Hardy's test provides 100 % contradiction with Local-Realism in GNST

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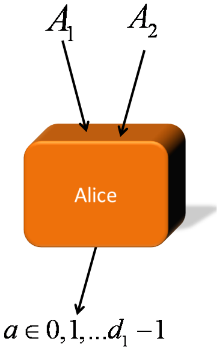
YouQu2015

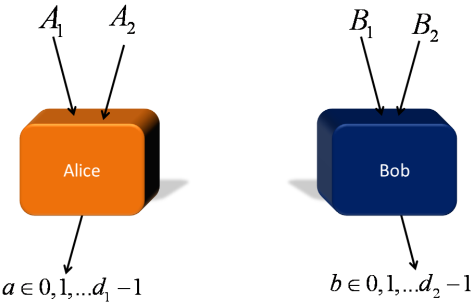
Outline

An Event



$(A_i = a)$: an Event.
Described by $p(A_i = a)$ for a given preparation procedure.

A Phenomenon



$(A_i = a, B_j = b)$: a Phenomenon.
Described by $p(A_i = a, B_j = b)$ for a given preparation procedure.

Bipartite Scenario
Local Realistic Hidden Variable Theory

Ontic state: $\lambda \in \Lambda$ is the complete specification of the properties of the system such that,

$$p(A_i = a, B_j = b) = \int d\lambda \rho_\lambda p(A_i = a, B_j = b|\lambda).$$

Local determinism[1]: Conjunction of the following two conditions

- Parameter Independence or Locality

$$p(A_i = a|B_j = b, \lambda) = p(A_i = a|b, \lambda)$$
$$p(B_j = b|A_i = a, \lambda) = p(B_j = b|a, \lambda)$$

- Determinism

$$p(A_i = a, B_j|\lambda) \in \{0, 1\}$$

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Hardy's Test in Two-outcome Scenario

Hardy's Argument:

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\begin{aligned}
p(A_1 = 0, B_2 = 1) &= 0 \\
p(A_2 = 1, B_2 = 0) &= 0 \\
p(A_2 = 0, B_1 = 1) &= 0 \\
p(A_1 = 0, B_1 = 1) &= q_H > 0 \qquad (\textit{Hardy}\ '92)
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Maximum allowed probability of non-local phenomena in Hardy's test in a theory

$q_H^{opt} \rightarrow$ The maximum probability of phenomena that cannot be explained by local theories.

In Quantum theory,

$q_H^{opt} = 0.09 \quad (Hardy\ '92)$

In GNST,

$q_H^{opt} = 0.500 \quad (Kukri\ '05)$

An observation

In two-outcome scenario, even the PR box, which is maximally non-local(i.e. BI violation is algebraic maximum), provides only 50% probability of observing non-local events in Hardy's test under GNST.

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A question

Can this test of Hardy's, be extended in many-outcome scenario such that the maximum probability of observing non-local events in Hardy's test is increased in GNST?

Hardy's Test in Many-outcome Scenario

Conventional Extension of Hardy's Argument:

$$p(A_1 = 0, B_1 = 0) = 0$$
$$p(A_1 \neq 0, B_2 = 0) = 0$$
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[1] S. K. Chudhary, S. Ghosh, G. Kar, S. Kunkri, R. Rahaman, and A. Roy, Quant. Inf. Comp. 10, 0859 (2010)

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Maximum success probability of conventional extention

In Quantum theory

$$q_H^{opt} = 0.09 \text{ for } d \otimes d \quad (\textit{Kukri}\ '05)$$
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An observation

In the conventional scheme of many-outcome extension $\rightarrow$ assign zero probability to a set of outcomes and retain the paradox in an effective two dimensional subspace.
But the recently proposed generalized extension of Hardy's test by Chen et. al. doesn't look that trivial.

Hardy's Test in Many-outcome Scenario

New Generalized Hardy Argument:

$$\begin{aligned} p(A_2 < B_1) &= 0 \\ p(B_1 < A_1) &= 0 \\ p(A_1 < B_2) &= 0 \\ p(A_2 < B_2) &= q_{RH} > 0 \qquad\qquad (Chen\,'13) \end{aligned} \tag{1}$$

where

$$p(A_2 < B_1) = \sum_{m<n} p(A_2 = m, B_1 = n)$$

.

