Harish Chandra Research Institute

Quantum Field Theory II

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Assignment # 1
Due - February 25

1. Compute the quantity $Z(J)$ for a free real scalar field in d spacetime dimensions.

[10 points]

2. Consider a free complex scalar φ in d dimension which coupled to an external current $J(x)$ by an interaction term in the Lagrangian

$$\mathcal{L}_{\text{int}} = J^\dagger(x)\varphi(x) + J(x)\varphi^\dagger(x). \tag{1}$$

Compute $Z(J, J^\dagger)$. Obtain the statement of Wick's theorem in this case.

[10 points]

3. Consider a quantum simple harmonic oscillator of mass m and angular frequency ω in its ground state ($t = -\infty$) which is subjected to a time dependent external force $f(t)$.The function $f(t)$ is vanishes at early and late times. What is the probability that the SHO is in its ground state at $t = +\infty$. Express your answer in terms of the Fourier transform of $f(t)$, perform all integrals that you encounter. You can assume that the Fourier transform of $f(t)$ falls of zero for large values of its argument.

[10 points]

4. Consider electron-electron, electron-positron, and positron-electron scattering at tree level in QED (i.e to leading order in an expansion in e^2). In each case, obtain the potential between the particles at low energies by taking a suitable limit of the scattering amplitudes and determine if they are attractive or repulsive.

[10 points]

5. Perform the integral

$$\int d^n y \;\; \exp(-y_i M_{ij} y_j + b_i y_i)$$

where y_i ($i = 1....n$) are real variables with the integration range $-\infty$ to $+\infty$ for all of them, M_{ij} is a constant real symmetric matrix with positive eigenvalues and b_i are real constants. The Einstein summation convention is followed. Make comments comparing the final result with the final result of problem 1.

[10 points]