USE OF CONCEPTS OF QUANTUM OPTICS IN QUANTUM INFORMATION THEORY

Invited Talk by

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Meeting on Quantum Information Processing and Applications,
Harish-Chandra Research Institute, Allahabad
07-13 December, 2015

Hari Prakash
Indian Institute of Information Technology, Allahabad, India

COHERENT STATE IN QUANTUM OPTICS

a – annihilation operator

Coherent state $|\alpha\rangle$ defined by

$$a|\alpha\rangle = \alpha|\alpha\rangle \quad \text{, for any complex number } \alpha$$

$$|\alpha\rangle = \sum_n e^{-\frac{1}{2}|\alpha|^2} \frac{\alpha^n}{\sqrt{n!}}|n\rangle$$

Properties

$$\langle\alpha|\alpha\rangle = 1, \quad \frac{1}{\pi}\int d^2\alpha |\alpha\rangle\langle\alpha| = 1, \quad d^2\alpha \equiv d\alpha_r d\alpha_i$$

Coherent states are in fact not complete but overcomplete since

$$\int d^2\alpha\, f(\alpha)\, |\alpha\rangle = 0$$

Sudarshan Glauber Representation and Nonclassicality

Coherent states- closet to stable classical light

Uncertainty principle brings in uncertainty.

Classical Coherent Functions: $\langle v^* v \rangle$ for complex signal $v = v_r + i v_i$

Normally Ordered Coherent Function: $\Gamma_N^{(m,n)}\ \mathrm{Tr}[\rho a^{+m} a^n]$

Antinormally Ordered Coherent Function : $\Gamma_N^{(m,n)}\ \mathrm{Tr}[\rho a^m a^{+n}]$

For $|\alpha\rangle$, $\rho = |\alpha\rangle\langle\alpha|$

Sudarshan Glauber Representation

$$\rho = \int d^2\alpha\, P(\alpha)\, |\alpha\rangle\langle\alpha|,\ \int d^2\alpha\, P(\alpha) = 1,\quad P(\alpha) \text{ -real}$$

gives

$$\Gamma_N = \int d^2\alpha\, P(\alpha)\, \alpha^{*m} \alpha^{*n}$$

Since most experiments involve Γ_N and not Γ_A ,

comparing this with classical result

$$\langle v^* v \rangle = \int d^2 v\, f(v)\, v^{*m}\, v^n$$

where $f(v)\, d^2 v$ is probability for signal to lie between v and $v + dv$.

Shows that if $P(\alpha)$ is non-negative definite, it can be considered to be $f(v)$ and classical interpretation is possible.

However if $P(\alpha)$ is not non-negative definite, results can be outside classical optics.

These are called NON-CLASSICAL FEATURES.

ENTANGLEMENT OF TWO-QUBITS

Coherent state $|\alpha\rangle$ - closest to classical stable light

Superposition of coherent states -

$$|\psi\rangle = a|\alpha\rangle + b|\beta\rangle$$ - Non-classical

Simple examples-

$|\mathrm{EVEN},\alpha\rangle \sim |\alpha\rangle + |-\alpha\rangle$ contains $|n\rangle$ only with n even

$|\mathrm{ODD},\alpha\rangle \sim |\alpha\rangle - |-\alpha\rangle$ contains $|n\rangle$ only with n odd

$|\mathrm{EVEN},\alpha\rangle$ exhibits squeezing

$|\mathrm{ODD},\alpha\rangle$ exhibits antibunching

Similarity in Quantum Information:

If two-qubit pure state can be written as product

$$|\psi\rangle = |\psi\rangle_1 |\psi\rangle_2$$

It is separable but if it cannot be written in this form in any basis it exhibites the non-classical feature, entanglement.

CONCEPT OF SQUEEZING

In quantum optics, if we write

$$a = X_1 + iX_2$$

relation $[X_1, X_2] = \frac{1}{2}$ gives

$$\langle (\Delta X_1)^2 \rangle \langle (\Delta X_2)^2 \rangle = \frac{1}{16}$$

Since in P- representation

$$\langle (\Delta X_1)^2 \rangle = \frac{1}{4} + \int d^2\alpha \, P(\alpha) \, \alpha_r^2$$

$$\langle (\Delta X_2)^2 \rangle = \frac{1}{4} + \int d^2\alpha \, P(\alpha) \, \alpha_i^2$$

For classical fields $\langle (\Delta X_1)^2 \rangle$ and $\langle (\Delta X_2)^2 \rangle$ are separately $> \frac{1}{4}$.

Quantum considerations demand their product $> \frac{1}{16}$ and hence there are states for which no classical interpretation is possible. These states are called squeezed states in Quantum Optics. Both X_1 and X_2 cannot be squeezed.

GENERAL SQUEEZED STATES & ATOMIC or SPIN or POLARIZATION SQUEEZED STATES

If $ae^{i\theta} = X_{1\theta} + iX_{2\theta}$ and we look for condition of squeezing of say $X_{1\theta}$ for any possible θ we obtain

$$\left|\left\langle a^2\right\rangle - \left\langle a\right\rangle^2\right| > \left\langle a^+ a\right\rangle \quad .$$

Somewhat parallel situation for Dicke's collective operators or spin operator or Stoke's operator leads to Atomic or Spin or Polarization Squeezing.

For operators $S_{1,2,3}$,

$$[S_1, S_2] = iS_3 \quad , \quad [S_2, S_3] = iS_1 \quad , \quad [S_3, S_1] = iS_2$$

First relation gives

$$\left\langle (\Delta S_1)^2\right\rangle \left\langle (\Delta S_2)^2\right\rangle \geq \frac{1}{4}\left|\left\langle S_3\right\rangle\right|^2 \quad .$$

Hence Walls and Zoller defined squeezing if

$$\left\langle (\Delta S_1)^2\right\rangle < \frac{1}{2}\left|\left\langle S_3\right\rangle\right| \text{ or } \left\langle (\Delta S_2)^2\right\rangle < \frac{1}{2}\left|\left\langle S_3\right\rangle\right|$$

and similarly for the other two relations.

Thus S_1 is said to be squeezed if

$$\left\langle (\Delta S_1)^2 \right\rangle < \frac{1}{2}\left|\left\langle S_2 \right\rangle\right| \text{ or } \frac{1}{2}\left|\left\langle S_3 \right\rangle\right|$$

<u>Generalized definition of Atomic or Spin or Polarization squeezing</u>

This has been generalized by a number of authors. Rakesh Kumar and me. We considered

$$S_\theta = S_x \cos\theta + S_y \sin\theta \quad \text{and} \quad S_{\theta+\frac{\pi}{2}} = -S_x \cos\theta + S_y \sin\theta$$

Since $[S_\theta, S_{\theta+\frac{\pi}{2}}] = 2iS_z$, condition for squeezing of S_z is

$$\left\langle (\Delta S_z)^2 \right\rangle < 2\left|\left\langle S_x \right\rangle \cos\theta + \left\langle S_y \right\rangle \sin\theta\right|$$

Since maximum of RHS is $(\langle S_x \rangle^2 + \langle S_y \rangle^2)^{1/2}$ the squeezing condition becomes

$$\left\langle (\Delta S_z)^2 \right\rangle < 2(\langle S_x \rangle^2 + \langle S_y \rangle^2)^{1/2}$$

or for a general component

$$\left\langle (\Delta S_n)^2 \right\rangle < 2(\langle \overline{S} \rangle^2 - \langle S_{\hat{n}} \rangle^2)^{1/2}$$

This definition has been used to study polarization squeezing by R. Prakash & N. Shukla.

SPIN SQUEEZING OF ALL SPIN COMPONENTS

Usual impression is that two orthogonal components of spin cannot be squeezed just as in quantum optics, both X_1 and X_2 or q and p cannot be squeezed. Simple calculation shows that all 3 orthogonal components can be squeezed.

For state $|00\rangle$ and for a given direction $\hat{n} = (\theta, \phi, 1)$

$$\langle S_n \rangle = \cos\theta \quad ; \quad \langle S_n^2 \rangle = \cos^2\theta + \tfrac{1}{2}\sin^2\theta\cos^2\phi$$

Hence

$$\langle (\Delta S_n)^2 \rangle < \frac{1}{2}\sin^2\theta\cos^2\phi \ , \quad |\langle \Delta S_{n\perp} \rangle| = \sin\theta .$$

Obviously

$$\langle (\Delta S_n)^2 \rangle < \frac{1}{2}|\langle \Delta S_{n\perp} \rangle|$$

and only general component (except equatorial components, $\theta = 0$) are squeezed.

Coherent states, Superposed Coherent States and Entangled Coherent States

Coherent states are eigenstates of photon annihilation operator a

$$a|\alpha\rangle = \alpha|\alpha\rangle,\ |\alpha\rangle = \sum_n e^{-\frac{1}{2}|\alpha|^2} \frac{\alpha^n}{\sqrt{n!}}|n\rangle,$$

The commonly considered superposed coherent states are single mode states

$$|I\rangle = \varepsilon_+|\alpha\rangle + \varepsilon_-|-\alpha\rangle$$

$$= A_+|\mathrm{EVEN},\alpha\rangle + A_-|\mathrm{ODD},\alpha\rangle$$

where

$$|\mathrm{EVEN},\alpha\rangle = \frac{|\alpha\rangle + |-\alpha\rangle}{\sqrt{2(1+x^2)}} = \sum_{n=0}^{\infty} \sqrt{\frac{2x}{(1+x^2)}} \frac{\alpha^{2n}}{2n!}|2n\rangle, \quad x \equiv e^{-|\alpha|^2}$$

$$|\mathrm{ODD},\alpha\rangle = \frac{|\alpha\rangle - |-\alpha\rangle}{\sqrt{2(1-x^2)}} = \sum_{n=0}^{\infty} \sqrt{\frac{2x}{(1-x^2)}} \frac{\alpha^{2n+1}}{2n+1!}|2n+1\rangle.$$

To abbreviate we write these states as $|\pm\rangle$ whenever there is no chance of confusion.

The coefficients satisfy

$$\varepsilon_{\pm} = \frac{A_{+}}{\sqrt{2(1+x^2)}} \pm \frac{A_{-}}{\sqrt{2(1-x^2)}} \quad \text{or} \quad A_{\pm} = (\varepsilon_{+} \pm \varepsilon_{-})\sqrt{\frac{1 \pm x^4}{2}}.$$

$$|A_{+}|^2 + |A_{-}|^2 = |\varepsilon_{+}|^2 + |\varepsilon_{-}|^2 + x^2(\varepsilon_{+}\varepsilon_{-} + \varepsilon_{-}\varepsilon_{+}) = 1$$

Since $\langle -\alpha | \alpha \rangle = \mathrm{e}^{-2|\alpha|^2} \neq 0$, states $|\pm\alpha\rangle$ are not orthogonal and do not form a useful basis. To cope with this problem, we use the basis of even and odd coherent states.

The state is represented on Bloch sphere by angles θ and φ defined by

$$A_{+} = \cos\tfrac{\theta}{2}, A_{-} = \sin\tfrac{\theta}{2} e^{i\varphi}$$

Entanglement and Concurrence has already been introduced by earlier speakers, and for pure and mixed states we write

$$C(\psi)_{AB} = \left| \langle \psi | \sigma_y \otimes \sigma_y | \psi^* \rangle \right| = \left| \langle \psi | \tilde{\psi} \rangle \right|$$

$$C(\rho_{AB}) = \max\{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\}$$

It is worthwhile to investigate what happens if a non-maximally state is taken as the resource. The common belief is a reduction in fidelity and that happens for the case of atomic qubits. Verma and the speaker investigated this problem in detail for pure non-maximally entangled sates (Q. Inf. Proc. 10 (2011) 1951-59).

We found that for a general non-maximally entangled resource information state dependent fidelity is obtained and that the Minimum Assured Fidelity defined a the minimum fidelity for any possible information state is given by

$$\mathrm{MASFI} = \frac{2\mathrm{C}}{1+\mathrm{C}}$$

Here C is concurrence.

We also considered the question of identifying the parameter which gives best judgement about the quality of an imperfect teleportation and concluded that MASFI is better than concurrence or Minimum Average Fidelity.

Teleportation of Superposed Coherent State Using Non-Maximally Entangled Resource

Hirota et al [Phys. Rev. A **64,** 022313 **(**2001**)**]: Showed how to teleport a SCS encoded with one qubit using ECS with success probability equal to 0.5.

Wang [Phys. Rev. A **64**, 022302 (2001)]: Showed how to teleport a bipartite ECS encoded with one qubit using ECS with success probability equal to 0.5.

Prakash et al [Phys. Rev. A 75, 044305 (2007)]: Modified the photon counting scheme and reported almost perfect teleportation for an appreciable mean photon number.

Many other schemes proposed the teleportation of SCS using ECS. However most of the schemes used maximally entangled coherent state (MECS) as quantum channel.

$$\left|E\right\rangle_{1,2} \sim \left[\left|\alpha,\alpha\right\rangle - \left|-\alpha,-\alpha\right\rangle\right]_{1,2}$$

We consider more practical problem of teleporting SCS using non-maximally entangled coherent state (NMECS) and study the effect of entanglement on the quality of teleportation.

For teleportation we use the bipartite ECS,

$$|E\rangle_{1,2} = N\left[\cos\frac{\theta}{2}|\alpha,\alpha\rangle + \sin\frac{\theta}{2}e^{i\varphi}|-\alpha,-\alpha\rangle\right]_{1,2}$$

$$\theta \in [0,\pi], \varphi \in [0,2\pi)$$

$$N = (1 + x^4 \sin\theta\cos\varphi)^{-1/2} \quad x \equiv \exp(-|\alpha|^2)$$

Glauber coherent states 180^0 out of phase:

$$|\pm\alpha\rangle = e^{-\frac{1}{2}|\alpha|^2}\sum_{n=0}^{\infty}\frac{(\pm\alpha)^n}{\sqrt{n!}}|n\rangle$$

Even coherent state (state with even number of photons) and Odd coherent state (state with odd number of photons):

$$\left|\pm\right\rangle = \frac{1}{\sqrt{2(1\pm x^2)}}[\left|\alpha\right\rangle \pm \left|-\alpha\right\rangle]$$

Coherent states in terms of Even and Odd coherent states:

$$\left|\pm\alpha\right\rangle = \tfrac{1}{\sqrt{2}}[\sqrt{(1+x^2)}\left|+\right\rangle \pm \sqrt{(1-x^2)}\left|-\right\rangle]$$

Entangled coherent state in terms of Even and Odd coherent states:

$$\left|E\right\rangle_{1,2} = \tfrac{N}{2}[C_+(1+x^2)\left|+,+\right\rangle + C_-\sqrt{(1-x^4)}(\left|+,-\right\rangle + \left|-,+\right\rangle)$$

$$+ C_+(1-x^2)\left|-,-\right\rangle].$$

$$C_\pm = \cos\frac{\theta}{2} \pm \sin\frac{\theta}{2}e^{i\varphi}$$

Concurrence [Wootters, Phys. Rev. Lett. 80, 2245 (1998)] of a pure bipartite state is given by relation,

$$C = \left|\langle \psi | \tilde{\psi} \rangle\right| \qquad |\tilde{\psi}\rangle = \sigma_y |\psi^*\rangle$$

Concurrence of ECS is

$$C = \left|\frac{(1-x^4)\sin\theta\cos\varphi}{(1+x^4\sin\theta\cos\varphi)}\right|$$

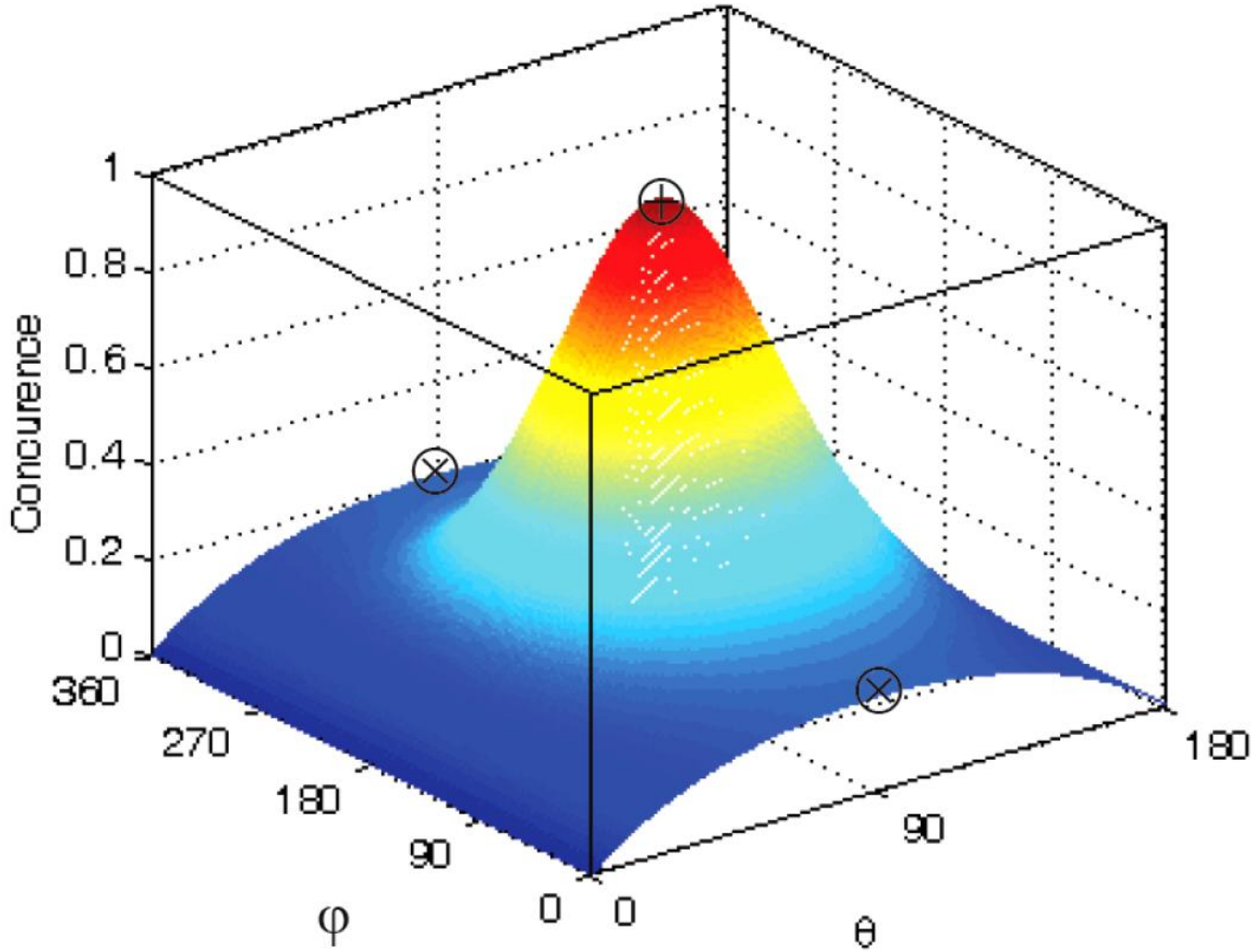
For $\theta = \pi/2 \;\; \varphi = \pi$: $\left|E_{(\theta=\pi/2,\varphi=\pi)}\right\rangle_{1,2} = \frac{1}{\sqrt{2}}[|+,-\rangle + |-,+\rangle]_{1,2}$ and C = 1.

This MECS was used in most of the previously proposed schemes for teleportation of SCS.

