

HARISH CHANDRA RESEARCH INSTITUTE

QUANTUM FIELD THEORY II

Anshuman Maharana

Summer 2022

Assignment # 7

Due - June 10

1. Consider N complex scalar described by the Lagrangian

$$\mathcal{L} = -\partial_\mu \varphi_a^\dagger \partial_\mu \varphi_a - m^2 \varphi_a^\dagger \varphi_a - \lambda \left(\varphi_a^\dagger \varphi_a \right)^2,$$

$a = 1, \dots, N$ (repeated indices are summed). Determine the associated global symmetry.

[5 points]

2. Given a representation R of $SU(N)$ one has associated hermitian generators T_R^a . These can be used to construct the matrix $A_\mu^R(x) = A_\mu^a(x) T_R^a$. Consider the transformation of the matrix given by

$$\tilde{A}_\mu^R(x) = U(x) A_\mu^R(x) U^{-1}(x) + \frac{i}{g} U(x) \partial_\mu U^{-1}(x),$$

where $U(x)$ is a representation matrix in the representation R ; $U(x) = \exp(-ig\theta^a(x) T_R^a)$ with $\theta^a(x)$ real. Show that this corresponds to a transformation for A_μ^a which is independent of the representation. Hint: Start by looking at the cases in which $U(x)$ is close to the identity.

[10 points]

3. Show that the field strength $F_{\mu\nu}^a$ has transformation properties corresponding to the adjoint representation. Hence define the action of covariant derivative on it.

[5 points]

4. Show that the field strength satisfies the Bianchi identity

$$(D_\mu F_{\rho\sigma}^a) + (\text{cyclic}) = 0$$

[5 points]

5. Consider a $SU(2)$ gauge theory whose matter content is Dirac fermions in the 2 dimensional representation of the theory and complex scalars in the 3 dimensional representation of the theory. The gauge coupling of the theory is g . Write down the Lagrangian of the theory, using the explicit form of the representation matrices. Carry out the sums over all gauge indices. All particles are massless, there are no potentials.

[5 points]

6. Consider the Standard Model Lagrangian in the the absence of the Higgs (electroweak symmetry remains unbroken). Write down the explicit form of all kinetic terms and couplings

between gauge bosons and matter fields. Provide the explicit form the representation matrices involved, you do not have to carry out the sum over gauge indices.

[5 points]

7. Let T_R^a be the representation matrices of a the representation R of a compact Lie group. If

$$\text{Tr}(T_R^a T_R^b) = \frac{1}{2} \delta_{ab}$$

show that the structure constant f^{abc} defined by

$$[T_R^a, T_R^b] = i f^{abc} T_R^c$$

are fully antisymmetric.

[5 points]

8. Working with the quantities defined in the above problem, show that the Casimir of a representation

$$\sum_a T_R^a T_R^a$$

commutes with all the generators T_R^a . Hence argue that it is proportional to the identity matrix.

[5 points]

9. Draw the Feynman diagrams that would contribute to quark - antiquark annihilation in QCD at weak coupling. Simplify each diagram as much as possible

[10 points]

10. Obtain the gluon propagator in QCD motivated by the gauge $A_3^a = 0$ condition. Follow a procedure similar to that we use for the gauge $\partial^\mu A_\mu^a = 0$ i.e choose the gauge fixing function to be

$$A_3^a - w^a(x) = 0,$$

multiply the path integral by $\exp\left(-\frac{i}{2\xi} w_a w^a\right)$ and integrate over w_a . What is the ghost gluon interaction ?

[10 points]