[1] J-L. Chen, A. Cabello, Z.P.Xu, H.Y.Su, C. Wu, and L. C. Kwek, Phys. Rev. A 88, 062116 (2013).

Hardy's Test in three-outcome Scenario

$$
\begin{aligned}
p(A_2 = 0, B_1 = 1) &+ p(A_2 = 0, B_1 = 2) \\
&+ p(A_2 = 1, B_1 = 2) = 0 \\
p(A_1 = 1, B_1 = 0) &+ p(A_1 = 2, B_1 = 0) \\
&+ p(A_1 = 2, B_1 = 1) = 0 \\
p(A_1 = 0, B_2 = 1) &+ p(A_1 = 0, B_2 = 2) \\
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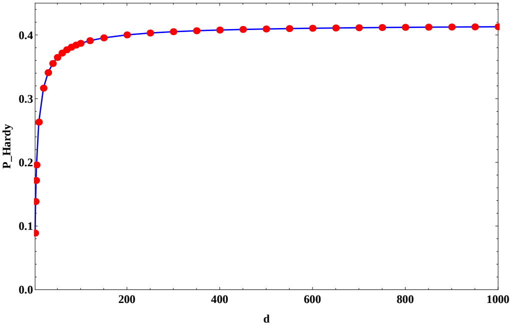
Hardy's Test in Many-outcome Scenario

In Quantum world

Dimension of system	q_{RH}^{opt}
$2 \otimes 2$	0.09
$3 \otimes 3$	0.14
$4 \otimes 4$	0.17
$5 \otimes 5$	0.20
$6 \otimes 6$	0.22
$7 \otimes 7$	0.24

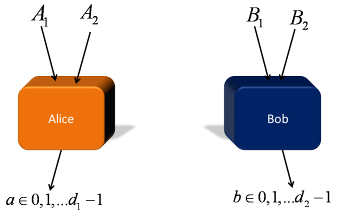


The maximum allowed probability of non-local events is increasing with system dimension. (Plot from Chen et. al. '13)

A Question

How to understand the structural difference between these two possible extensions in many-outcome scenario?

General no-signaling correlations in many-outcome scenario



Normalization:

$$\sum_{a,b \in \{0,\ldots,d_1-1\}} p(A_i = a, B_j = b) = 1 \forall A_i, B_i$$

No-signaling:

$$\sum_{b=0}^{d_2-1} P(X_1 = x_1, B_1 = b) = \sum_{b'=0}^{d_2-1} P(X_1 = x_1, B_2 = b')$$

$$\forall X_1 \in \{A_1, A_2\} \text{ and } x_1 \in \{0, d_1 - 1\}$$

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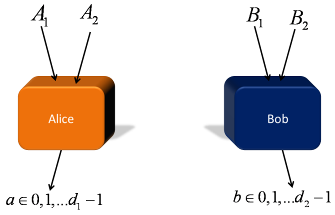

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Hardy's Test in Many-outcome Scenario

No-Signaling Polytopes $\boldsymbol{P(2, d)}$

Extreme Points/Vertices

1. Partial-output vertices $\rightarrow$ at least one of the conditions $P(X = a) = 0$ or $P(Y = b) = 0$ hold.
2. Full-output vertices $\rightarrow$ all $P(X = a) \neq 0$ and $P(Y = b) \neq 0$.

Partial-output vertices of $\boldsymbol{P}$ correspond to the full-output vertices of some other polytope $\tilde{\boldsymbol{P}}$ with fewer local dimension (i.e. $d'^A < d^A$ or $d'^B < d^B$).

Local Box

$$P_L^{\alpha\beta\gamma\delta} = \begin{cases} 1 & \text{if } a = \alpha X \oplus \beta, b = \gamma Y \oplus \delta \\ 0 & \text{otherwise} \end{cases}$$

Non-local Box

$$P_{NL}^{\alpha\beta\gamma} = \begin{cases} \frac{1}{d} & \text{if } (b \ominus a) = XY \oplus \alpha X \oplus \beta Y \oplus \gamma, \\ & a, b \in \{1, ..., d\} \\ 0 & \text{otherwise} \end{cases}$$

$\alpha, \beta, \gamma, \delta \in \{0, ..., \min(d_1, d_2) - 1\}$, $d = min\{d_1, d_2\}$.