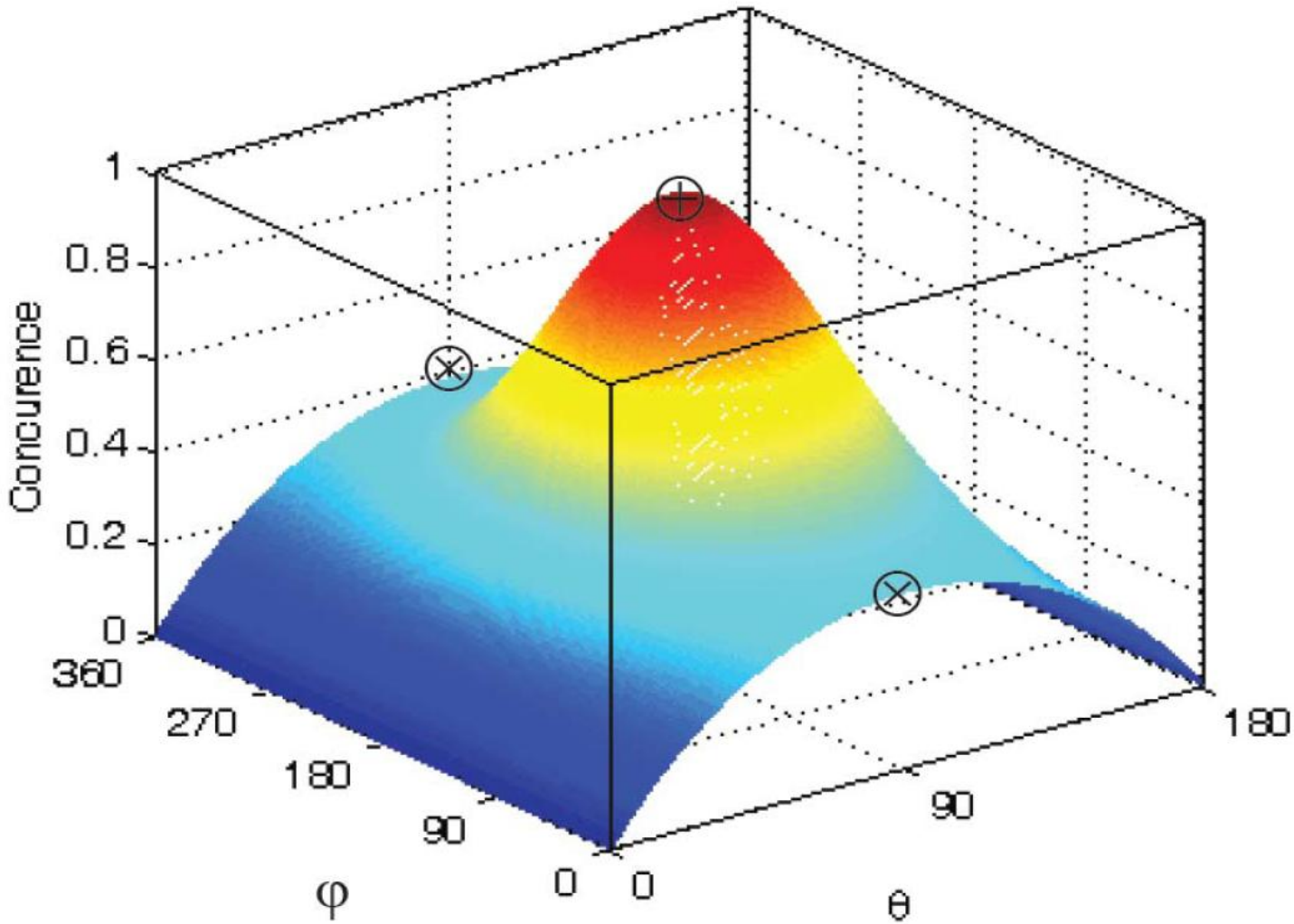
For $\theta = \pi/2 \;\; \varphi = 0 \text{ or } 2\pi$

$$\left|E_{(\theta=\pi/2,\varphi=0,2\pi)}\right\rangle_{1,2} = \frac{1}{\sqrt{2(1+x^4)}}[(1+x^2)|+,+\rangle + (1-x^2)|-,-\rangle]_{1,2}$$

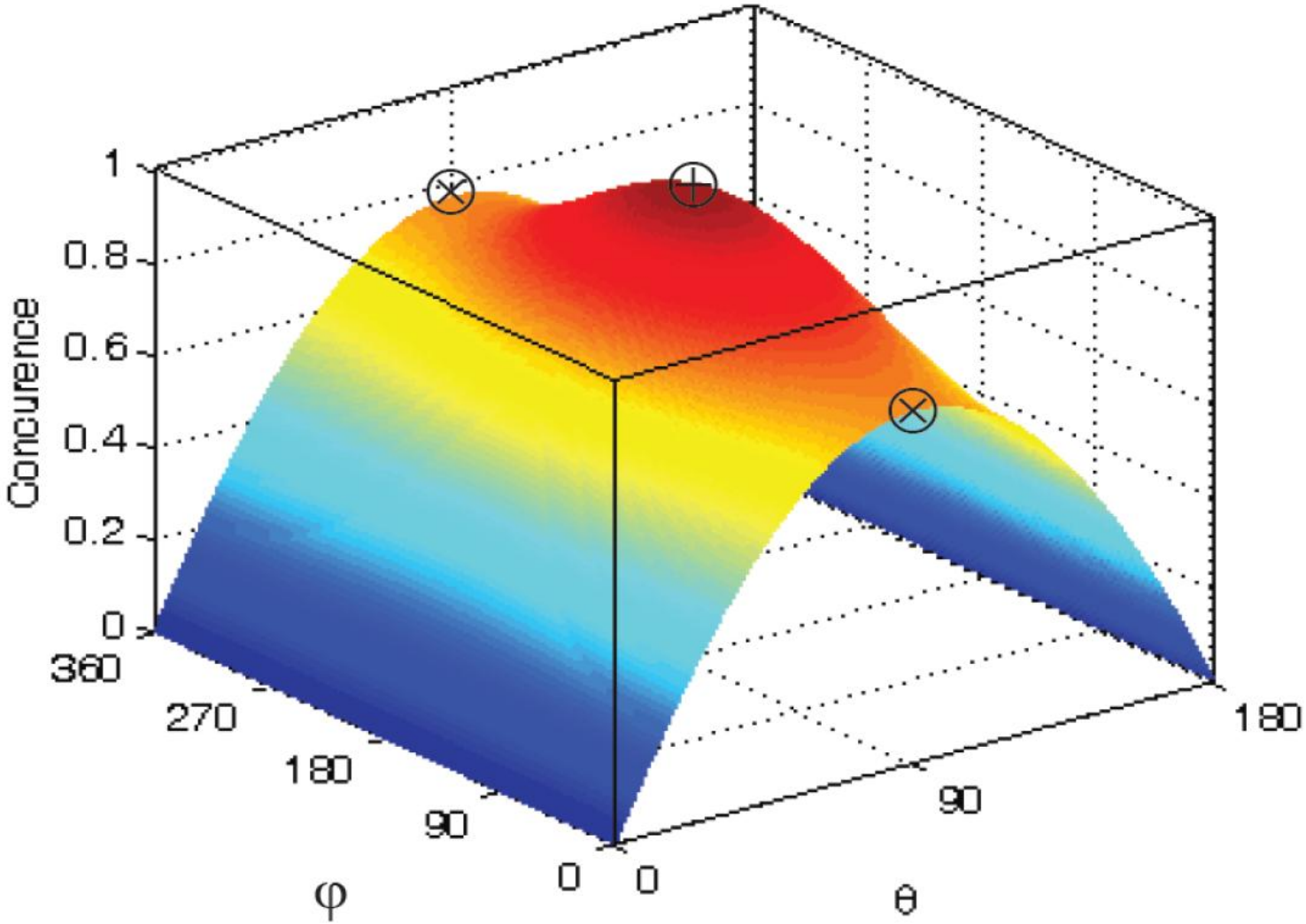
NMECS with $C = (1-x^4)(1+x^4)^{-1}$



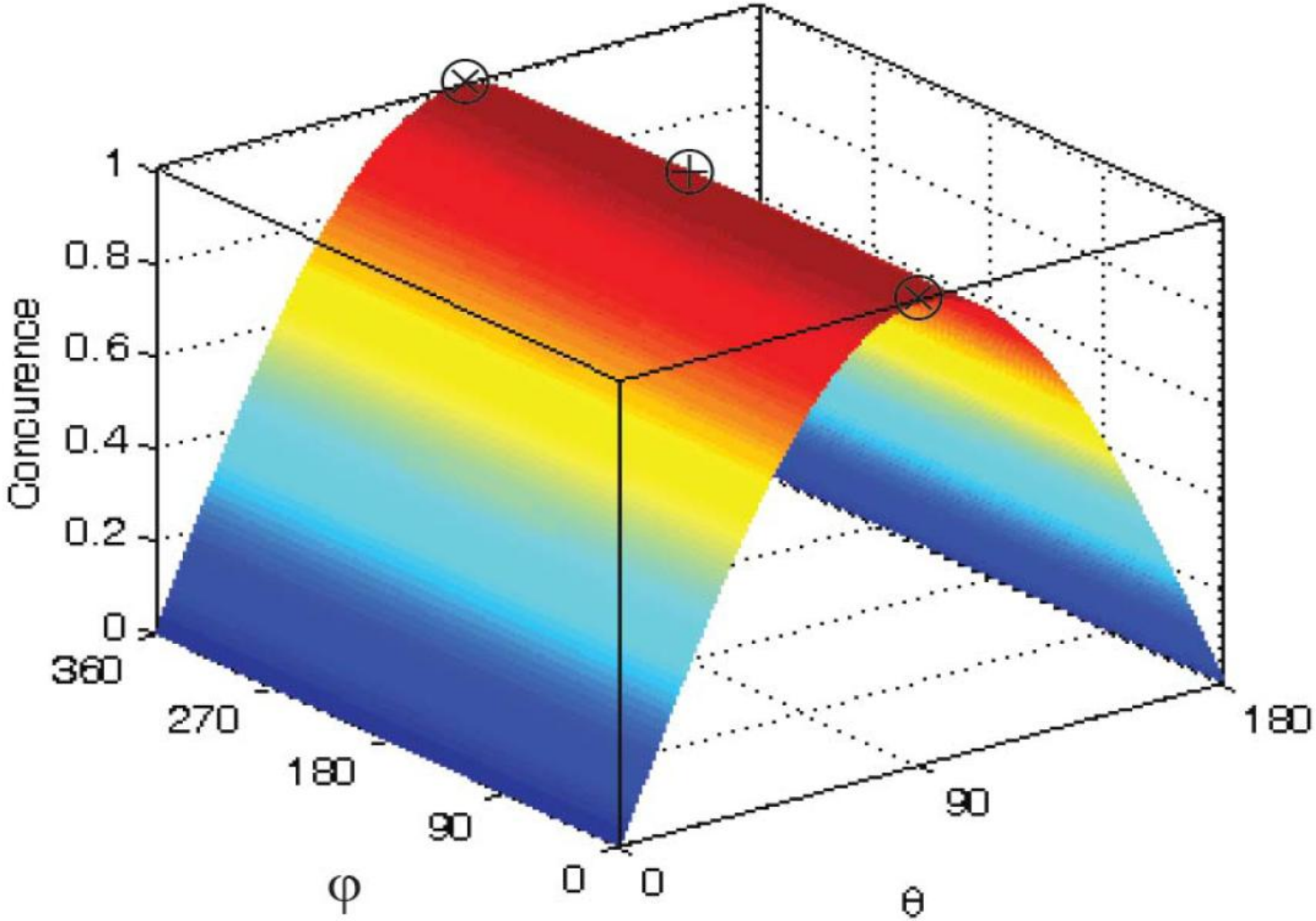
(a) $|\alpha|^2 = 0.1$



(b) $|\alpha|^2 = 0.2$



(c) $|\alpha|^2 = 0.5$



(d) $|\alpha|^2 = 1.5$

$$\left| E_{(\theta=\pi/2,\varphi=\pi)} \right\rangle_{1,2}$$

Maximally entangled coherent state (MECS), which has been extensively used as quantum channel for teleportation of SCS in many previously proposed schemes.

$$\left| E_{(\theta=\pi/2,\varphi=0,2\pi)} \right\rangle_{1,2}$$

Non-maximally entangled coherent state (NMECS) of which entanglement depends on the mean photon number which is of the order of $|\alpha|^2$.

In previous figures, '$\oplus$' sign represents the MECS with unit concurrence and two '$\otimes$' signs represents a particular NMECS with concurrence lesser than unit for low value of $|\alpha|^2$.

However, we see that concurrence of particular NMECS becomes almost equal to unity for appreciable value of $|\alpha|^2$. Since for low values of $|\alpha|^2$, ECS is a NMECS except at points represented by '$\oplus$', therefore it will be interesting to study how the quality of teleportation of SCS is affected by the amount of entanglement contained in ECS.

Let Alice desire to teleport information state (a SCS),

$$|I\rangle_0 = [\varepsilon_+|\alpha\rangle + \varepsilon_-|-\alpha\rangle]_0$$

Complex coefficients $\varepsilon_\pm$ satisfy the normalization condition,

$$[|\varepsilon_+|^2 + |\varepsilon_-|^2 + 2x^2\,\mathrm{Re}(\varepsilon_+^*\varepsilon_-)] = 1$$

In terms of Even and Odd coherent state information state is

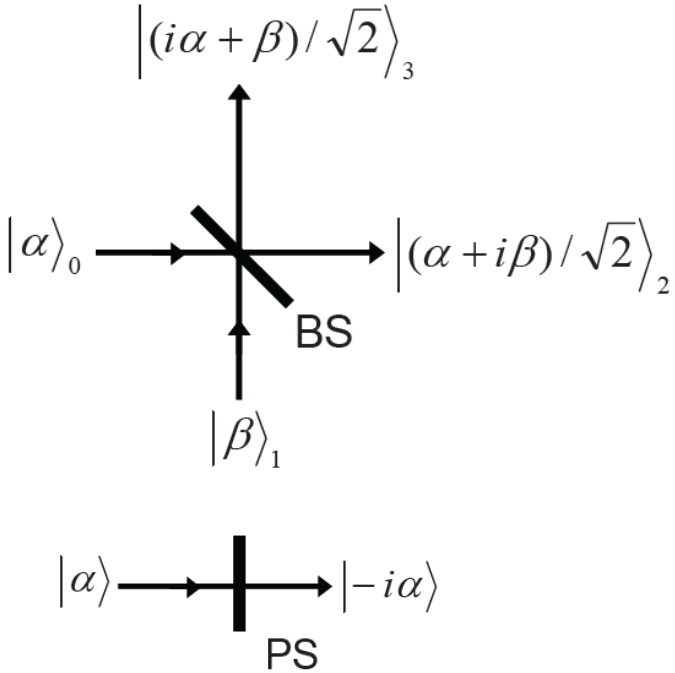
$$|I\rangle_0 = [A_+|+\rangle + A_-|-\rangle]_0 \qquad |A_+|^2 + |A_-|^2 = 1$$

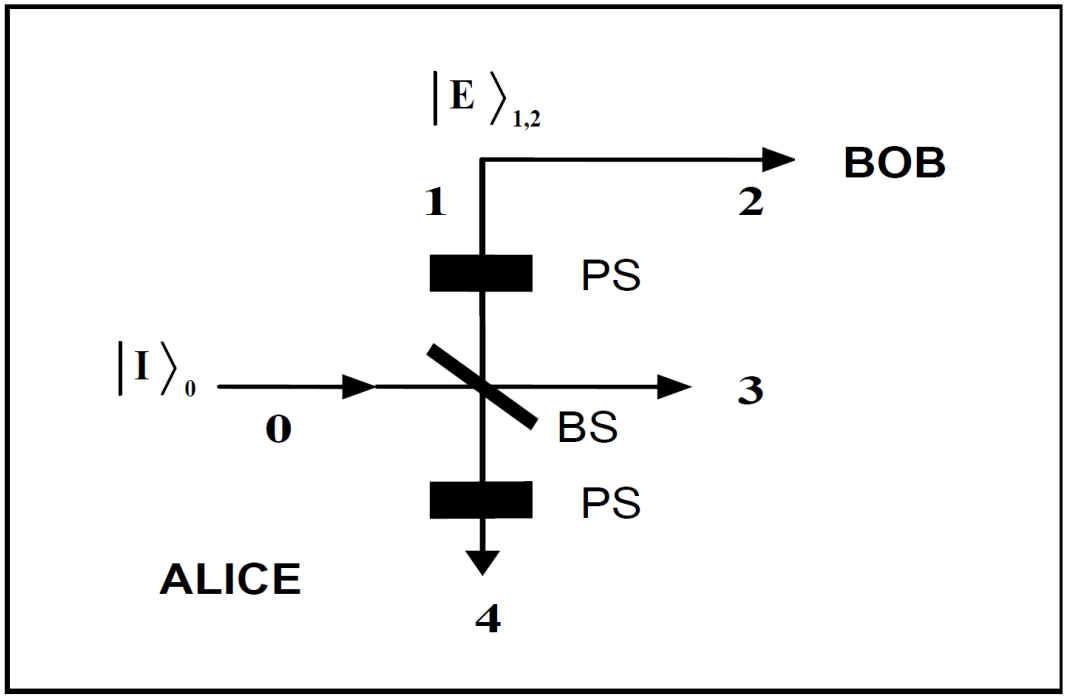
The interrelationship between coefficients $A_\pm$ and $\varepsilon_\pm$ are given by

$$A_\pm = \sqrt{\frac{(1\pm x^2)}{2}}(\varepsilon_+ \pm \varepsilon_-) \qquad \varepsilon_\pm = \frac{A_+}{\sqrt{2(1+x^2)}} \pm \frac{A_-}{\sqrt{2(1-x^2)}}$$

Quantum Channel shared by Alice and Bob

$$|E\rangle_{1,2} = N\left[\cos\frac{\theta}{2}|\alpha,\alpha\rangle + \sin\frac{\theta}{2}e^{i\varphi}|-\alpha,-\alpha\rangle\right]_{1,2} \quad \theta\in[0,\pi], \varphi\in[0,2\pi)$$





Shows the effect of 50:50 symmetric beam splitter (BS) and $-\pi/2$ phase shifter (PS) on a coherent state.

Shows the teleportation scheme.

Information modes 0 and entangled mode 1 are with Alice (sender) while entangled mode 2 is with Bob (receiver).

Initial joint state of the system is

$$\left|\psi\right\rangle_{0,1,2} = \left|I\right\rangle_0 \left|E\right\rangle_{12} = N[\varepsilon_+\{\cos\frac{\theta}{2}\left|\alpha,\alpha,\alpha\right\rangle + \sin\frac{\theta}{2}e^{i\varphi}\left|\alpha,-\alpha,-\alpha\right\rangle\}$$

$$+\varepsilon_-\{\cos\frac{\theta}{2}\left|-\alpha,\alpha,\alpha\right\rangle + \sin\frac{\theta}{2}e^{i\varphi}\left|-\alpha,-\alpha,-\alpha\right\rangle\}]_{0,1,2}\,.$$

The final output state is found to be

$$\left|\psi\right\rangle_{3,4,2} = N[\varepsilon_+\{\cos\frac{\theta}{2}\left|\sqrt{2}\alpha,0,\alpha\right\rangle + \sin\frac{\theta}{2}e^{i\varphi}\left|0,\sqrt{2}\alpha,-\alpha\right\rangle\}$$

$$+\varepsilon_-\{\cos\frac{\theta}{2}\left|0,-\sqrt{2}\alpha,\alpha\right\rangle + \sin\frac{\theta}{2}e^{i\varphi}\left|-\sqrt{2}\alpha,0,-\alpha\right\rangle\}]_{3,4,2}\,.$$

Alice performs photon counting (PC) in modes 3 and 4.

It is clear that one of the counts is always zero.

