Harish Chandra Research Institute

Quantum Field Theory II

Anshuman Maharana Summer 2022

Assignment # 5

Due - May 6, 5:30 pm

1. Consider the scaling behaviour of Green's functions in the massive $\lambda\varphi^4$ scalar field theory in four dimensions

$$G^{(N)}(sq_1, sq_2...., sq_N, \lambda(\mu_0), m^2(\mu_0), \mu_0^2) = \exp\left(N \int_{\lambda(\mu_0)}^{\lambda(s)} d\lambda' \frac{\gamma(\lambda')}{\beta(\lambda')}\right) s^{(4-3N)} F\left(q_1, ...q_N, \lambda(s), \frac{m^2(s)}{s^2}\right)$$

Take the classical limit of the formula. Explain the structure of the classical limit of the formula in terms of a property of the classical theory.

[10 points]

2. Consider an asymptotically free theory with

$$\beta(g^2) = \frac{dg^2}{d\ln\mu} = -bg^4 \qquad b > 0.$$

at one loop. Let μ_0 be a scale at which the theory is weakly coupled i.e $g^2(\mu_0) \ll 1$. Assume that for $g^2 < g^2(\mu_0)$

$$\gamma_m(g^2) \approx cg^2$$

Express $m(\mu)$ in terms of $m(\mu_0), g(\mu_0)$ and $g(\mu)$. Argue that the high energy limit of Green's functions (if it exists, or an observable determined in terms of the high energy limit of Green's functions)

$$\lim_{s\to\infty} G^{(N)}(sp_1, sp_2, ..., sp_N, g(\mu_0), m^2(\mu_0), \mu_0^2)$$

is independent of $m(\mu_0)$.

[5+10 points]

3. Compute the β and γ functions at one loop in the $\overline{\text{MS}}$ scheme for the φ^4 theory in four dimensions. [20 points]

4. Compute the β and γ functions at one loop in the $\overline{\text{MS}}$ scheme for the theory of a complex scalar field in four dimensions with interaction term $\mathcal{L}_{\text{int}} = -\frac{1}{4}g(\varphi\varphi^\dagger)^2$

[20 points]

5. Compute the β and γ functions in the $\overline{\text{MS}}$ scheme at one loop for a four dimensional theory involving a real scalar and a Dirac fermions with a Yukawa interaction: $\lambda\phi\bar{\psi}\psi$. [20 points]