

HARISH CHANDRA RESEARCH INSTITUTE

QUANTUM FIELD THEORY II

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Assignment # 8

Due - June 20

1. Consider BRST quantisation of free Maxwell theory as discussed in the lectures.
 - a) Show that all one particle physical states have positive norm.
 - b) Making use of the explicit form of Q_{BRST} show that $Q_{\text{BRST}}^2 = 0$
 - c) Compute the commutators of Q_{ghost} with the creation and annihilation operators of the ghost fields (i.e with $c(k)$, $c^\dagger(k)$ and $\bar{c}(k)$, $\bar{c}^\dagger(k)$). Choose a normalisation for the ghost charge such that it is hermitian.
 - d) Suppose we consider Maxwell theory in D dimensions. Use BRST quantisation to determine the number of physical polarisations.

[5 + 5 + 5 + 5 points]

4. Consider QED at one loop regularised by dimensional regularisation and in the on shell scheme. Show that $Z_2 = Z_1$ (where Z_2 and Z_1 are the renormalisation coefficients of the fermion kinetic term and the photon-fermion interaction terms in the renormalised Lagrangian).

[15 points]

5. Compute the beta function of pure Yang-Mills theory (no matter fields) based on the group $SU(N)$.

[40 points]

6. What is the form of the BRST symmetry when matter fields are included in a gauge theory Lagrangian? Show that “acting the symmetry twice” gives zero in this case also.

[10 points]

7. What does QED Ward identity (in the \overline{MS} scheme, all fields and couplings below are the renormalised ones)

$$i\partial_\mu \left(\frac{\delta\Gamma}{\delta A_\mu(x)} \right) = e \left(\bar{\psi}(x) \frac{\delta\Gamma}{\delta \bar{\psi}(x)} + \frac{\delta\Gamma}{\delta \psi(x)} \psi(x) \right) + \frac{i}{\xi} \square \partial_\mu A^\mu(x),$$

imply for the 1PI 4 photon vertex ? What are the allowed tensor structures (as functions of the external momenta) for this vertex ? Which of these can have a divergent coefficient ? What does the Ward identity imply for this coefficient ?

[10 points]

8. Compute the Ward identity (in the form of a functional relation that the 1PI effective action obeys) associated with the BRST symmetry of QED. Compare with the Ward identity obtained in the lectures.

[20 points]