For better treatment of all possible PC results, we expand the coherent states on Alice side into zero-photon state (the vacuum state), state with nonzero even numbers of photons and state with odd numbers of photons as

$$\left|\pm\sqrt{2}\alpha\right\rangle = x\left|0\right\rangle + \tfrac{1}{\sqrt{2}}(1-x^2)\left|NZE,\sqrt{2}\alpha\right\rangle \pm \sqrt{\tfrac{1}{2}(1-x^4)}\left|ODD,\sqrt{2}\alpha\right\rangle$$

where $$\left|NZE,\sqrt{2}\alpha\right\rangle = \frac{1}{\sqrt{2}(1-x^2)}[(\left|\sqrt{2}\alpha\right\rangle + \left|-\sqrt{2}\alpha\right\rangle) - 2x\left|0\right\rangle]$$

$$\left|ODD,\sqrt{2}\alpha\right\rangle = \frac{1}{\sqrt{2(1-x^4)}}(\left|\sqrt{2}\alpha\right\rangle - \left|-\sqrt{2}\alpha\right\rangle)$$

And expanding coherent state on Bob side in terms of Even and Odd coherent state

$$\left|\pm\right\rangle = \frac{1}{\sqrt{2(1\pm x^2)}}[\left|\alpha\right\rangle \pm \left|-\alpha\right\rangle]$$

This gives

$$\left|\psi\right\rangle_{3,4,2} = \tfrac{N}{\sqrt{2}}\Big[\left|0,0\right\rangle_{3,4} x(\varepsilon_+ + \varepsilon_-)\{p^{-1}C_+\left|+\right\rangle + q^{-1}C_-\left|-\right\rangle\}_2$$

$$+\tfrac{1}{2}\{q^{-2}\left|NZE,0\right\rangle_{3,4}\{p^{-1}(C_+A_+p + C_-A_-q)\left|+\right\rangle + q^{-1}(C_-A_+p + C_+A_-q)\left|-\right\rangle\}_2$$

$$+q^{-2}\left|0,NZE\right\rangle_{3,4}\{p^{-1}(C_+A_+p - C_-A_-q)\left|+\right\rangle + q^{-1}(C_-A_+p - C_+A_-q)\left|-\right\rangle\}_2$$

$$+(pq)^{-1}\left|ODD,0\right\rangle_{3,4}\{p^{-1}(C_-A_+p + C_+A_-q)\left|+\right\rangle + q^{-1}(C_+A_+p + C_-A_-q)\left|-\right\rangle\}_2$$

$$+(pq)^{-1}\left|0,ODD\right\rangle_{3,4}\{p^{-1}(C_-A_+p - C_+A_-q)\left|+\right\rangle + q^{-1}(C_+A_+p - C_-A_-q)\left|-\right\rangle\}_2\}\Big].$$

$$p \equiv (1+x^2)^{-1/2} \qquad q \equiv (1-x^2)^{-1/2} \qquad C_\pm = \cos\frac{\theta}{2} \pm \sin\frac{\theta}{2}e^{i\varphi}$$

There are five possible photon counting results:

I (*modes 3 and 4 count zero number of photon*),
II (*mode 3 count zero and mode 4 count non-zero even numbers of photon*),
III (*mode 3 count non-zero even and mode 4 count zero numbers of photon*),
IV (*mode 3 count odd and mode 4 count zero numbers of photon*) and
V (*mode 3 count zero and mode 4 count odd numbers of photon*).

Residual states with Bob in mode 2 after measurement corresponding to each PC results are

$$\left|B_I\right\rangle_2 \sim \{p^{-1}C_+\left|+\right\rangle + q^{-1}C_-\left|-\right\rangle\}_2,$$

$$\left|B_{II}\right\rangle_2 \sim \{p^{-1}(C_+A_+p + C_-A_-q)\left|+\right\rangle + q^{-1}(C_-A_+p + C_+A_-q)\left|-\right\rangle\}_2,$$

$$\left|B_{III}\right\rangle_2 \sim \{p^{-1}(C_+A_+p - C_-A_-q)\left|+\right\rangle + q^{-1}(C_-A_+p - C_+A_-q)\left|-\right\rangle\}_2,$$

$$\left|B_{IV}\right\rangle_2 \sim \{p^{-1}(C_-A_+p + C_+A_-q)\left|+\right\rangle + q^{-1}(C_+A_+p + C_-A_-q)\left|-\right\rangle\}_2,$$

$$\left|B_V\right\rangle_2 \sim \{p^{-1}(C_-A_+p - C_+A_-q)\left|+\right\rangle + q^{-1}(C_+A_+p - C_-A_-q)\left|-\right\rangle\}_2\}.$$

The residual state with Bob contains complex factors $C_{\pm}$ that depend on entanglement parameters (θ and φ), therefore required unitary operation should depend upon entanglement parameters to recover the replica of information state with as large fidelity as possible.

Strategy 1: For $|C_+| \leq |C_-|$ (i.e., for $\cos\varphi \geq 0$), Bob performs the following unitary operations:

$$U_I = U_{II} = I, \quad U_{III} = |+\rangle\langle+| - |-\rangle\langle-|,$$

$$U_{IV} = |+\rangle\langle-| + |-\rangle\langle+|, \quad U_V = |+\rangle\langle-| - |-\rangle\langle+|.$$

Applying these unitary operations on Bob's state, the teleported states with Bob are given by

$$\left|T_I^{(1)}\right\rangle_2 = N_I\{C_+ p^{-1}\left|+\right\rangle + C_- q^{-1}\left|-\right\rangle\}_2,$$

$$\left|T_{II}^{(1)}\right\rangle_2 = N_{II}\{(C_+ A_+ p + C_- A_- q)p^{-1}\left|+\right\rangle + (C_+ A_- q + C_- A_+ p)q^{-1}\left|-\right\rangle\}_2,$$

$$\left|T_{III}^{(1)}\right\rangle_2 = N_{III}\{(C_+ A_+ p - C_- A_- q)p^{-1}\left|+\right\rangle + (C_+ A_- q - C_- A_+ p)q^{-1}\left|-\right\rangle\}_2,$$

$$\left|T_{IV}^{(1)}\right\rangle_2 = N_{IV}\{(C_+ A_+ p + C_- A_- q)q^{-1}\left|+\right\rangle + (C_+ A_- q + C_- A_+ p)p^{-1}\left|-\right\rangle\}_2,$$

$$\left|T_V^{(1)}\right\rangle_2 = N_V\{(C_+ A_+ p - C_- A_- q)q^{-1}\left|+\right\rangle + (C_+ A_- q - C_- A_+ p)p^{-1}\left|-\right\rangle\}_2,$$

Normalization factors in terms of θ, φ and ω, ξ defined by $A_+ = \cos\frac{\omega}{2}$ and $A_- = \sin\frac{\omega}{2}e^{i\xi}$ are

$$N_I = [2(1 + x^2 \sin\theta\cos\varphi)]^{-1/2},$$

$$N_{II} = [(1+\sin\theta\cos\varphi) + (1-x^4)^{-1}(1-\sin\theta\cos\varphi)(1-2x^2\cos\omega + x^4)$$
$$+ 2(1-x^4)^{-1/2}\sin\omega(\cos\theta\cos\xi + 2x^2\sin\theta\sin\varphi\sin\xi)]^{-1/2},$$

$$N_{III} = [(1+\sin\theta\cos\varphi) + (1-x^4)^{-1}(1-\sin\theta\cos\varphi)(1-2x^2\cos\omega + x^4)$$
$$- 2(1-x^4)^{-1/2}\sin\omega(\cos\theta\cos\xi + 2x^2\sin\theta\sin\varphi\sin\xi)]^{-1/2},$$

$$N_{IV} = [(1-\sin\theta\cos\varphi) + (1-x^4)^{-1}(1+\sin\theta\cos\varphi)(1-2x^2\cos\omega + x^4)$$
$$+ 2(1-x^4)^{-1/2}\sin\omega(\cos\theta\cos\xi - 2x^2\sin\theta\sin\varphi\sin\xi)]^{-1/2},$$

$$N_V = [(1-\sin\theta\cos\varphi) + (1-x^4)^{-1}(1+\sin\theta\cos\varphi)(1-2x^2\cos\omega + x^4)$$
$$- 2(1-x^4)^{-1/2}\sin\omega(\cos\theta\cos\xi - 2x^2\sin\theta\sin\varphi\sin\xi)]^{-1/2}.$$

Putting $A_+ = \cos\frac{\omega}{2}$, $A_- = \sin\frac{\omega}{2}e^{i\xi}$, the fidelity of teleported states are

$$F_I^{(1)} = |N_I|^2(1-x^4)^{-1}[(1-x^2\sin\theta\cos\varphi)+\cos\omega(\sin\theta\cos\varphi - x^2)$$

$$+(1-x^4)^{-1/2}\sin\omega(\cos\theta\cos\xi - 2\sin\theta\sin\varphi\sin\xi)],$$

$$F_{II}^{(1)} = |N_{II}|^2[(1+\sin\theta\cos\varphi)+2(1-x^4)^{-1/2}\sin\omega(\cos\theta\cos\xi + 2x^2\sin\theta\sin\varphi\sin\xi)$$

$$+(1-x^4)^{-1}(1-\sin\theta\cos\varphi)\sin^2\omega(\cos^2\xi - x^4\sin^2\xi)],$$

$$F_{III}^{(1)} = |N_{III}|^2[(1+\sin\theta\cos\varphi)-2(1-x^4)^{-1/2}\sin\omega(\cos\theta\cos\xi + 2x^2\sin\theta\sin\varphi\sin\xi)$$

$$+(1-x^4)^{-1}(1-\sin\theta\cos\varphi)\sin^2\omega(\cos^2\xi - x^4\sin^2\xi)],$$

$$F_{IV}^{(1)} = |N_{IV}|^2[(1-x^4)^{-1}(1+\sin\theta\cos\varphi)(1-x^2\cos\omega)^2 + 2(1-x^4)^{-1/2}$$

$$\times\sin\omega\cos\theta\cos\xi(1-x^2\cos\omega)+(1-\sin\theta\cos\varphi)\sin^2\omega\cos^2\xi],$$

$$F_V^{(1)} = |N_V|^2[(1-x^4)^{-1}(1+\sin\theta\cos\varphi)(1-x^2\cos\omega)^2 - 2(1-x^4)^{-1/2}$$

$$\times\sin\omega\cos\theta\cos\xi(1-x^2\cos\omega)+(1-\sin\theta\cos\varphi)\sin^2\omega\cos^2\xi].$$

Average fidelity $F_{av.} = \sum_{i=I}^{V} F_i P_i$ is given by

$$F_{av}^{(1)} = \tfrac{1}{4} N^2 [2x^2(1+\cos\omega)(1+x^2)^{-1}\{1 + x^2 \sin\theta\cos\varphi + \cos\omega(\sin\theta\cos\varphi + x^2)$$

$$+ (1-x^4)^{1/2} \sin\omega(\cos\theta\cos\xi - \sin\theta\sin\varphi\sin\xi)\} + (1+\sin\theta\cos\varphi)(1-x^2\cos\omega)^2$$

$$+ (1-x^2)^2\{1+\sin\theta\cos\varphi + (1-x^4)^{-1}\sin^2\omega(1-\sin\theta\cos\varphi)(\cos^2\xi + x^4\sin^2\xi)\}$$

$$+ (1-x^4)(1-\sin\theta\cos\varphi)\sin^2\omega\cos^2\xi].$$

The probability of occurrence of different PC results are given by

$$P_I = [x^2(1+\cos\omega)]/[(1+x^2)(1+x^4\sin\theta\cos\varphi)|N_I|^2],$$

$$P_{II} = (1-x^2)^2/[8(1+x^4\sin\theta\cos\varphi)|N_{II}|^2],$$

$$P_{III} = (1-x^2)^2/[8(1+x^4\sin\theta\cos\varphi)|N_{III}|^2],$$

$$P_{IV} = (1-x^4)/[8(1+x^4\sin\theta\cos\varphi)|N_{IV}|^2],$$

$$P_V = (1-x^4)/[8(1+x^4\sin\theta\cos\varphi)|N_V|^2].$$

Strategy 2: For $|C_+| > |C_-|$ (i.e., for $\cos\varphi < 0$), Bob performs the following unitary operations:

$$U_I = U_{IV} = I, \quad U_{II} = |+\rangle\langle -| + |-\rangle\langle +|,$$

$$U_{III} = |+\rangle\langle -| - |-\rangle\langle +|, \quad U_V = |+\rangle\langle +| - |-\rangle\langle -|.$$

Applying these unitary operations on Bob's state, the teleported states with Bob are given by

$$\left|T_I^{(2)}\right\rangle_2 = N_I\{C_+ p^{-1}|+\rangle + C_- q^{-1}|-\rangle\}_2,$$

$$\left|T_{II}^{(2)}\right\rangle_2 = N_{II}\{(C_- A_+ p + C_+ A_- q)q^{-1}|+\rangle + (C_- A_- q + C_+ A_+ p)p^{-1}|-\rangle\}_2,$$

$$\left|T_{III}^{(2)}\right\rangle_2 = N_{III}\{(C_- A_+ p - C_+ A_- q)q^{-1}|+\rangle + (C_- A_- q - C_+ A_+ p)p^{-1}|-\rangle\}_2,$$

$$\left|T_{IV}^{(2)}\right\rangle_2 = N_{IV}\{(C_- A_+ p + C_+ A_- q)p^{-1}|+\rangle + (C_- A_- q + C_+ A_+ p)q^{-1}|-\rangle\}_2,$$

$$\left|T_V^{(2)}\right\rangle_2 = N_V\{(C_- A_+ p - C_+ A_- q)p^{-1}|+\rangle + (C_- A_- q - C_+ A_+ p)q^{-1}|-\rangle\}_2.$$

Putting $A_+ = \cos\frac{\omega}{2}$, $A_- = \sin\frac{\omega}{2} e^{i\xi}$, the fidelity of teleported states are

$$F_I^{(2)} = F_I^{(1)}$$

$$F_{II}^{(2)} = |N_{II}|^2 [(1-x^4)^{-1}(1-\sin\theta\cos\varphi)(1-x^2\cos\omega)^2 + 2(1-x^4)^{-1/2}$$

$$\times \sin\omega\cos\theta\cos\xi(1-x^2\cos\omega) + (1+\sin\theta\cos\varphi)\sin^2\omega\cos^2\xi],$$

$$F_{III}^{(2)} = |N_{III}|^2 [(1-x^4)^{-1}(1-\sin\theta\cos\varphi)(1-x^2\cos\omega)^2 - 2(1-x^4)^{-1/2}$$

$$\times \sin\omega\cos\theta\cos\xi(1-x^2\cos\omega) + (1+\sin\theta\cos\varphi)\sin^2\omega\cos^2\xi],$$

$$F_{IV}^{(2)} = |N_{IV}|^2 [(1-\sin\theta\cos\varphi) + 2(1-x^4)^{-1/2}\sin\omega(\cos\theta\cos\xi - 2x^2\sin\theta\sin\varphi\sin\xi)$$

$$+ (1-x^4)^{-1}(1+\sin\theta\cos\varphi)\sin^2\omega(\cos^2\xi + x^4\sin^2\xi)],$$

$$F_V^{(2)} = |N_V|^2 [(1-\sin\theta\cos\varphi) - 2(1-x^4)^{-1/2}\sin\omega(\cos\theta\cos\xi - 2x^2\sin\theta\sin\varphi\sin\xi)$$

$$+ (1-x^4)^{-1}(1+\sin\theta\cos\varphi)\sin^2\omega(\cos^2\xi + x^4\sin^2\xi)].$$

Average fidelity

$$F_{av}^{(2)} = \tfrac{1}{4} N^2 [2x^2(1+\cos\omega)(1+x^2)^{-1}\{1+x^2\sin\theta\cos\varphi+\cos\omega(\sin\theta\cos\varphi+x^2)$$

$$+(1-x^4)^{1/2}\sin\omega(\cos\theta\cos\xi-\sin\theta\sin\varphi\sin\xi)\}+(1-x^2)^2\{(1-x^4)^{-1}$$

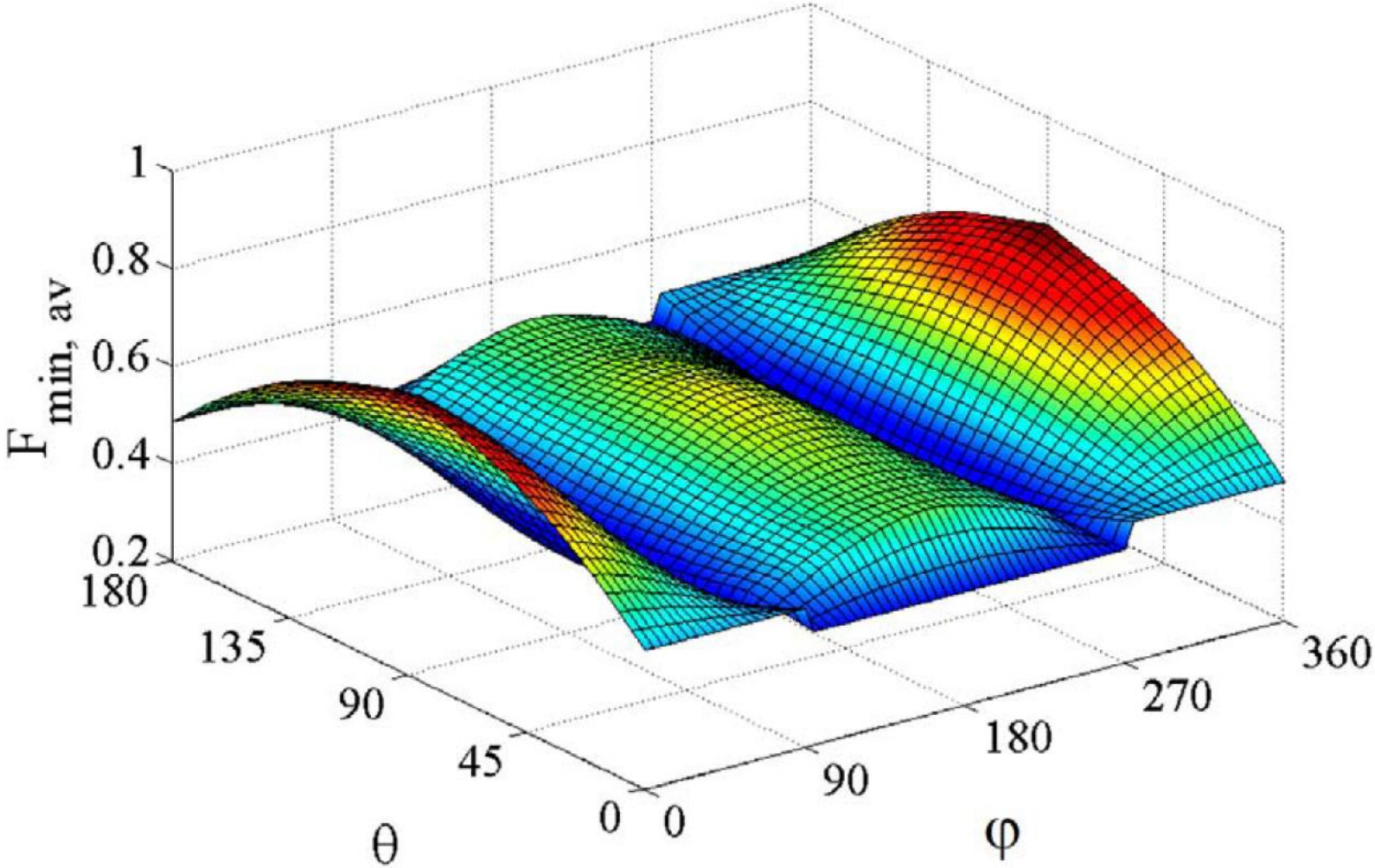
$$\times(1-\sin\theta\cos\varphi)(1-x^2\cos\omega)^2+(1+\sin\theta\cos\varphi)\sin^2\omega\cos^2\xi\}+(1-x^4)$$

$$\times(1-\sin\theta\cos\varphi)+\sin^2\omega(1+\sin\theta\cos\varphi)(\cos^2\xi+x^4\sin^2\xi)].$$