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$$\alpha,\beta,\gamma,\delta\in\{0,...,\min(d_1,d_2)-1\},\ d=min\{d_1,d_2\}.$$

Hardy's Test in Many-outcome Scenario

The maximum value of q_{RH} that can be achieved by a full-output vertex of $\boldsymbol{P(2, d)}$

$$q_{RH}^{full} = \frac{d-1}{d}$$

where $d = \min\{d_1, d_2\}$.

Hardy's Test in Many-outcome Scenario

An observation

The conventional scheme of extending Hardy's argument for many-outcomes, assigns zero probability to a set of outcomes $\rightarrow$ corresponds to a partial-output vertex in $\boldsymbol{P(2,d)}$ $\rightarrow$ equivalent to a full-output vertex of $\boldsymbol{P(2,2)}$.

The maximum value for q_H, achieved by any non-local full-output vertex of $\boldsymbol{P(2,2)}$ is

$$q_H^{full} = \frac{1}{2}$$

Hardy's Test in Many-outcome Scenario

An observation

The conventional scheme of extending Hardy's argument for many-outcomes, assigns zero probability to a set of outcomes $\rightarrow$ corresponds to a partial-output vertex in $\boldsymbol{P(2,d)}$ $\rightarrow$ equivalent to a full-output vertex of $\boldsymbol{P(2,2)}$.

The maximum value for q_H, achieved by any non-local full-output vertex of $\boldsymbol{P(2,2)}$ is

$$q_H^{full} = \frac{1}{2}$$

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Results

Using Convexity of the no-signaling polytope,

In GNST:

1. $q_{RH}^{opt} = \frac{\min\{d_1,d_2\}-1}{\min\{d_1,d_2\}}$ for any $(d_1 \otimes d_2)$ system.
2. $q_{H}^{opt} = \frac{1}{2}$ for any $(d_1 \otimes d_2)$ system.

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Results

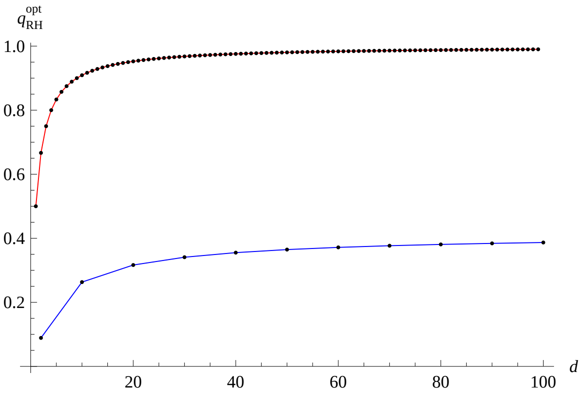

Figure: The line in blue shows the increase of q_{RH}^{opt} with increasing system dimension for quantum systems (Chen et. al. '13). The red line shows q_{RH}^{opt} for generalized no-signaling correlations.

Summary

- We observe that in any theory that only respects relativistic causality, the maximum probability of observing non-local events in the generalized Hardy argument increases with local dimensions of the two subsystems.
- We provide a proof which emphasizes a simple functional dependence of the maximum probability of non-local events in the generalized non-locality argument on local dimensions.
- This result also suggests that the non-locality argument proposed by Chen et. al. is the most natural higher-dimensional generalization of Hardy's argument in two-input scenario.

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Reference I

J. Barrett, N. Linden, S. Massar, S. Pironio, S. Popescu, and D. Roberts.
Non-local correlations as an information theoretic resource.
Phys. Rev. A, 71, 022101 (2005).

J.-L. Chen, A. Cabello, Z.-P. Xu, H.-Y. Su, C. Wu, and L. C. Kwek.
Hardy's Paradox for High-Dimensional Systems: Beyond Hardy's Limit.
Phys. Rev. A , 88, 062116 (2013).