Harish Chandra Research Institute

Quantum Field Theory II

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Assignment # 6
Due - 30 May

1. Compute the $\overline{MS}$ β function for scalar QED – the theory of a single massive complex scalar coupled to a gauge field.

[15 points]

2. Consider the massive $\lambda\varphi^4$ theory at one loop in four dimensions regularised using dimensional regularisation. Renormalise the theory using the renormalisation condition:

$$\Gamma^{(4)}(s,t,u)\big|_{s,t,u=-M^2} = -\lambda(M),$$

where s, t, u are Mandelstam variable associated with the 1PI four vertex and M an arbitrary scale. Compute the β function in this scheme. Examine the $M >> m$ and $M << m$ limits of this function (m being the mass of the scalar field). Compare and contrast with the $\overline{MS}$ scheme β function and provide a physical explanation of the results of your comparisons.

[25 points]

3. Consider the correction to the photon propagator from a fermion loop in QED -

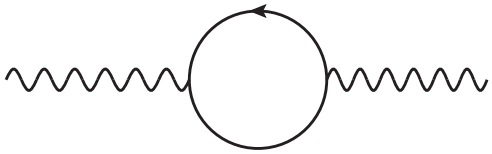

Figure 1: A fermion loop correcting the photon propagator

Show that if this is evaluated using *cut-off regularisation* one gets a contribution to $\Pi^{\mu\nu}(k)$ which is not transverse. Thus cut-off regularisation is not well suited for QED.

[15 points]

4. Consider the 1PI 4 vertex in QED involving four external photons at one loop (using dimensional regularisation in $4-\epsilon$ dimensions). Show that the contribution to this from the relevant loop diagram(s) is finite in the $\epsilon \to 0$ limit.

[15 points]

5. Consider the scattering of electrons off an external electromagnetic field ($A_\mu(x)$) at 1 loop in QED. By taking the suitable limit of the amplitude and comparing with the non-relativistic Born approximation result, determine the gyromagnetic ratio of the electron.

[20 points]