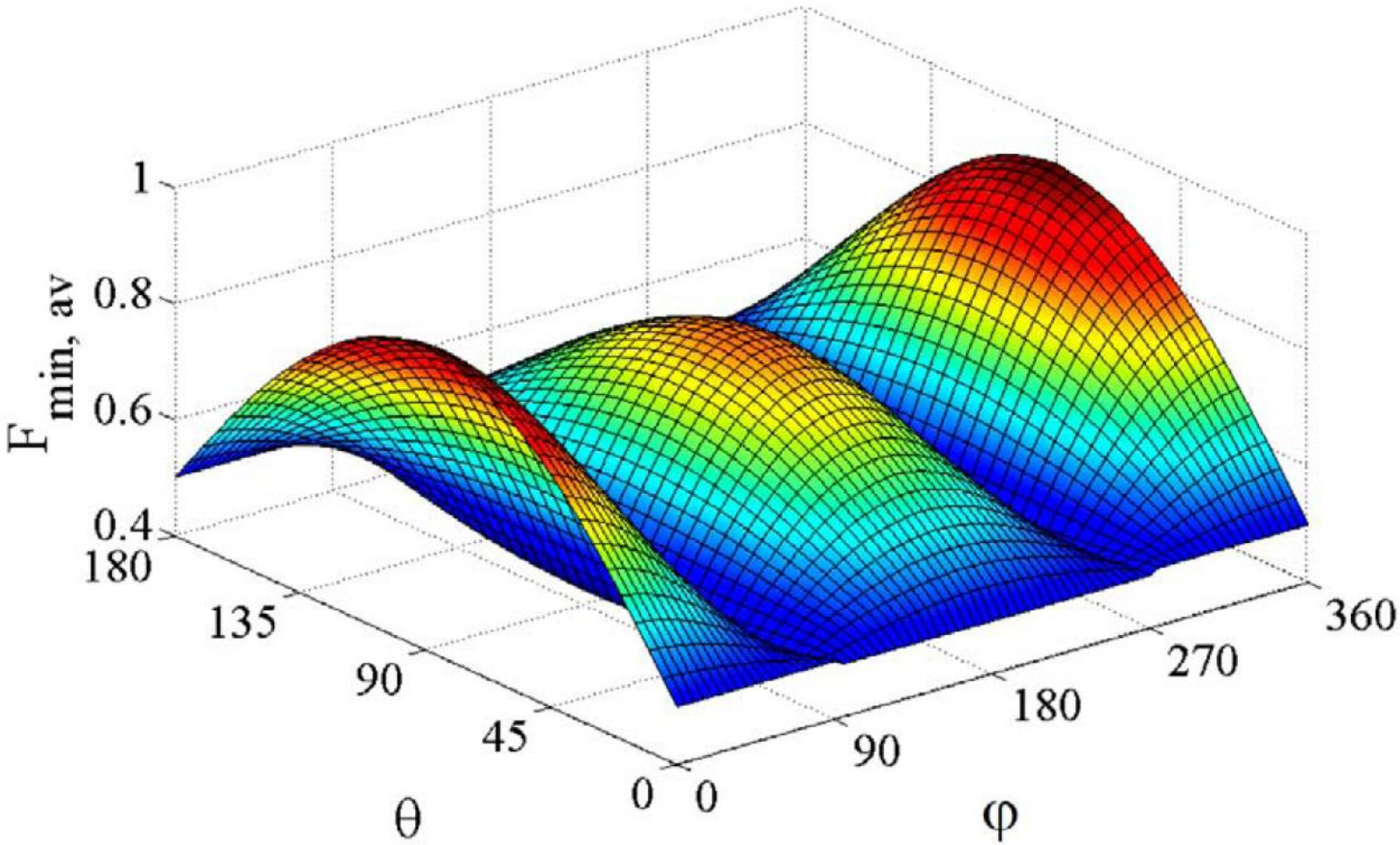
Minimum average fidelity is defined as minimum possible value of average fidelity over all possible information states.

We minimized average fidelity, over all possible information states, i.e., over angles ω and ξ, then plotted it with respect to entanglement parameters θ and φ.

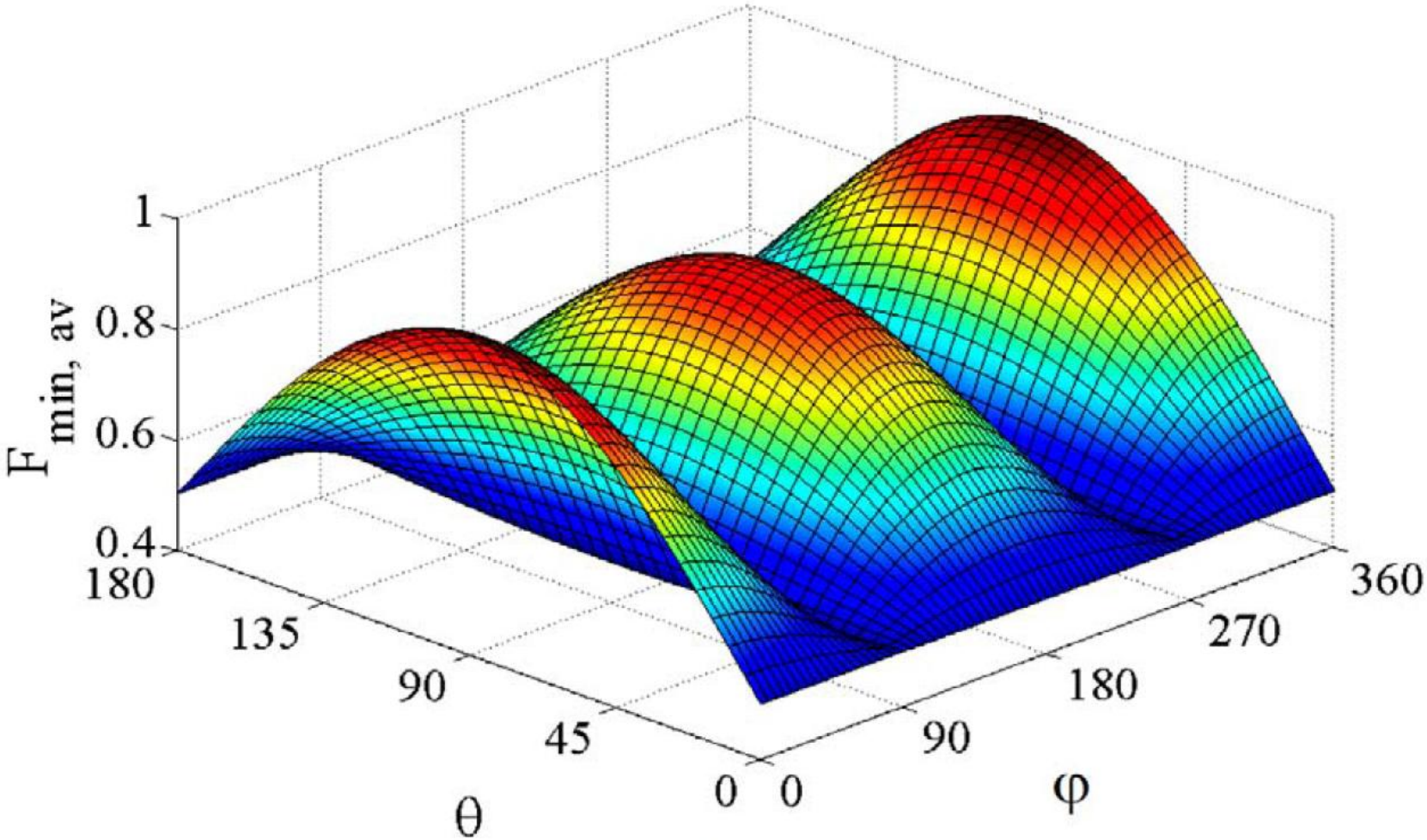
It is to be noted that for the process of minimization, a Mat-Lab code can be written in such a way that it gives minimum value of $F_{av}^{(1)}$ as $F_{\min,av}$ if, $|C_+| \le |C_-|$ otherwise it gives minimum value of $F_{av}^{(2)}$ as $F_{\min,av}$.



(a) $|\alpha|^2 = 0.5$



(b) $|\alpha|^2 = 1$



(c) $|\alpha|^2 = 1.5$

At low $|\alpha|^2$, the values of $F_{\min,av}$ for the two maxima at $\theta = \pi/2$, $\varphi = 0$ & 2π

That corresponds to NMECS $\left| E_{(\theta=\pi/2,\varphi=0,2\pi)} \right\rangle_{1,2}$, is higher than that at $\theta = \pi/2$,

$\varphi = \pi$ which corresponds to MECS $\left| E_{(\theta=\pi/2,\varphi=\pi)} \right\rangle_{1,2}$, used in previously proposed schemes.

Minimum average fidelity for NMECS is,

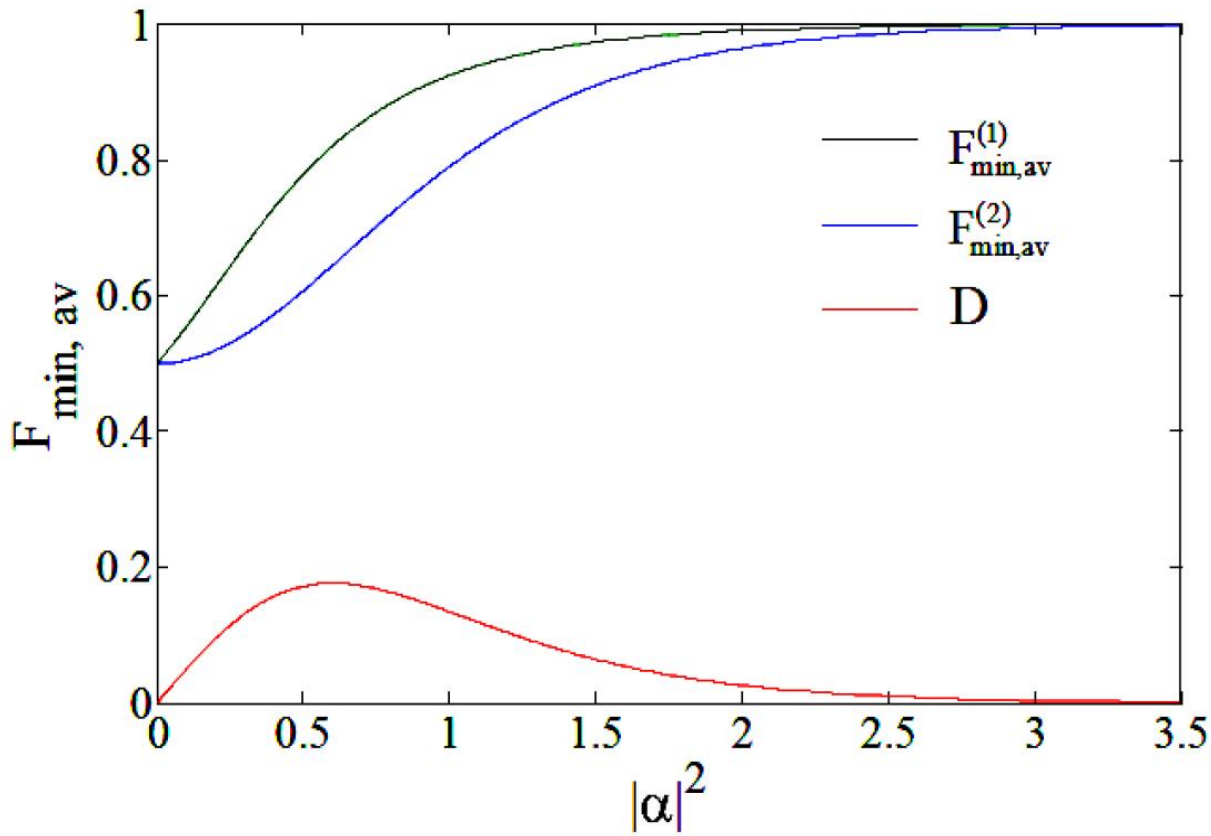
$$F^{(1)}_{\min.,av} = 1 - \frac{x^2(1+x^2)}{2(1+x^4)}$$

Minimum average fidelity for MECS

$$F^{(2)}_{\min,av} = 1 - \frac{2x^2}{(1+x^2)^2}$$

Difference between these two minimum average fidelities is given as

$$D = F^{(1)}_{\min,av} - F^{(2)}_{\min,av} = \frac{x^2(3+x^4)(1-x^2)}{2(1+x^4)(1+x^2)^2}.$$



Shows variation of $F^{(1)}_{\min,av}$ for non-maximally ECS, $F^{(2)}_{\min,av}$ for maximally ECS and difference *D* with respect to mean photon number $|\alpha|^2$.

The maximum difference ≈ 0.17 is at $|\alpha|^2 \approx 0.6$

Teleportation of One Ququat Encoded in Single Mode Superposed Coherent State

Recently, entangled qudits have arrested much more attention than the entangled qubits for their stronger non-locality and capacity of information transmission.

Higher dimensional quantum system (qudits), like qutrit and ququat defined in 3D & 4D Hilbert space, show advantage in secure quantum communication systems and in investigations on foundations of quantum mechanics.

To date single mode SCS have been employed only for encoding a qubit using even and odd coherent state as logical states.

However, various advantages of higher dimensional quantum states, makes it necessary to investigate the possibility of encoding one ququat in single mode SCS, generation of entangled ququats based on coherent states, and methods for quantum teleportation of SCS encoded with one ququat.

Here we show that superposition of four non-orthogonal coherent states $|\pm\alpha\rangle$ and $|\pm i\alpha\rangle$, that are 90° out of phase can be employed for encoding one ququat.

We propose a scheme to generate newly defined orthogonal states $|\alpha_j\rangle$ with $4n+j$ numbers of photons, where, $j=0,1,2,3$.

These multi-photonic states when fall upon a 50-50 beam splitter, the resulting state is a bipartite four component entangled coherent state equivalently represents an entangled ququat.

We also propose a linear optical scheme that gives almost perfect teleportation (minimum average fidelity > 0.99) of single ququat encoded in singlr mode SCS with the aid of entangled ququat based on coherent states and using a 9-bit classical channel with almost perfect success rate.

Define four multi-photonic states as

$$|\alpha_0\rangle = N_0[|\alpha\rangle + |i\alpha\rangle + |-\alpha\rangle + |-i\alpha\rangle]$$
$$|\alpha_1\rangle = N_1[|\alpha\rangle - i|i\alpha\rangle - |-\alpha\rangle + i|-i\alpha\rangle]$$
$$|\alpha_2\rangle = N_2[|\alpha\rangle - |i\alpha\rangle + |-\alpha\rangle - |-i\alpha\rangle]$$
$$|\alpha_3\rangle = N_3[|\alpha\rangle + i|i\alpha\rangle - |-\alpha\rangle - i|-i\alpha\rangle]$$
$$\langle\alpha_j|\alpha_k\rangle = \delta_{jk} \qquad j,k = 0,\ 1,\ 2,\ 3$$

States, $|\alpha_j\rangle$ with $j = 0,\ 1,\ 2,\ 3$ are the multi-photonic states having 4n, 4n+1, 4n+2, and 4n+3, numbers of photons, respectively.

Normalization constants are given by

$$N_0 = [2(1 + x^2 + 2x\cos|\alpha|^2)^{1/2}]^{-1}$$
$$N_1 = [2(1 - x^2 + 2x\sin|\alpha|^2)^{1/2}]^{-1}$$
$$N_2 = [2(1 + x^2 - 2x\cos|\alpha|^2)^{1/2}]^{-1}$$
$$N_3 = [2(1 - x^2 - 2x\sin|\alpha|^2)^{1/2}]^{-1}$$

Since $|\alpha_j\rangle$ are orthogonal, we can easily write $|\pm\alpha\rangle$ and $|\pm i\alpha\rangle$ in terms of $|\alpha_j\rangle$

In terms of states $|\alpha_j\rangle$ coherent states $|\pm\alpha\rangle$ and $|\pm i\alpha\rangle$ can be written as

$$|\pm\alpha\rangle = \tfrac{1}{2}[r_0|\alpha_0\rangle \pm r_1|\alpha_1\rangle + r_2|\alpha_2\rangle \pm r_3|\alpha_3\rangle]$$

$$|\pm i\alpha\rangle = \tfrac{1}{2}[r_0|\alpha_0\rangle \pm ir_1|\alpha_1\rangle - r_2|\alpha_2\rangle \mp ir_3|\alpha_3\rangle]$$

where, $r_j = (2N_j)^{-1}$

Thus, any coherent state defined in an infinite dimensional Hilbert space spanned by photon Fock states, can equivalently be defined in a 4D Hilbert space spanned by the four ortho-normal multi-photonic states, $|\alpha_j\rangle$.

We define four entangled ququat states based on coherent state, say, four bipartite four-component entangled coherent states (BFECS) as

$$|E_0\rangle = N_{E0}[|\alpha,\alpha\rangle + |i\alpha,i\alpha\rangle + |-\alpha,-\alpha\rangle + |-i\alpha,-i\alpha\rangle]$$

$$|E_1\rangle = N_{E1}[|\alpha,\alpha\rangle - i|i\alpha,i\alpha\rangle - |-\alpha,-\alpha\rangle + i|-i\alpha,-i\alpha\rangle]$$

$$|E_2\rangle = N_{E2}[|\alpha,\alpha\rangle - |i\alpha,i\alpha\rangle + |-\alpha,-\alpha\rangle - |-i\alpha,-i\alpha\rangle]$$

$$|E_3\rangle = N_{E3}[|\alpha,\alpha\rangle + i|i\alpha,i\alpha\rangle - |-\alpha,-\alpha\rangle - i|-i\alpha,-i\alpha\rangle]$$

Where

$$N_{E0} = [2(1+x^2+2x\cos 2|\alpha|^2)^{1/2}]^{-1}$$

$$N_{E1} = [2(1-x^2+2x\sin 2|\alpha|^2)^{1/2}]^{-1}$$

$$N_{E2} = [2(1+x^2-2x\cos 2|\alpha|^2)^{1/2}]^{-1}$$

$$N_{E3} = [2(1-x^2-2x\sin 2|\alpha|^2)^{1/2}]^{-1}$$

In terms of orthogonal states $|\alpha_j\rangle$, these BFECS can also be written as

$$|E_0\rangle = N_{E0}[r_0^2|\alpha_0,\alpha_0\rangle + r_1r_3|\alpha_1,\alpha_3\rangle + r_2^2|\alpha_2,\alpha_2\rangle + r_3r_1|\alpha_3,\alpha_1\rangle]$$

$$|E_1\rangle = N_{E1}[r_0r_1|\alpha_0,\alpha_1\rangle + r_1r_0|\alpha_1,\alpha_0\rangle + r_2r_3|\alpha_2,\alpha_3\rangle + r_3r_2|\alpha_3,\alpha_2\rangle]$$

$$|E_2\rangle = N_{E2}[r_0r_2|\alpha_0,\alpha_2\rangle + r_1^2|\alpha_1,\alpha_1\rangle + r_2r_0|\alpha_2,\alpha_0\rangle + r_3^2|\alpha_3,\alpha_3\rangle]$$

$$|E_3\rangle = N_{E3}[r_0r_3|\alpha_0,\alpha_3\rangle + r_1r_2|\alpha_1,\alpha_2\rangle + r_2r_1|\alpha_2,\alpha_1\rangle + r_3r_0|\alpha_3,\alpha_0\rangle]$$

BFECS are the non-maximally entangled ququats.

However, for appreciably large coherent amplitude, i.e., in the limit, $|\alpha| \to \infty$, the coefficients N_{Ej} and r_j, become almost equal to ½ and unity, respectively, therefore BFECS becomes maximally entangled in the limit of large coherent amplitude.

