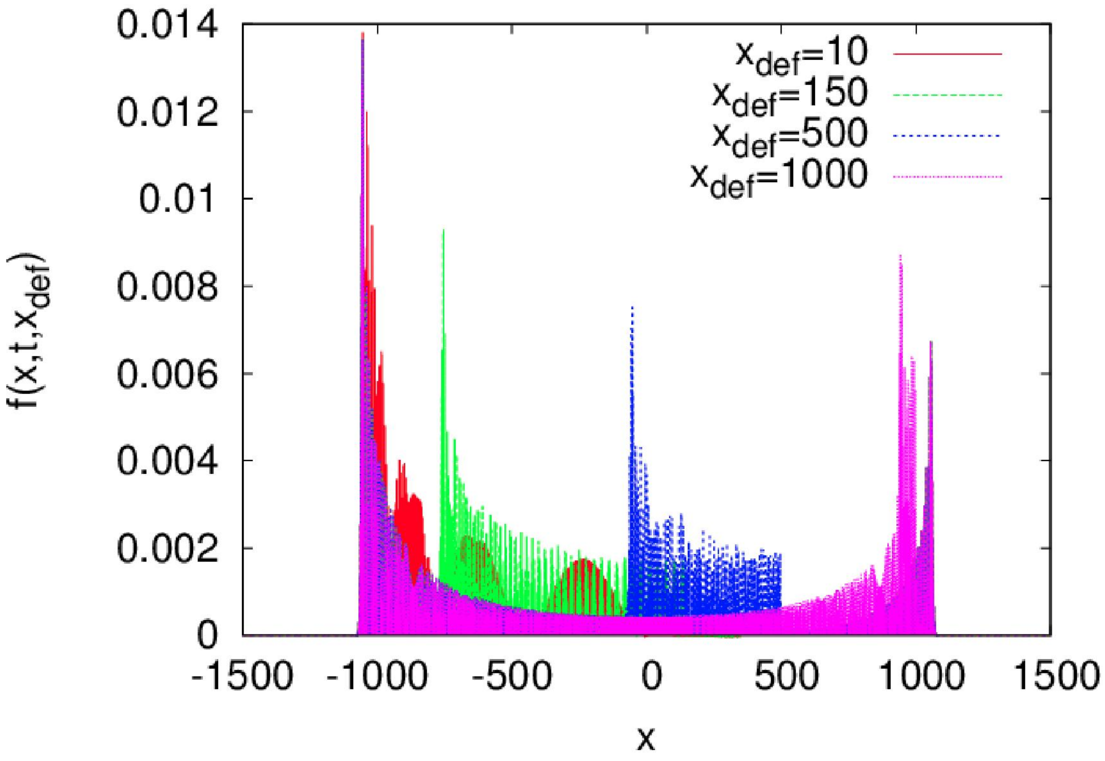






x_D=10,t_i=50
t_i=100
x_D=20,t_i=100
x^2
10
1
0.1
0.01
0.001
0.0001
1e-05
1e-06
1e-07
1
10
100
1000
10000

Introduction of defect:



Highlights

❑Quenching :

➢The detector is present up to t_R, the quenched walker cannot go beyond x_D, and the undetected walker will move away from x_D.

➢For $t > t_R$, walker is free once again, it can move towards and beyond x_D. Most of the contributions to x_D and beyond come from the density of walkers closer to it. The occupation probability profile approaches the IW picture, local hill like structures smoothens out. For IW, the walker has moved reasonably away from x_D at t_R, such that the ratio can be greater than unity close to x_D.

➢When t_R is greater than t_R^{lim}, the ratio can no longer exceed unity.

❑Exponential decay :

➢The occurrence probability is enhanced here also. This is not possible in the classical case.

➢For small $x=x_D$, the saturation value can never cross 1. For larger $x=x_D$, it first increases beyond 1 at small λ and then approaches a constant.

❑Power law decay :

➢The ratio is always less than 1.

❑Detector introduced at t_i :

➢If the detector is introduced for one case at time t and for the other case at a much later time t', then the second case will be closer to the IW picture.

➢As presence of the detector before measurement is made smaller, the gap in the probability picture becomes larger which is linear in x_D.

❑Defect :

➢Introduction of defect at a particular site leads to some sort of decoherence among the probability amplitudes which then leads to asymmetry.

- The observation that the occurrence probability of a QRW may actually be enhanced by quenching is one of the main results of these studies. This is a purely ***quantum mechanical*** effect.

- The ratio saturates as the "memory" of the detector gets erased in time. Memory effects are strong for $x << x_D$. ***The effect of quenching is rather local***.

- Peaks or secondary peaks are always at $|t/\sqrt{2}|$.