The BFECS generation scheme

Consider two even coherent states in mode 0 and 1, respectively,

$$|+\rangle_0 = N_e[|\alpha\rangle + |-\alpha\rangle]_0 \qquad |+'\rangle_1 = N_e[|-i\alpha\rangle + |i\alpha\rangle]_1$$

Where $N_e = [2(1+x^2)]^{-1/2}$

The initial state of the system is written as

$$|\psi\rangle_{0,1} = |+\rangle_0 |+'\rangle_1 = N_e{}^2[|\alpha,-i\alpha\rangle + |\alpha,i\alpha\rangle + |-\alpha,-i\alpha\rangle + |-\alpha,i\alpha\rangle]_{0,1}$$

Following given scheme which uses a Mach Zehnder se up final output state in modes 4,5 is

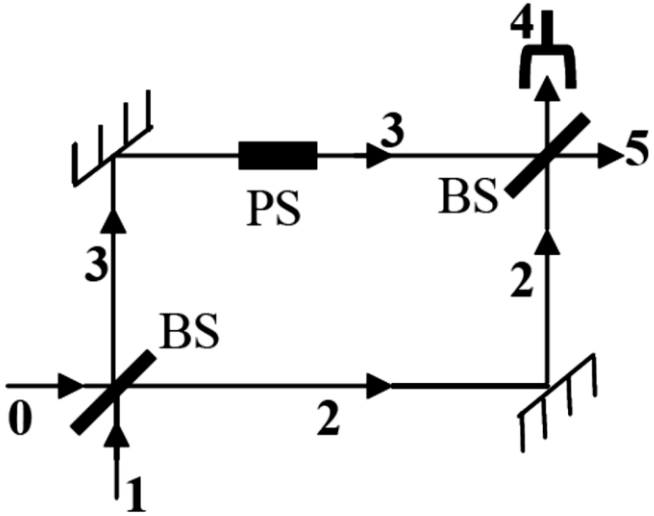
$$|\psi\rangle_{4,5} = N_e^{\,2}[|\alpha, i\alpha\rangle + |i\alpha, \alpha\rangle$$

$$+|-i\alpha, -\alpha\rangle + |-\alpha, -i\alpha\rangle]_{4,5}$$

In terms of states $|\alpha_j\rangle$, this can be written as

$$|\psi\rangle_{4,5} = N_e^{\,2}[r_0^{\,2}|\alpha_0, \alpha_0\rangle + ir_1^{\,2}|\alpha_1, \alpha_1\rangle$$

$$-r_2^{\,2}|\alpha_2, \alpha_2\rangle - ir_3^{\,2}|\alpha_3, \alpha_3\rangle]_{4,5}$$



Scheme for generating BFCECS

It is clear that photon counting (PC) in mode 4 gives four possible PC results: 4n, 4n+1, 4n+2 or 4n+3 numbers of photon count corresponding to which states $|\alpha_0\rangle$,$|\alpha_1\rangle$,$|\alpha_2\rangle$ and $|\alpha_3\rangle$ gets generated in mode 5.

The probability of generation of state $|\alpha_j\rangle$ is given by $P_j = N_e^{\,4} r_j^{\,4}$ which becomes equal to 0.25 for appreciable value of coherent amplitude $|\alpha|$.

After illuminating a 50-50 BS by state $|\alpha_j\rangle$,the resulting state is an entangled ququat similar to BFECS $|E_j\rangle$.

Teleportation of one ququat encoded in superposition of coherent state

The information state to be teleported is given by

$$|I\rangle_1 = [\varepsilon_0|\alpha\rangle + \varepsilon_1|i\alpha\rangle + \varepsilon_2|-\alpha\rangle + \varepsilon_3|-i\alpha\rangle]_1$$

where ε_j are the complex coefficients with normalization condition

$$\sum_{j=0}^{3}[|\varepsilon_j|^2 + x^2\varepsilon_j^*\varepsilon_{\underline{j+2}} + x(\varepsilon_j^*\varepsilon_{\underline{j+1}}x^{-i} + \varepsilon_j^*\varepsilon_{\underline{j+1}}x^i)] = 1$$

$$\underline{j+1} = \mathrm{mod}(j+1,\ 3) \qquad \underline{j+2} = \mathrm{mod}(j+2,\ 3)$$

This can also be written as

$$|I\rangle_1 = [c_0|\alpha_0\rangle + c_1|\alpha_1\rangle + c_2|\alpha_2\rangle + c_3|\alpha_3\rangle]_1$$

$$c_0 = \tfrac{1}{2}r_0(\varepsilon_0 + \varepsilon_1 + \varepsilon_2 + \varepsilon_3) \quad c_1 = \tfrac{1}{2}(\varepsilon_0 + i\varepsilon_1 - \varepsilon_2 - i\varepsilon_3)$$

$$c_2 = \tfrac{1}{2}r_2(\varepsilon_0 - \varepsilon_1 + \varepsilon_2 - \varepsilon_3) \quad c_3 = \tfrac{1}{2}r_3(\varepsilon_0 - i\varepsilon_1 - \varepsilon_2 + i\varepsilon_3)$$

$$|c_0|^2 + |c_1|^2 + |c_2|^2 + |c_3|^2 = 1$$

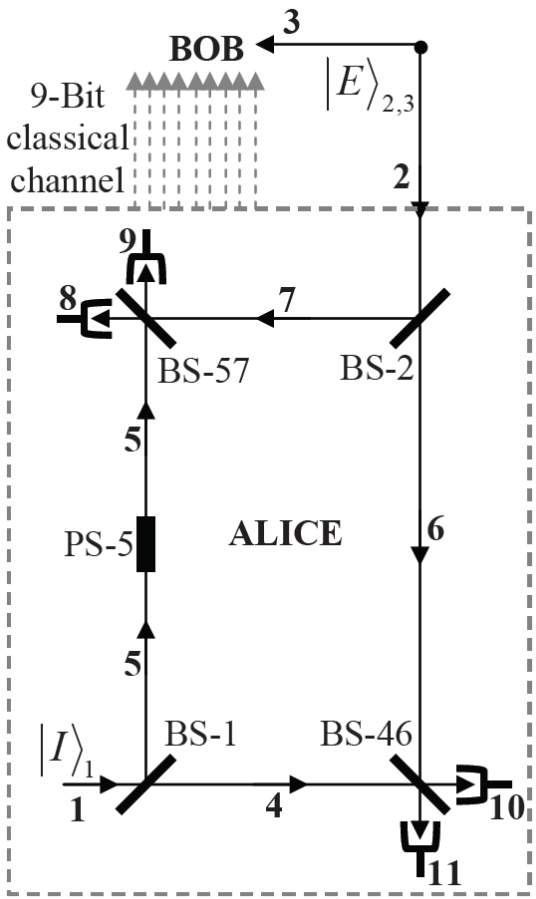
This represents an arbitrary ququat defined in a 4D Hilbert space.

In principle any of the four BFECS (entangled ququat) can be used for teleportation. The BFECS (entangled ququat) to be used as quantum channel

$$\left|E_0\right\rangle = N_{E0}[\left|\alpha,\alpha\right\rangle + \left|i\alpha,i\alpha\right\rangle + \left|-\alpha,-\alpha\right\rangle + \left|-i\alpha,-i\alpha\right\rangle]$$

Information mode 1 is with Alice. Entangled mode 2 is with Alice and mode 3 is sent to Bob. Initial joint state is given by

$$\begin{aligned}
\left|\psi\right\rangle_{1,2,3} &= \left|I\right\rangle_1 \otimes \left|E_0\right\rangle_{2,3} \\
&= N_{E0}[\varepsilon_0\{\left|\alpha,\alpha,\alpha\right\rangle + \left|\alpha,i\alpha,i\alpha\right\rangle + \left|\alpha,-\alpha,-\alpha\right\rangle + \left|\alpha,-i\alpha,-i\alpha\right\rangle\} \\
&+ \varepsilon_1\{\left|i\alpha,\alpha,\alpha\right\rangle + \left|i\alpha,i\alpha,i\alpha\right\rangle + \left|i\alpha,-\alpha,-\alpha\right\rangle + \left|i\alpha,-i\alpha,-i\alpha\right\rangle\} \\
&+ \varepsilon_2\{\left|-\alpha,\alpha,\alpha\right\rangle + \left|-\alpha,i\alpha,i\alpha\right\rangle + \left|-\alpha,-\alpha,-\alpha\right\rangle + \left|-\alpha,-i\alpha,-i\alpha\right\rangle\} \\
&+ \varepsilon_3\{\left|-i\alpha,\alpha,\alpha\right\rangle + \left|-i\alpha,i\alpha,i\alpha\right\rangle + \left|-i\alpha,-\alpha,-\alpha\right\rangle + \left|-i\alpha,-i\alpha,-i\alpha\right\rangle\}]_{1,2,3}
\end{aligned}$$



Scheme for teleporting one ququat encoded in superposition of coherent states with the aid of entangled ququat based on coherent state called BFECS. BS and PS stands for 50-50 beam splitter and -π/2 phase shifter, respectively. Bold numbers represent the quantum mode.

The final joint state of Alice and Bob system is described by

$$|\psi\rangle_{1,2,3} = N_{E0}[\varepsilon_0\{|i\alpha,0,\beta,\beta,\alpha\rangle + |i\beta,-i\beta,0,i\alpha,i\alpha\rangle + |0,\alpha,-i\beta,i\beta,-\alpha\rangle + |\beta,\beta,\alpha,0,-i\alpha\rangle\}$$

$$+\varepsilon_1\{|i\beta,i\beta,i\alpha,0,\alpha\rangle + |-\alpha,0,i\beta,i\beta,i\alpha\rangle + |-\beta,\beta,0,-\alpha,-\alpha\rangle + |0,i\alpha,\beta,-\beta,-i\alpha\rangle\}$$

$$+\varepsilon_2\{|0,-\alpha,i\beta,-i\beta,\alpha\rangle + |-\beta,-\beta,-\alpha,0,i\alpha\rangle + |-i\alpha,0,-\beta,-\beta,-\alpha\rangle + |-i\beta,i\beta,0,-i\alpha,-i\alpha\rangle\}$$

$$+\varepsilon_3\{|\beta,-\beta,0,\alpha,\alpha\rangle + |0,-i\alpha,-\beta,\beta,i\alpha\rangle + |-i\beta,-i\beta,-i\alpha,0,-\alpha\rangle + |\alpha,0,-i\beta,-i\beta,-i\alpha\rangle\}]_{8,9,10,11,3}$$

where $|\beta\rangle = |\frac{1}{2}(1+i)\alpha\rangle$

Alice performs the photon counting (PC) in modes 8, 9, 10, and 11 and conveys her PC result to Bob, on the basis of which Bob performs an appropriate unitary operation on his mode 3 to get faithful replica of the original information state.

One mode always counts zero photon.

Since coherent states are the superposition of all possible photon number states, therefore, there will be many possible PC results.

For better understanding of all possible PC results it is appropriate to expand coherent states, $|\pm\alpha\rangle$ and $|\pm i\alpha\rangle$, into states $|0\rangle$, $|\alpha_1\rangle$, $|\alpha_2\rangle$, $|\alpha_3\rangle$ and $|\alpha_4\rangle$ with 0, 4n+1, 4n+2, 4n+3 and 4n+4 numbers of photon, respectively.

$$|\pm\alpha\rangle = a_0|0\rangle \pm a_1|\alpha_1\rangle + a_2|\alpha_1\rangle \pm a_3|\alpha_3\rangle + a_4|\alpha_4\rangle$$

$$|\pm i\alpha\rangle = a_0|0\rangle \pm ia_1|\alpha_1\rangle - a_2|\alpha_1\rangle \mp ia_3|\alpha_3\rangle + a_4|\alpha_4\rangle$$

where, $a_0 = \sqrt{x}$ $a_{1,2,3} = \frac{1}{2} r_{1,2,3}$ $a_4 = \frac{1}{2}\sqrt{(r_0{}^2 - 4x)}$

In the very similar way, coherent states, $|\pm\beta\rangle$ and $|\pm i\beta\rangle$ can be expanded into states $|0\rangle$, $|\beta_1\rangle$, $|\beta_2\rangle$, $|\beta_3\rangle$ and $|\beta_4\rangle$ with 0, 4m+1, 4m+2, 4m+3 and 4m+4 numbers of photon, respectively.

$$|\pm\beta\rangle = b_0|0\rangle \pm b_1|\beta_1\rangle + b_2|\beta_1\rangle \pm b_3|\beta_3\rangle + b_4|\beta_4\rangle$$

$$|\pm i\beta\rangle = b_0|0\rangle \pm ib_1|\beta_1\rangle - b_2|\beta_1\rangle \mp ib_3|\beta_3\rangle + b_4|\beta_4\rangle$$

Coefficients $b_{i=0,1,2,3,4}$ can be obtained by substituting $\frac{1}{2}|\alpha|^2$ instead of $|\alpha|^2$ in expressions for coefficients $a_{i=0,1,2,3,4}$

Using these expansions one can verify that one of the modes 8, 9, 10, and 11 always has vacuum state and of the other three modes can give any of the five results, zero or nonzero, which is 0, 1, 2 or 3 (modulo 4).

Thus, there are ${}^4C_1 4^3 + {}^4C_2 4^2 + {}^4C_3 4^1 + {}^4C_4 4^0 = 369$ different PC results.

These results can be transmitted to Bob on a 9-bit classical channel. Since Bob has to know only the required unitary transformation and there are only 64 distinct unitary transformations, even 8 c-bit channel is sufficient.

We write these PC results as 0, 1, 2, 3 and 4, the last one being the nonzero-result (0 modulo 4) written as 4 to distinguish it from the result of 0 counts.

These results can be classified into four groups:

Group I (*All modes count zero photon*),

Group II (*Any three modes count zero and one mode count non-zero photon*),

Group III (*Any two modes count zero photon and rest two modes count non-zero photon*), and

Group IV (*Only one mode count zero and rest three modes count non-zero photons*).

Group I (All modes count zero photon); only one case of result: The teleported state is seen to be $\sim|\alpha\rangle+|i\alpha\rangle+|-\alpha\rangle+-i|\alpha\rangle$ irrespective of the information, if the information is in this state F=1 and if the information is orthogonal to it F=0. Thus MASFI is 0 and we say that the **Teleportation Fails. This case is however important for small $|\alpha|^2$, and for probability for occurrence of this case is nearly zero.**

Group II (Any three modes count zero and one mode counts non-zero photons): This group has 16 possible PC results as the non-zero photon mode may be any one of the four modes and non-zero photon counts may be any of 4n+1, 4n+2, 4n+3 or 4n+4. For this case, the teleported state is

$$\sim[a_4 b_0{}^2|\alpha_4,0,0,0\rangle\{c_0|\alpha_0\rangle+c_1|\alpha_1\rangle+c_2|\alpha_2\rangle+c_3|\alpha_3\rangle\}$$

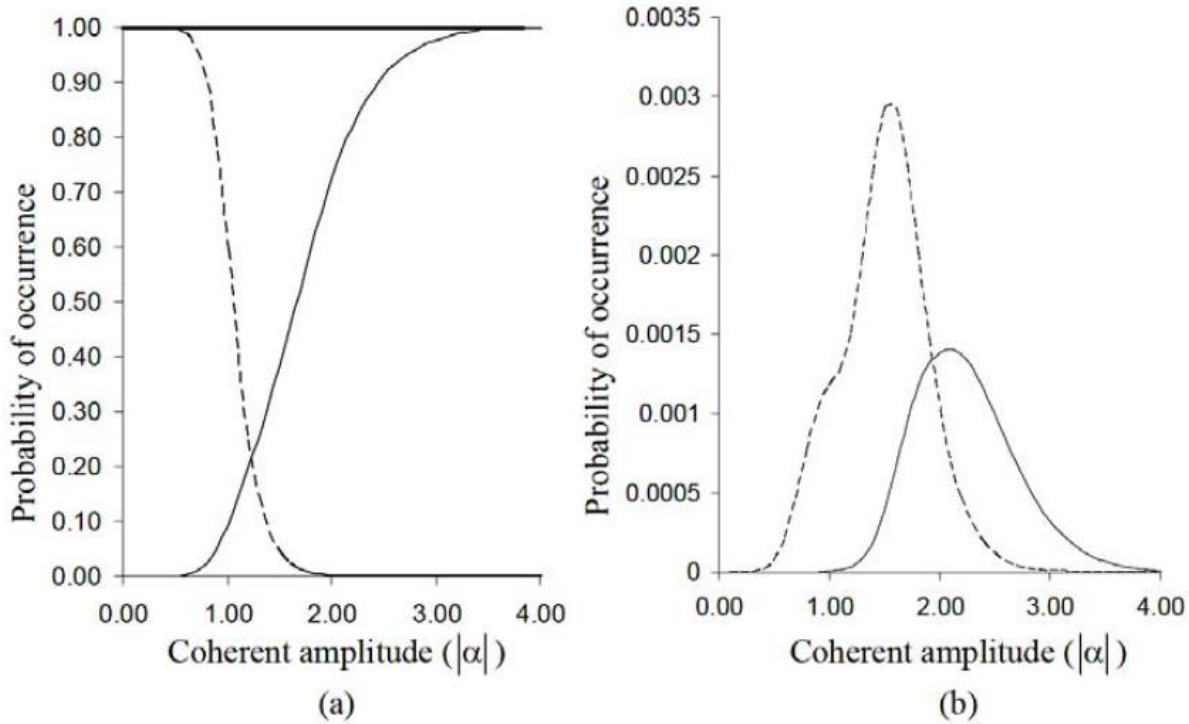
$$+2a_0 b_0 b_4|\beta_4,0,0,0\rangle\{c_0|\alpha_0\rangle-c_2|\alpha_2\rangle\}$$

Since nonzero counts may be obtained both for $|\alpha_4\rangle$ and for $|\beta_4\rangle$, one cannot devise a prescription for the required unitary transformation to be performed by Bob. Similar results come for the rest 15 cases in this group. Hence **Teleportation Fails.**

Group III (Any two modes count zero photon and rest two modes count non-zero photons) : This group has ${}^4C_2 4^2 = 96$ PC results, which may further be divided into two subgroups, Subgroup III.I and Subgroup III.II.

Subgroup III.I (Pair of modes '8 and 10' or '8 and 11' or '9 and 10' or '9 and 11' show zero counts, while the rest two modes show non-zero photons): This subgroup has ${}^4C_1 4^2 = 64$ PC results. The situation for this case is exactly similar to that discussed for Group II and **Teleportation Fails**.

Figures on next slide shows variation of maximum probability of occurrence for PC result of different groups. From where it is clear that probability of maximum probability of occurrence for PC result belonging to groups I, II and III becomes zero for appreciable coherent amplitude.



(a) Dashed curve shows variation of maximum probability of occurrence for photon counting result (0000) of group I, with respect to coherent amplitude . Continuous curve shows the variation of summation of probabilities of occurrence for all 256 photon counting results belonging to Group IV. (b) Dashed and continuous curves shows variations of maximum probability of occurrence with respect to coherent amplitude for typical cases of group II and III of the photon counting result (4000) and (4040) respectively.

Thus Occurrence of PC results belonging to Groups I, II, III will not degrade the average fidelity for $|\alpha| \geq 3.2$

Subgroup III.II (modes '8 and 9' or '10 and 11' counts zero, while rest mode count non-zero photons): This subgroup has 32 PC results. If we look at the states with Bob for the 32 PC results, it is seen that the Bob's state is invariably in the form

$B^{(j,k,m)} = \frac{1}{2}[B^{(j,k)} + i^m B^{(j+2,k)}]$ where $B^{(j,k)} = \sum_{l=0}^{3} c_{l+k}(r_l / r_{l+k}) i^{jl} |\alpha_l\rangle$

For 16 cases, a unitary transformation resulting in perfect or almost perfect teleportation exists, the required unitary transformations for the Bob's state

$B^{(j,k,m)}$ is

$U^{(j,k,m)} = \frac{1}{2}[U^{(j,k)} + (-i)^m U^{(j+2,k)}]$ where $U^{(j,k)} = \sum_{l=0}^{3} (-i)^{jl} |\alpha_{k+l}\rangle\langle\alpha_l|$

For the cases where no unitary transformation giving F =1 is possible and MASFI=0, we admit failure, but prescribe unitary transformations $U^{(j,k)}$ which give F=1 for certain cases of special information states, although MASFI=0. There are 16 such cases.

Table in next slide shows all 32 PC results belonging to subgroup III.II, corresponding state with Bob, the unitary operations, teleported state and the fidelity. For 16 cases for which fidelity is F5 or F6, MASFI≈1 for $|\alpha \geq 1.7|$.

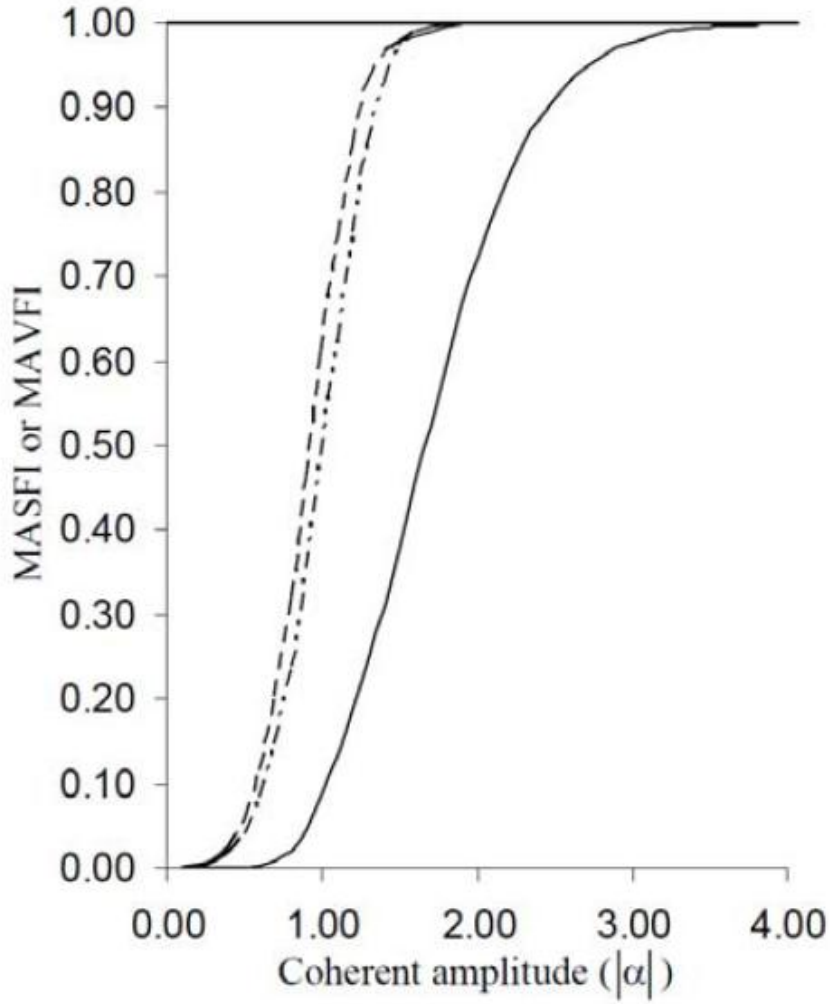
PC result	$B^{(j,k,m)}$	$U^{(j,k)}$ or $U^{(j,k,m)}$	$\lvert T\rangle$	F
(4,4,0,0) (2,2,0,0)	$B^{(1,0,0)}$	$U^{(1,0)}$	$\lvert T_1\rangle = c_0\lvert\alpha_0\rangle + c_2\lvert\alpha_2\rangle$	$F_1 = \lvert c_0\rvert^2 + \lvert c_2\rvert^2$
(0,0,4,4) (0,0,2,2)	$B^{(2,0,0)}$	$U^{(2,0)}$		
(1,3,0,0) (3,1,0,0)	$-B^{(1,0,2)}$	$-U^{(1,0)}$	$\lvert T_2\rangle = c_1\lvert\alpha_1\rangle + c_3\lvert\alpha_3\rangle$	$F_2 = \lvert c_1\rvert^2 + \lvert c_3\rvert^2$
(0,0,1,3) (0,0,3,1)	$-B^{(2,0,2)}$	$-U^{(2,0)}$		
(4,2,0,0) (2,4,0,0)	$-B^{(1,2,2)}$	$-U^{(1,2)}$	$\lvert T_3\rangle = c_3(r_1/r_3)\lvert\alpha_3\rangle + c_1(r_3/r_1)\lvert\alpha_1\rangle$	$F_3 = [\lvert c_3\rvert^2 r_1^2 + \lvert c_1\rvert^2 r_3^2]^2 / [\lvert c_3\rvert^2 r_1^4 + \lvert c_1\rvert^2 r_3^4]$
(0,0,4,2) (0,0,2,4)	$-B^{(2,2,2)}$	$-U^{(2,2)}$		
(1,1,0,0) (3,3,0,0)	$B^{(1,2,0)}$	$U^{(1,2)}$	$\lvert T_4\rangle = c_2(r_0/r_2)\lvert\alpha_2\rangle + c_0(r_2/r_0)\lvert\alpha_0\rangle$	$F_4 = [\lvert c_2\rvert^2 r_0^2 + \lvert c_0\rvert^2 r_2^2]^2 / [\lvert c_2\rvert^2 r_0^4 + \lvert c_0\rvert^2 r_2^4]$
(0,0,1,1) (0,0,3,3)	$B^{(2,2,0)}$	$U^{(2,2)}$		
(4,1,0,0) (2,3,0,0)	$\sqrt{2}B^{(3,1,3)}$	$\sqrt{2}U^{(3,1,3)}$	$\lvert T_5\rangle = c_1(r_0/r_1)\lvert\alpha_1\rangle + c_2(r_1/r_2)\lvert\alpha_2\rangle + c_3(r_2/r_3)\lvert\alpha_3\rangle + c_0(r_3/r_0)\lvert\alpha_0\rangle$	$F_5 = [\lvert c_1\rvert^2 r_0^2 r_2 r_3 + \lvert c_2\rvert^2 r_1^2 r_3 r_0 + \lvert c_3\rvert^2 r_2^2 r_0 r_1 + \lvert c_0\rvert^2 r_3^2 r_1 r_2]^2 / [\lvert c_1\rvert^2 r_0^4 r_2^2 r_3^2 + \lvert c_2\rvert^2 r_1^4 r_3^2 r_0^2 + \lvert c_3\rvert^2 r_2^4 r_0^2 r_1^2 + \lvert c_0\rvert^2 r_3^4 r_1^2 r_2^2]$
(0,0,4,1) (0,0,2,3)	$\sqrt{2}B^{(0,1,1)}$	$\sqrt{2}U^{(0,1,1)}$		
(1,4,0,0) (3,2,0,0)	$\sqrt{2}B^{(3,1,1)}$	$\sqrt{2}U^{(3,1,1)}$		
(0,0,1,4) (0,0,3,2)	$\sqrt{2}B^{(0,1,3)}$	$\sqrt{2}U^{(0,1,3)}$		
(4,3,0,0) (2,1,0,0)	$\sqrt{2}B^{(3,3,1)}$	$\sqrt{2}U^{(3,3,1)}$	$\lvert T_6\rangle = c_3(r_0/r_3)\lvert\alpha_3\rangle + c_0(r_1/r_0)\lvert\alpha_0\rangle + c_1(r_2/r_1)\lvert\alpha_1\rangle + c_2(r_3/r_2)\lvert\alpha_2\rangle$	$F_6 = [\lvert c_3\rvert^2 r_0^2 r_1 r_2 + \lvert c_0\rvert^2 r_1^2 r_2 r_3 + \lvert c_1\rvert^2 r_2^2 r_3 r_0 + \lvert c_2\rvert^2 r_3^2 r_0 r_1]^2 / [\lvert c_3\rvert^2 r_0^4 r_1^2 r_2^2 + \lvert c_0\rvert^2 r_1^4 r_2^2 r_3^2 + \lvert c_1\rvert^2 r_2^4 r_3^2 r_0^2 + \lvert c_2\rvert^2 r_3^4 r_0^2 r_1^2]$
(0,0,4,3) (0,0,2,1)	$\sqrt{2}B^{(0,3,3)}$	$\sqrt{2}U^{(0,3,3)}$		
(1,2,0,0) (3,4,0,0)	$\sqrt{2}B^{(3,3,3)}$	$\sqrt{2}U^{(3,3,3)}$		
(0,0,1,2) (0,0,3,4)	$\sqrt{2}B^{(0,3,1)}$	$\sqrt{2}U^{(0,3,1)}$		

The fidelities are seen to occur in one of six different forms $F_{1,2,3,4,5,6}$.

It is found that $F_{1,2,3,4}^{MASFI} = 0$

Plot given below shows that $F_{5,6}^{MASFI}$ reaches unity for $|\alpha| \geq 1.7$.

Thus 16 PC results of this subgroup fails to teleport the information state, while rest 16 PC results gives almost perfect teleportation for $|\alpha| \geq 1.7$.



Dashed curve shows variation of $F_{5,6}^{MASFI}$

Group IV (Only one mode count zero and rest three modes count non-zero photons): This group has ${}^4C_1 4^3 = 256$ PC results.

For this group of PC results the Bob's state and unitary transformation are seen to occur in the form

$$B^{(j,k)} = \sum_{l=0}^{3} c_{l+k}(r_l / r_{l+k}) i^{jl} \left| \alpha_l \right\rangle \text{ and } U^{(j,k)} = \sum_{l=0}^{3} (-i)^{jl} \left| \alpha_{k+l} \right\rangle \left\langle \alpha_l \right|$$

respectively, defined earlier.

For all 256 PC results corresponding Bob's state and required unitary transformation, are tabulated in table given in next slide.

PC result in modes 8, 9, 10 & 11	$B^{(j,k)}$	$U^{(j,k)}$
(0,4,4,4) (0,4,2,2) (0,2,1,1) (0,2,3,3)	$B^{(2,0)}$	$U^{(2,0)}$
(0,4,3,1) (0,4,1,3) (0,2,2,4) (0,2,4,2)	$-B^{(2,0)}$	$-U^{(2,0)}$
(0,1,3,4) (0,1,1,2) (0,3,4,1) (0,3,2,3)	$iB^{(2,0)}$	$-iU^{(2,0)}$
(0,1,2,1) (0,1,4,3) (0,3,1,4) (0,3,3,2)	$-iB^{(2,0)}$	$iU^{(2,0)}$
(4,0,4,4) (4,0,3,1) (4,0,2,2) (4,0,4,3)	$B^{(0,0)}$	$U^{(0,0)}$
(1,0,3,4) (1,0,2,1) (1,0,1,2) (1,0,4,3)	$iB^{(0,0)}$	$-iU^{(0,0)}$
(2,0,2,4) (2,0,1,1) (2,0,4,2) (2,0,3,3)	$-B^{(0,0)}$	$-U^{(0,0)}$
(3,0,1,4) (3,0,4,1) (3,0,3,2) (3,0,2,3)	$-iB^{(0,0)}$	$iU^{(0,0)}$
(2,2,0,4) (2,4,0,2) (3,4,0,1) (3,2,0,3)	$B^{(1,0)}$	$U^{(1,0)}$
(4,4,0,4) (4,2,0,2) (1,2,0,1) (1,4,0,3)	$B^{(1,0)}$	$U^{(1,0)}$
(2,1,0,1) (2,3,0,3) (3,1,0,4) (3,3,0,2)	$-B^{(1,0)}$	$-U^{(1,0)}$
(4,3,0,1) (4,1,0,3) (1,3,0,4) (1,1,0,2)	$-B^{(1,0)}$	$-U^{(1,0)}$
(4,4,4,0) (4,3,1,0) (4,2,2,0) (4,1,3,0)	$B^{(3,0)}$	$U^{(3,0)}$
(2,2,4,0) (2,1,1,0) (2,4,2,0) (2,3,3,0)	$B^{(3,0)}$	$U^{(3,0)}$
(1,3,4,0) (1,2,1,0) (1,1,2,0) (1,4,3,0)	$B^{(3,0)}$	$U^{(3,0)}$
(3,1,4,0) (3,4,1,0) (3,3,2,0) (3,2,3,0)	$B^{(3,0)}$	$U^{(3,0)}$
(0,1,4,4) (0,1,2,2) (0,3,1,1) (0,3,3,3)	$B^{(2,1)}$	$U^{(2,1)}$
(0,1,3,1) (0,1,1,3) (0,3,2,4) (0,3,4,2)	$-B^{(2,1)}$	$-U^{(2,1)}$
(0,4,1,4) (0,4,3,2) (0,2,2,1) (0,2,4,3)	$-iB^{(2,1)}$	$iU^{(2,1)}$
(0,4,4,1) (0,4,2,3) (0,2,1,2) (0,2,3,4)	$iB^{(2,1)}$	$-iU^{(2,1)}$
(4,0,1,4) (4,0,4,1) (4,0,3,2) (4,0,2,3)	$B^{(0,1)}$	$U^{(0,1)}$
(3,0,2,4) (3,0,1,1) (3,0,4,2) (3,0,3,3)	$-iB^{(0,1)}$	$iU^{(0,1)}$
(2,0,3,4) (2,0,2,1) (2,0,1,2) (2,0,4,3)	$-B^{(0,1)}$	$-U^{(0,1)}$
(1,0,4,4) (1,0,3,1) (1,0,2,2) (1,0,1,3)	$iB^{(0,1)}$	$-iU^{(0,1)}$
(2,3,0,4) (2,1,0,2) (3,1,0,1) (3,3,0,3)	$B^{(1,1)}$	$U^{(1,1)}$
(4,1,0,4) (4,3,0,2) (1,3,0,1) (1,1,0,3)	$B^{(1,1)}$	$U^{(1,1)}$
(2,2,0,1) (2,4,0,3) (3,2,0,4) (3,4,0,2)	$-B^{(1,1)}$	$-U^{(1,1)}$
(4,4,0,1) (4,2,0,3) (1,4,0,4) (1,2,0,2)	$-B^{(1,1)}$	$-U^{(1,1)}$
(4,1,4,0) (4,4,1,0) (4,3,2,0) (4,2,3,0)	$B^{(3,1)}$	$U^{(3,1)}$
(2,3,4,0) (2,2,1,0) (2,1,2,0) (2,4,3,0)	$B^{(3,1)}$	$U^{(3,1)}$
(1,4,4,0) (1,3,1,0) (1,2,2,0) (1,1,3,0)	$B^{(3,1)}$	$U^{(3,1)}$
(3,2,4,0) (3,1,1,0) (3,4,2,0) (3,3,3,0)	$B^{(3,1)}$	$U^{(3,1)}$

(0,1,1,4) (0,1,3,2) (0,3,2,1) (0,3,4,3)	$B^{(2,2)}$	$U^{(2,2)}$
(0,1,4,1) (0,1,2,3) (0,3,3,4) (0,3,1,2)	$-B^{(2,2)}$	$-U^{(2,2)}$
(0,4,2,4) (0,4,4,2) (0,2,3,1) (0,2,1,3)	$-iB^{(2,2)}$	$iU^{(2,2)}$
(0,4,4,1) (0,4,3,3) (0,2,4,4) (0,2,2,2)	$iB^{(2,2)}$	$-iU^{(2,2)}$
(4,0,2,4) (4,0,1,1) (4,0,4,2) (4,0,3,3)	$B^{(0,2)}$	$U^{(0,2)}$
(1,0,1,4) (1,0,4,1) (1,0,3,2) (1,0,2,3)	$iB^{(0,2)}$	$-iU^{(0,2)}$
(2,0,4,4) (2,0,3,1) (2,0,2,2) (2,0,1,3)	$-B^{(0,2)}$	$-U^{(0,2)}$
(3,0,3,4) (3,0,2,1) (3,0,1,2) (3,0,4,3)	$-iB^{(0,2)}$	$iU^{(0,2)}$
(2,4,0,4) (2,2,0,2) (3,2,0,1) (3,4,0,3)	$B^{(1,2)}$	$U^{(1,2)}$
(4,2,0,4) (4,4,0,2) (1,4,0,1) (1,2,0,3)	$B^{(1,2)}$	$U^{(1,2)}$
(2,3,0,1) (4,4,0,2) (3,3,0,4) (3,1,0,2)	$-B^{(1,2)}$	$-U^{(1,2)}$
(4,1,0,1) (2,1,0,3) (1,1,0,4) (1,3,0,2)	$-B^{(1,2)}$	$-U^{(1,2)}$
(4,2,4,0) (4,1,1,0) (4,4,2,0) (4,3,3,0)	$B^{(3,2)}$	$U^{(3,2)}$
(2,4,4,0) (2,3,1,0) (2,2,2,0) (2,1,3,0)	$B^{(3,2)}$	$U^{(3,2)}$
(1,1,4,0) (1,4,1,0) (1,3,2,0) (1,2,3,0)	$B^{(3,2)}$	$U^{(3,2)}$
(3,3,4,0) (3,2,1,0) (3,1,2,0) (3,4,3,0)	$B^{(3,2)}$	$U^{(3,2)}$
(0,1,2,4) (0,1,4,2) (0,3,3,1) (0,3,1,3)	$B^{(2,3)}$	$U^{(2,3)}$
(0,1,1,1) (0,1,3,3) (0,3,4,4) (0,3,2,2)	$-B^{(2,3)}$	$-U^{(2,3)}$
(0,4,3,4) (0,4,1,2) (0,2,4,1) (0,2,2,3)	$-iB^{(2,3)}$	$iU^{(2,3)}$
(0,4,2,1) (0,4,4,3) (0,2,1,4) (0,2,3,2)	$iB^{(2,3)}$	$-iU^{(2,3)}$
(4,0,3,4) (4,0,2,1) (4,0,1,2) (4,0,4,3)	$B^{(0,3)}$	$U^{(0,3)}$
(1,0,2,4) (1,0,1,1) (1,0,4,2) (1,0,3,3)	$iB^{(0,3)}$	$-iU^{(0,3)}$
(2,0,1,4) (2,0,4,1) (2,0,3,2) (2,0,2,3)	$-B^{(0,3)}$	$-U^{(0,3)}$
(3,0,4,4) (3,0,3,1) (3,0,2,2) (3,0,1,3)	$-iB^{(0,3)}$	$iU^{(0,3)}$
(2,1,0,4) (2,3,0,2) (3,3,0,1) (3,1,0,3)	$B^{(1,3)}$	$U^{(1,3)}$
(4,3,0,4) (4,1,0,2) (1,1,0,1) (1,3,0,3)	$B^{(1,3)}$	$U^{(1,3)}$
(2,4,0,1) (2,2,0,2) (3,4,0,4) (3,2,0,2)	$-B^{(1,3)}$	$-U^{(1,3)}$
(4,2,0,1) (4,4,0,3) (1,2,0,4) (1,4,0,2)	$-B^{(1,3)}$	$-U^{(1,3)}$
(4,3,4,0) (4,2,1,0) (4,1,2,0) (4,4,3,0)	$B^{(3,3)}$	$U^{(3,3)}$
(2,1,4,0) (2,4,1,0) (2,3,2,0) (2,2,3,0)	$B^{(3,3)}$	$U^{(3,3)}$
(1,2,4,0) (1,1,1,0) (1,4,2,0) (1,3,3,0)	$B^{(3,3)}$	$U^{(3,3)}$
(3,4,4,0) (3,3,1,0) (3,2,2,0) (3,1,3,0)	$B^{(3,3)}$	$U^{(3,3)}$

It is seen that, for all PC results belonging to k=0, teleported state is

$$|T_7\rangle = [c_0|\alpha_0\rangle + c_1|\alpha_1\rangle + c_2|\alpha_2\rangle + c_3|\alpha_4\rangle]$$

with fidelity F=1 , and the teleportation is perfect.

For PC results belonging to k=1, 2 and 3 teleported states are

$$|T_8\rangle = [\frac{c_0 r_3}{r_0}|\alpha_0\rangle + \frac{c_1 r_0}{r_1}|\alpha_1\rangle + \frac{c_2 r_1}{r_2}|\alpha_2\rangle + \frac{c_3 r_2}{r_3}|\alpha_4\rangle]$$

$$|T_9\rangle = [\frac{c_0 r_2}{r_0}|\alpha_0\rangle + \frac{c_1 r_3}{r_1}|\alpha_1\rangle + \frac{c_2 r_0}{r_2}|\alpha_2\rangle + \frac{c_3 r_1}{r_3}|\alpha_4\rangle]$$

and

$$|T_{10}\rangle = [\frac{c_0 r_1}{r_0}|\alpha_0\rangle + \frac{c_1 r_2}{r_1}|\alpha_1\rangle + \frac{c_2 r_3}{r_2}|\alpha_2\rangle + \frac{c_3 r_0}{r_3}|\alpha_4\rangle]$$

with fidelities $F_8 = F_5$,$F_{10} = F_6$ and

$$F_9 = \frac{[|c_0|^2 r_2^2 r_1 r_3 + |c_1|^2 r_3^2 r_0 r_2 + |c_2|^2 r_0^2 r_1 r_3 + |c_3|^2 r_1^2 r_0 r_2]^2}{[|c_0|^2 r_2^4 r_1^2 r_3^2 + |c_1|^2 r_3^4 r_0^2 r_2^2 + |c_2|^2 r_0^4 r_1^2 r_3^2 + |c_3|^2 r_1^4 r_0^2 r_2^2]}$$

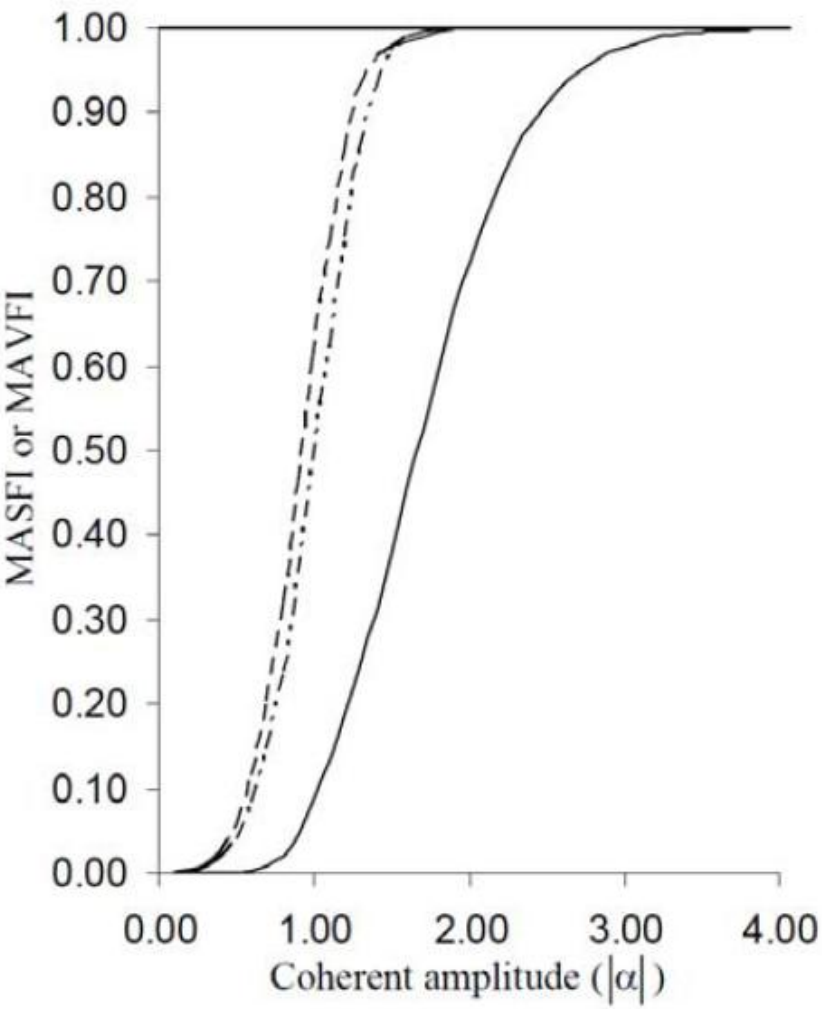
Dashed curve shows variation of minimum assured fidelities (MASFI) $F_{8,10}^{MASFI}$ against coherent amplitude.

Dash-dotted curve shows variation of minimum assured fidelity F_9^{MASFI} against coherent amplitude.

Thus, out of all 256 PC results belonging to Group IV, 64 PC results gives perfect teleportation for any value of $|\alpha|$, while rest 192 PC results gives almost perfect teleportation for $|\alpha| \geq 1.7$.

Continuous curve shows minimum average fidelity (MAVFI).

It is clear that $F_{av.\min} \geq 0.99$ for $|\alpha| \geq 3.2$.



Thus almost perfect teleportation with perfect success rate is achieved for $|\alpha| \geq 3.2$

Long Distance Atomic Teleportation Using Entangled Coherent States and Cavity Assisted Interaction

Large numbers of schemes for teleportation of qubits based on single photon and superposed coherent states (SCS) have been proposed.

However, single-photon or SCS are not ideal for long term storage of quantum information as they are very difficult to keep in a certain place.

On the other hand, it has been demonstrated that a single atom can be trapped for a few seconds inside an optical cavity. Thus, atoms are ideal for quantum information storage.

Numbers of schemes for atomic teleportation using atom-cavity interactions and atoms as flying qubit have been proposed.

Since atoms move slowly and interact strongly with their environment, these schemes are unable to perform long distance atomic teleportation and hence can not be used as link between two quantum processors working distant apart.

Long distance teleportation is of particular importance because of its applicability in secure quantum communication and future satellite based quantum communication.

S Bose [Phys. Rev. Lett. **83**, 5158 (1999)], have presented a novel scheme for teleporting quantum state of an atom trapped in an optical-cavity to second atom in another distant optical-cavity.

This scheme involves mapping of atomic state to a cavity state with Alice, followed by the detection of photons leaking out from Alice's cavity and Bob's cavity (initially in maximally entangled atom-cavity state) by mixing over a beam splitter.

The main shortcoming of this scheme is that the teleportation fidelity and success rate in this depends on the state to be teleported. Under reasonable cavity parameters and cavity decay time, success rate is near ½.

Further Chimczak [Phys. Rev. A **79**, 042311 (2009)] pointed out the inefficiency of scheme proposed by Bose due to large damping values of currently available cavities that reduces the fidelity of state mapping from atom to cavity and discussed a modification using non-maximally entangled atom-cavity state with amplitudes chosen in such a way that compensates the damping factors due to state mapping.

Although this resolves the effect of damping but gives very low success rate. In case of failure, in both schemes the message state is destroyed. Moreover, both schemes are expected to suffer decoherence due to photon absorption while propagating toward beam splitter.

For all these reasons, a *dream scheme for long distance atomic teleportation is required that,*

(i) gives state independent teleportation fidelity and

(ii) high success rate and

(iii) conserves message state on failure thus permitting repeated attempts and

(iv) does not need efficient single photon detection ability, and

(v) many matter-light interaction stages.

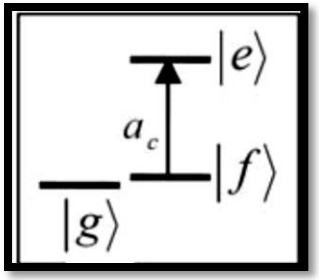
Along with these requirements, the scheme should use

(i) quantum channel that can be deterministically prepared and

(ii) must be robust against photon absorption.

Since ECS are more robust against decoherence due to photon absorption than the SBBS [Hirota et al, quant-ph/0101096v1] and trapped atom in an optical cavity are ideal for quantum information storage, we propose here a scheme for long distance atomic teleportation using ECS that fulfills most of the requirements mentioned above.

Effect of an atom-cavity coupling over an external input coherent pulse



B Wang and L M Duan, Phys. Rev A 72, 022320 (2005)

$|g\rangle$ and $|f\rangle$ are the ground levels with different hyperfine spins.

$|e\rangle$ is the excited level.

The transition $|f\rangle \leftrightarrow |e\rangle$ is resonantly coupled to the cavity mode a_c, which is resonantly driven by an input coherent pulse $|\alpha\rangle$.

The transition $|g\rangle \leftrightarrow |e\rangle$ is decoupled to the cavity mode a_c due to large detuning from the hyperfine frequency.

If initial joint state of atom and input pulse is $|g,\pm\alpha\rangle_{c,in}$, then input pulse is resonant with cavity and exact quantum optics calculation by [D F Walls, Quantum Optics (Springer Verlag, Berlin, 1994)], shows that input pulse reflects with a phase .

If initial state is $|f,\pm\alpha\rangle_{c,in}$, then due to strong atom cavity coupling, cavity mode *ac* is significantly detuned from the center frequency of the input pulse, thus input pulse reflects like from a mirror without any change in phase and pulse shape [Duan et al].

Mathmatically these can be written as

$$|g,\pm\alpha\rangle_{c,in} \rightarrow |g,\mp\alpha\rangle_{c,out} \,,\, |f,\pm\alpha\rangle_{c,in} \rightarrow |f,\pm\alpha\rangle_{c,out}.$$

Alice wishes to teleport message state of an atom in cavity C1 given by

$$|M\rangle_{C1} = [a|g\rangle + b|f\rangle]_{C1} \quad |a|^2 + |b|^2 = 1$$

to a second atom in a distant cavity C2, initially in state

$$|+\rangle_{C2} = \tfrac{1}{\sqrt{2}}[|g\rangle + |f\rangle]_{C2}$$

Alice and Bob shares an entangled coherent state

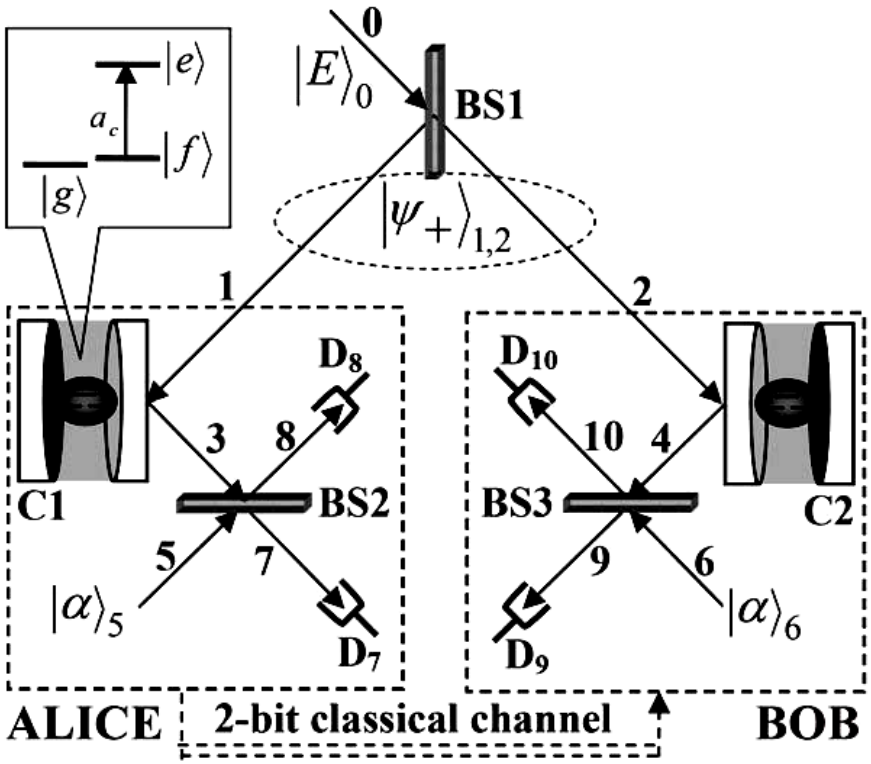
$$|\psi_+\rangle_{1,2} = \mathrm{N}_+[|\alpha,\alpha\rangle + |-\alpha,-\alpha\rangle]_{1,2}$$

$$\mathrm{N}_\pm = [2(1+\mathrm{x}^4)]^{-1/2}$$

$$x = \exp(-|\alpha|^2)$$

Initial state of the system is

$$|\Phi\rangle_{1,2,C1,C2} = |\psi_+\rangle_{1,2}|M\rangle_{C1}|+\rangle_{C2}$$



Scheme for teleportation of atomic-state trapped in cavity C1 to second atom in a distant cavity C2. Entangled coherent state ($|\psi_+\rangle_{1,2} = N_+[|\alpha,\alpha\rangle + |-\alpha,-\alpha\rangle]_{1,2}$) in modes 1 and 2 is produced by illuminating beam splitter BS1 with an even-coherent state ($|E\rangle_0 = N_+(|\sqrt{2}\alpha\rangle + |-\sqrt{2}\alpha\rangle)_0$) in mode 0. Inset shows level structure of atom. D1, 2, 3, 4 are photon detectors, atom in cavity C1 is measured in basis ($|\pm\rangle$). Encircled numbers represent the quantum mode.

Following this scheme the final output state is given by

$$
\begin{aligned}
\left|\Phi\right\rangle_{1,4,C1,C2} = \frac{1}{2}\Big[& \left\{\left|\sqrt{2}\alpha,0,\sqrt{2}\alpha,0\right\rangle + \left|0-\sqrt{2}\alpha,0,-\sqrt{2}\alpha\right\rangle\right\}_{7,8,9,10}\left|+\right\rangle_{C1}\left(\left|M\right\rangle\right)_{C2} \\
& + \left\{\left|\sqrt{2}\alpha,0,\sqrt{2}\alpha,0\right\rangle + \left|0-\sqrt{2}\alpha,0,-\sqrt{2}\alpha\right\rangle\right\}_{7,8,9,10}\left|-\right\rangle_{C1}\left(\sigma_Z\left|M\right\rangle\right)_{C2} \\
& + \left\{\left|\sqrt{2}\alpha,0,0,-\sqrt{2}\alpha\right\rangle + \left|0-\sqrt{2}\alpha,\sqrt{2}\alpha,0\right\rangle\right\}_{7,8,9,10}\left|+\right\rangle_{C1}\left(\sigma_X\left|M\right\rangle\right)_{C2} \\
& + \left\{\left|\sqrt{2}\alpha,0,0,-\sqrt{2}\alpha\right\rangle + \left|0-\sqrt{2}\alpha,\sqrt{2}\alpha,0\right\rangle\right\}_{7,8,9,10}\left|-\right\rangle_{C1}\left(-i\sigma_Y\left|M\right\rangle\right)_{C2}\Big].
\end{aligned}
$$

Now Alice performs photon counting in mode 7 & 8, and performs atomic measurement in diagonal basis in cavity C1. While Bob performs PC in modes 9 & 10.

It is clear form the above given output state that two modes always gives zero count.

For appreciable value of mean photon numbers of the order of $|\alpha|^2$ all possible measurement results are different and hence appropriate unitary operation can be prescribed to generate exact replica of original information state in cavity C2.

However, since coherent states are the superposition of vacuum state and all photon number state, thus there is nonzero probability to detect vacuum state even when light is present.

This results to some nonzero probability of failure at small mean photon numbers $|\alpha|^2$.

To estimate success rate and resolve the problem of failure at small values of $|\alpha|^2$, we expand coherent state $\left|\pm\sqrt{2}\alpha\right\rangle$ into vacuum state ($|0\rangle$) and state with nonzero numbers of photons ($|NZ_\pm\rangle$) given by

$$\left|\pm\sqrt{2}\alpha\right\rangle = x|0\rangle + \sqrt{1-x^2}\,|NZ_\pm\rangle$$

Using this the final output state becomes

$$\begin{aligned}
|\Phi\rangle_{7,8,9,10,C1,C2} = N_+\big[&\{2x^2|0,0,0,0\rangle + x\sqrt{(1-x^2)}(|0,0,NZ_+,0\rangle + |NZ_+,0,0,0\rangle \\
&+|0,0,0,NZ_-\rangle + |0,NZ_-,0,0\rangle)\}_{7,8,9,10}|M\rangle_{C1}|+\rangle_{C2} \\
&+\tfrac{1}{2}(1-x^2)\{(|NZ_+,0,NZ_+,0\rangle + |0,NZ_-,0,NZ_-\rangle)_{7,8,9,10} \\
&\times(|+\rangle_{C1}|M\rangle_{C2} + |-\rangle_{C1}(\sigma_Z|M\rangle_{C2}) \\
&+(|NZ_+,0,0,NZ_-\rangle + |0,NZ_-,NZ_+,0\rangle)_{7,8,9,10} \\
&\times(|+\rangle_{c1}(\sigma_X|M\rangle_{C2}) + |-\rangle_{C1}(-i\sigma_Y|M\rangle_{C2})\}\big].
\end{aligned}$$

It is clear that two modes of the 7, 8, 9, and 10 are always in vacuum state and measurement results can be classified into two groups:

Group I: Two field modes among 7-10 gives non-zero photon counts and atom in cavity C1 is detected in either of the states $|+\rangle$ or $|-\rangle$.

Group II: Three or all field modes among 7-10 are detected as OFF and atom in cavity C1 is detected in either of the states $|+\rangle$ or $|-\rangle$.

When measurement results falls into group I, Bob's atom can be transformed to the original message state just by applying an appropriate unitary operation. Group I gives perfect teleportation with unit fidelity.

The probability of successful teleportation P_s is given by summing the probability of occurrence of all measurement results corresponding to group I, and it is given by relation,

$$P_S = (1 - x^2)^2 (1 + x^4)^{-1}$$

Measurement results for successful teleportation. Tick stands for detection of non-zero photon and cross stands for detection of vacuum by detectors. ± stands for atomic state in basis $|\pm\rangle$ and σ's are Pauli matrices.

Alice			Bob		Bob's Atomic state	Unitary operation
D_7	D_8	$D_\pm$	D_9	D_{10}		
✓	×	+	✓	×	M	I
✓	×	-	✓	×	$\sigma_Z M$	σ_Z
×	✓	+	×	✓	M	I
×	✓	-	×	✓	$\sigma_Z M$	σ_Z
✓	×	+	×	✓	$\sigma_X M$	σ_X
✓	×	-	×	✓	$-i\sigma_Y M$	$i\sigma_Y$
×	✓	+	✓	×	$\sigma_X M$	σ_Z
×	✓	-	✓	×	$-i\sigma_Y M$	$i\sigma_Y$

However, for the measurement results corresponding to group II, teleportation fails. Probability of failure P_f is given by

$$P_f = 2x^2(1+x^4)^{-1} = 1 - P_s$$

But it is clear that in such case before measurement on atom in cavity C1, the joint state of atoms in cavity C1 and C2 is given by $|M\rangle_{C1}|+\rangle_{C2}$.

Thus message state of atoms in cavity C1 and initial state of the atom in cavity C2 remains conserved up to this stage.

We now summarize our scheme:

(*a*). Alice (Bob) detects field modes 7 & 8 (9 & 10) using detectors D7 & D8 (D9 & D10).

(*b*). Bob conveys his results to Alice using a two-bit classical channel.

(*c*). Alice looks over her measurement results and those conveyed by Bob, if any three or all modes among modes 7-10 are OFF, she rejects the complete process. As already mentioned in such case initial message state and Bob's atomic state remain unchanged. Therefore, Alice does not make measurement on her atom in cavity C1 and starts new process with a fresh copy of ECS.

(d). However, if Alice finds two modes in nonzero photon states and remaining two in vacuum, she measures her atom in cavity C1 and finally conveys two bit information to Bob about “atomic measurement result and the detector clicking result” through the same two-bit channel which was used earlier by Bob in step (b).

(e). Finally Bob performs the appropriate unitary operation on his atom in accordance of results conveyed by Alice and his own and generates exact replica of the information.

In this measurement scheme, step (*c*) avoids the failure by allowing us to repeat the complete process. In case if teleportation fails then in ‘*n*’ number of attempts probability of success becomes

$$P_S^{(n)} = 1 - (P_f)^n$$

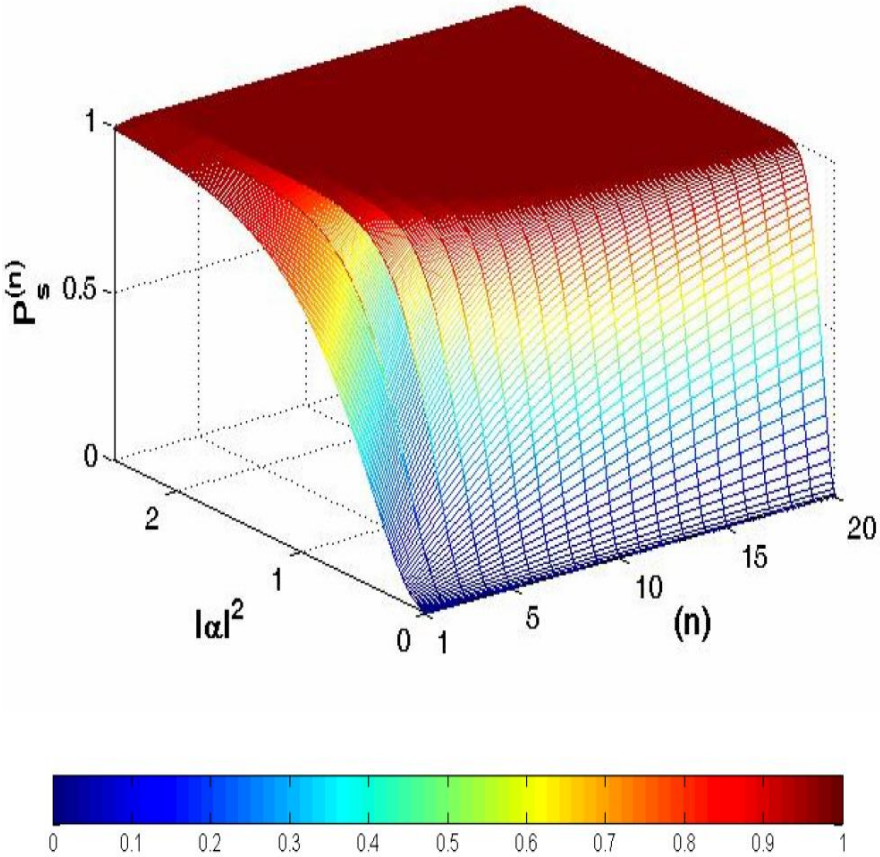
We plott $P_S^{(n)}$ with respect to $|\alpha|^2$ and 'n'

In a single attempt 'n =1', the probability of success increases as $|\alpha|^2$ increases and becomes almost equal to unity for $|\alpha|^2 \geq 2.5$.

This is due to the fact that for higher values $|\alpha|^2$ probability of detecting vacuum in coherent state becomes almost zero.

However for small $|\alpha|^2$, probability of success is appreciably less then unity but increases rapidly with increasing number of attempts 'n'.

For example, at $|\alpha|^2 = 1$, success is 0.734, 0.963 or 0.998 for one, two or three attempts.



Shows variation of the success probability ($P_S^{(n)}$) for different numbers of attempts 'n' with $|\alpha|^2$.

$|\alpha|^2 < 2.5$

Thus unit success can be obtained in a single attempt for $|\alpha|^2 \geq 2.5$ or in finite number of attempts for low value of $|\alpha|^2 < 2.5$.

We made calculations using a shared non-maximally entangled coherent state $|\psi_+\rangle_{1,2} = N_+[|\alpha,\alpha\rangle + |-\alpha,-\alpha\rangle]_{1,2}$. If we use the maximally entangled coherent stat e $|\psi_+\rangle_{1,2} = N_+[|\alpha,\alpha\rangle + |-\alpha,-\alpha\rangle]_{1,2}$, it is seen that the success rate increases by a multiplicative factor of $(1+x^2)/(1-x^2)$. For this, however, in case of a failure, the message states alter trivially and require transformation by Pauli matrix σ_z for restoration.

Quantum Discord Dynamics for Two-Level Atom Initially in Thermal Equilibrium Interacting with n-Photon State

Quantum entanglement is one of the most remarkable features of the quantum mechanics and it leads to applications like quantum teleportation, quantum cryptography, dense coding and quantum computing.

However a quantum state of a composed system may contain other types of non- classical correlations even if it is seprable (not entangled).

Aiming such correlation Zurek et al [PRL 88, 017901 (2001)], introduced quantum discord.

Consider a bipartite state ρ_{XY} in a composite Hilbert space $H = H_X \otimes H_Y$.

The von Neumann mutual information between X and Y is defined as

$$I(X:\mathrm{Y}) = S(\rho_X) + S(\rho_Y) - S(\rho_{X,Y})$$

where $S(\rho) = -Tr[\rho \log \rho]$ is the von Neumann entropy and $\rho_X = Tr_Y[\rho_{XY}]$.

Mutual information quantifies the total amount of correlations in quantum states.

A classically equivalent definition of mutual information is, $S(\rho_X) - S(\rho_{X|Y})$, where $\rho_{X|Y}$ is the state of X given a measurement in Y.

$S(\rho_{X|Y})$ is called conditional entropy. In quantum theory conditional entropy can be obtained by applying measurement over system Y.

Assuming perfect measurements of *Y* defined by a set of one-dimensional projectors $\{\Pi_j^Y\}$with $\sum_j \Pi_j^Y = 1$, state of subsystem X after this measurement is given by

$$\rho_{X|\Pi_j^Y} = (\Pi_j^Y \rho_{XY} \Pi_j^Y) / Tr(\Pi_j^Y \rho_{XY})$$

with probability $p_j = Tr(\Pi_j^Y \rho_{XY})$

Conditional entropy is defined as,

$$S(\rho_X | \{\Pi_j^Y\}) = \sum_j p_j S(\rho_{X|\Pi_j^Y})$$

where $S(\rho_{X|\Pi_j^Y}) = -Tr(\rho_{X|\Pi_j^Y} \log \rho_{X|\Pi_j^Y})$

Using these definitions, one can define mutual information as,

$$J(X:Y)_{\{\Pi_j^Y\}} = S(\rho_X) - S(\rho_X | \{\Pi_j^Y\})$$

Dispite both definations for the mutual information being equivalent for the classical systems, the quantum genralizations $I(X:Y)$and $J(X:Y)$ in genral do not coincide. Their difference defines quantum discord,

$$D(X:Y)_{\{\Pi_j^Y\}} = I(X:Y) - J(X:Y)_{\{\Pi_j^Y\}} = S(\rho_Y) - S(\rho_{X,Y}) + S(\rho_X | \{\Pi_j^Y\})$$

The minima of this is defined as,

$$\delta(X:Y)_{\{\Pi_j^Y\}} = \min_{\{\Pi_j^Y\}} [D(X:Y)_{\{\Pi_j^Y\}}]$$

Investigating QD in some systems is of important significance. On the one hand, it allows us to discover relevant quantum properties of systems. On the other hand, studying QD in physical systems helps and prompts us to explore the theory of QD. Most of the studies related to quantum discord remained focused on qubit systems.

We considered Werner-Like states formed by MECS's $\left|\psi^{\pm}\right\rangle_{XY}$ and NMECS $\left|\phi^{\pm}\right\rangle_{XY}$, which are defined by

$$\left|\psi^{\pm}\right\rangle_{XY} = n_{\pm}[\left|\alpha,\alpha\right\rangle \pm \left|-\alpha,-\alpha\right\rangle]_{XY}, \left|\phi^{\pm}\right\rangle_{XY} = n_{\pm}[\left|\alpha,-\alpha\right\rangle \pm \left|-\alpha,\alpha\right\rangle]_{XY}$$

These states are given by

$$\rho(\psi^{-},a) = (1-a)\frac{I}{4} + a\left|\psi^{-}\right\rangle\left\langle\psi^{-}\right| \qquad \rho(\phi^{-},a) = (1-a)\frac{I}{4} + a\left|\phi^{-}\right\rangle\left\langle\phi^{-}\right|$$

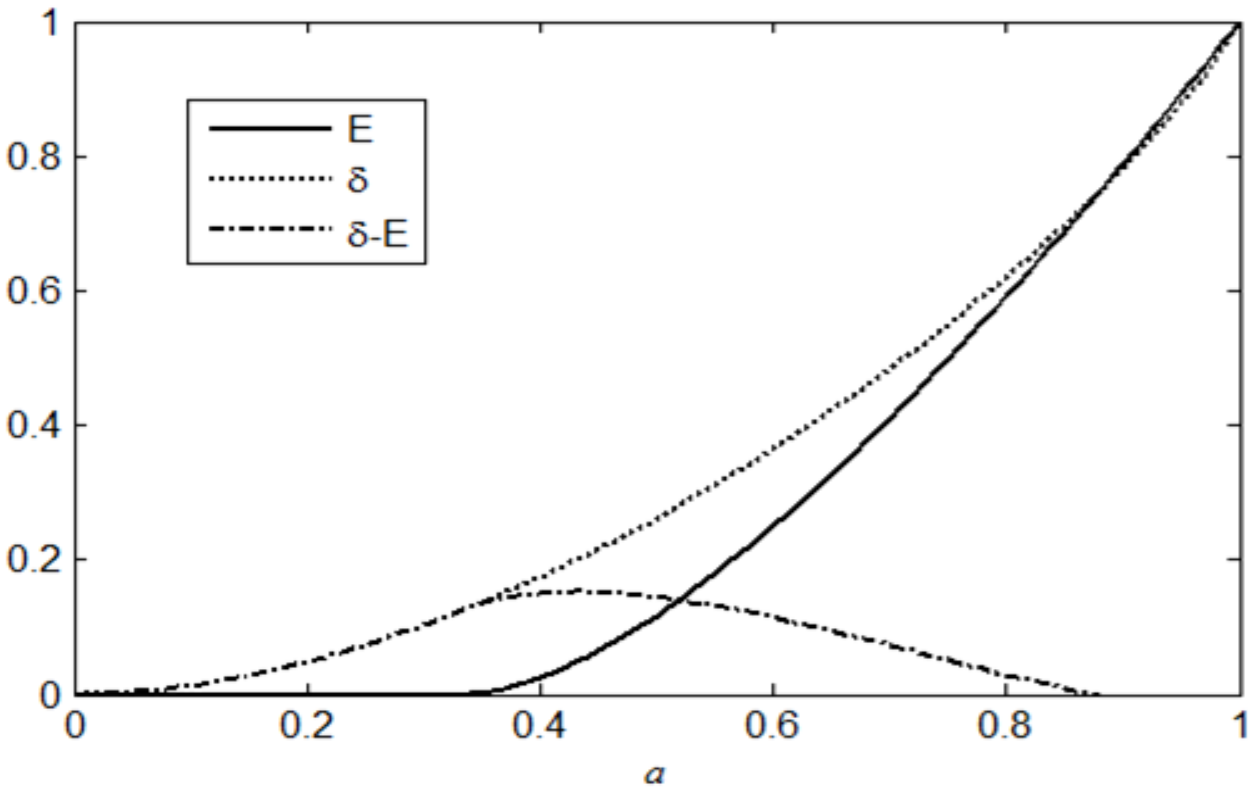
$$\rho(\psi^{+},a) = (1-a)\frac{I}{4} + a\left|\psi^{+}\right\rangle\left\langle\psi^{+}\right| \qquad \rho(\phi^{+},a) = (1-a)\frac{I}{4} + a\left|\phi^{+}\right\rangle\left\langle\phi^{+}\right|$$

We considered the measurement basis

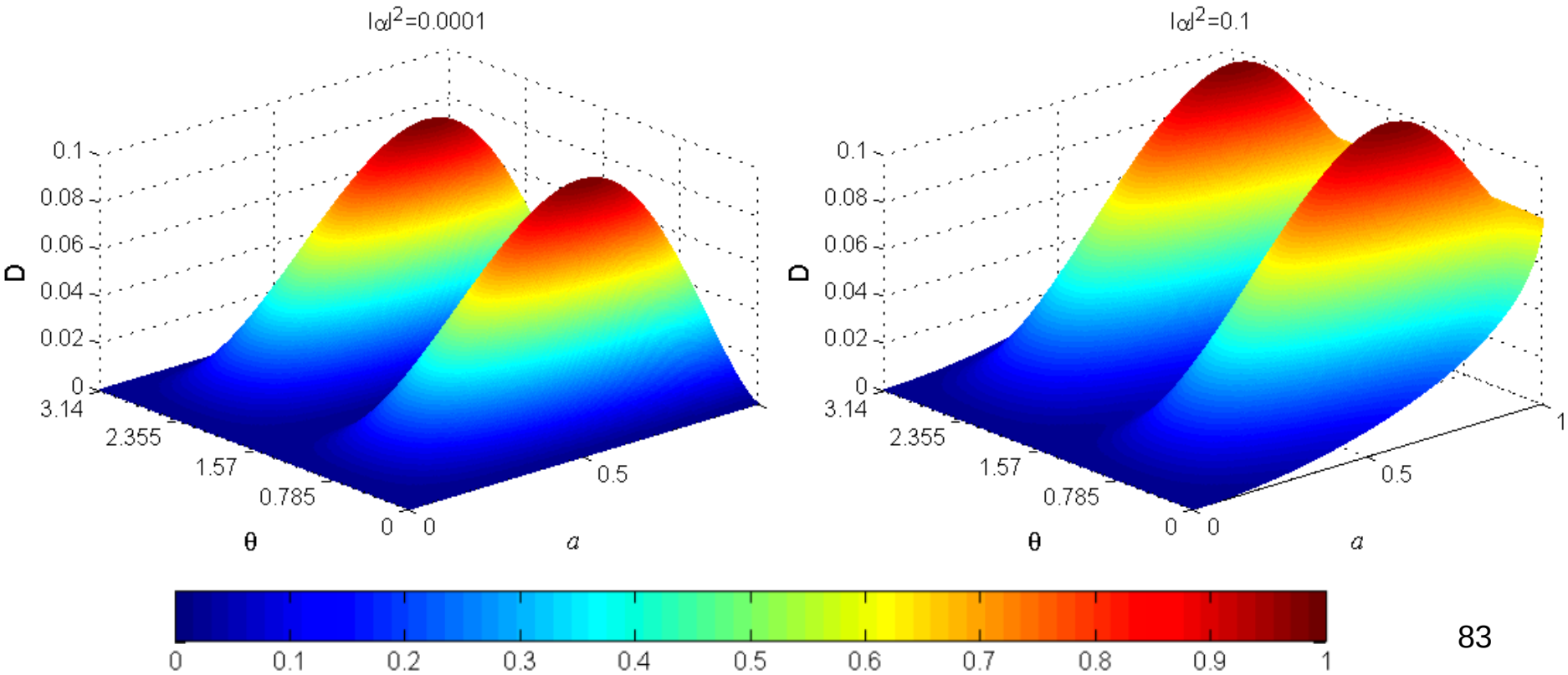
$$\left|\pi_0\right\rangle = \cos\theta\left|+\right\rangle + e^{i\phi}\sin\theta\left|-\right\rangle \qquad \left|\pi_1\right\rangle = \sin\theta\left|+\right\rangle - e^{i\phi}\cos\theta\left|-\right\rangle$$

And calculated the quantum discord

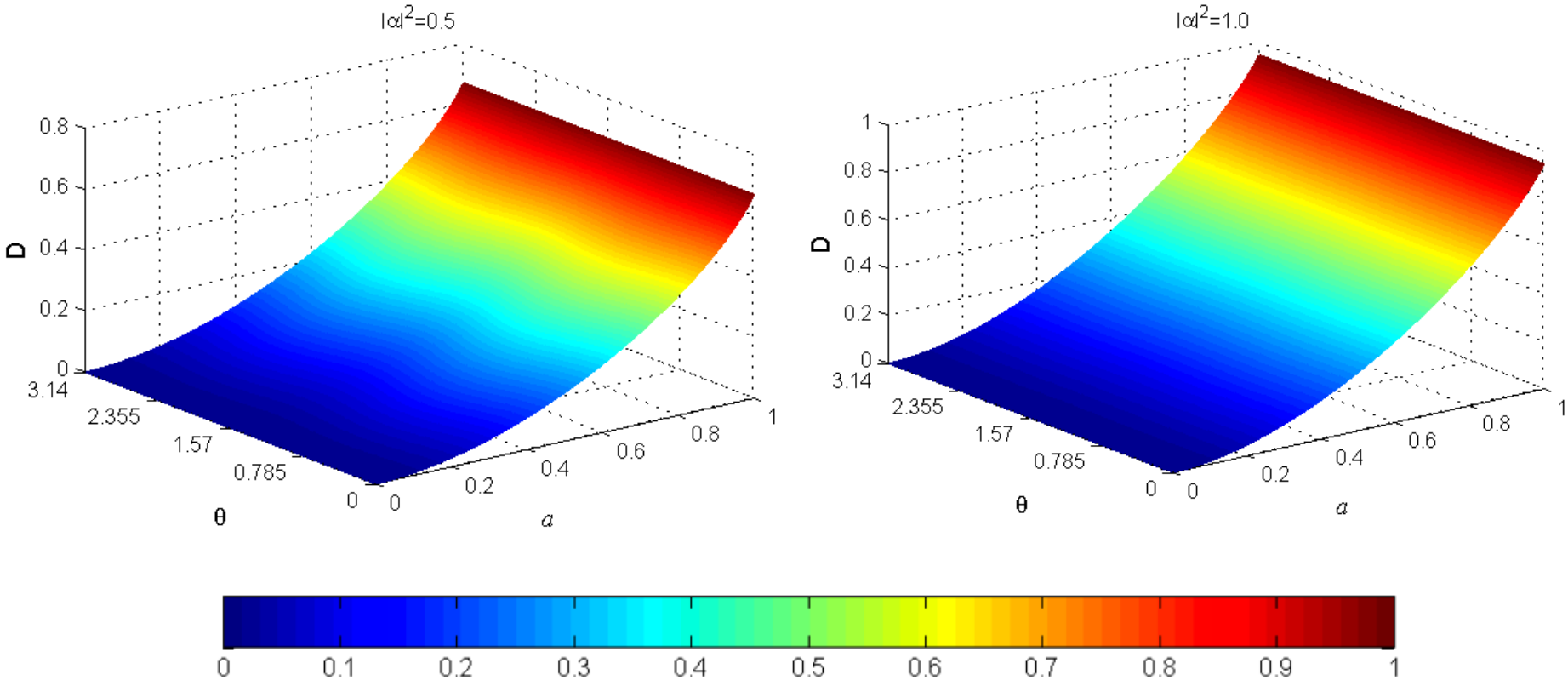
We find that the quantum discord for state $\rho(\psi^-, a)$ and $\rho(\phi^-, a)$ and found that this does not depend on angles θ, ϕ or α . We calculated the entanglement of formation of these states also and found that quantum discord δ is greater than or equal to the entanglement of formation E. The two are identical and equal to 0 at a=0 and identical and equal to 1 at a=1. D is zero in the beginning and then picks up.



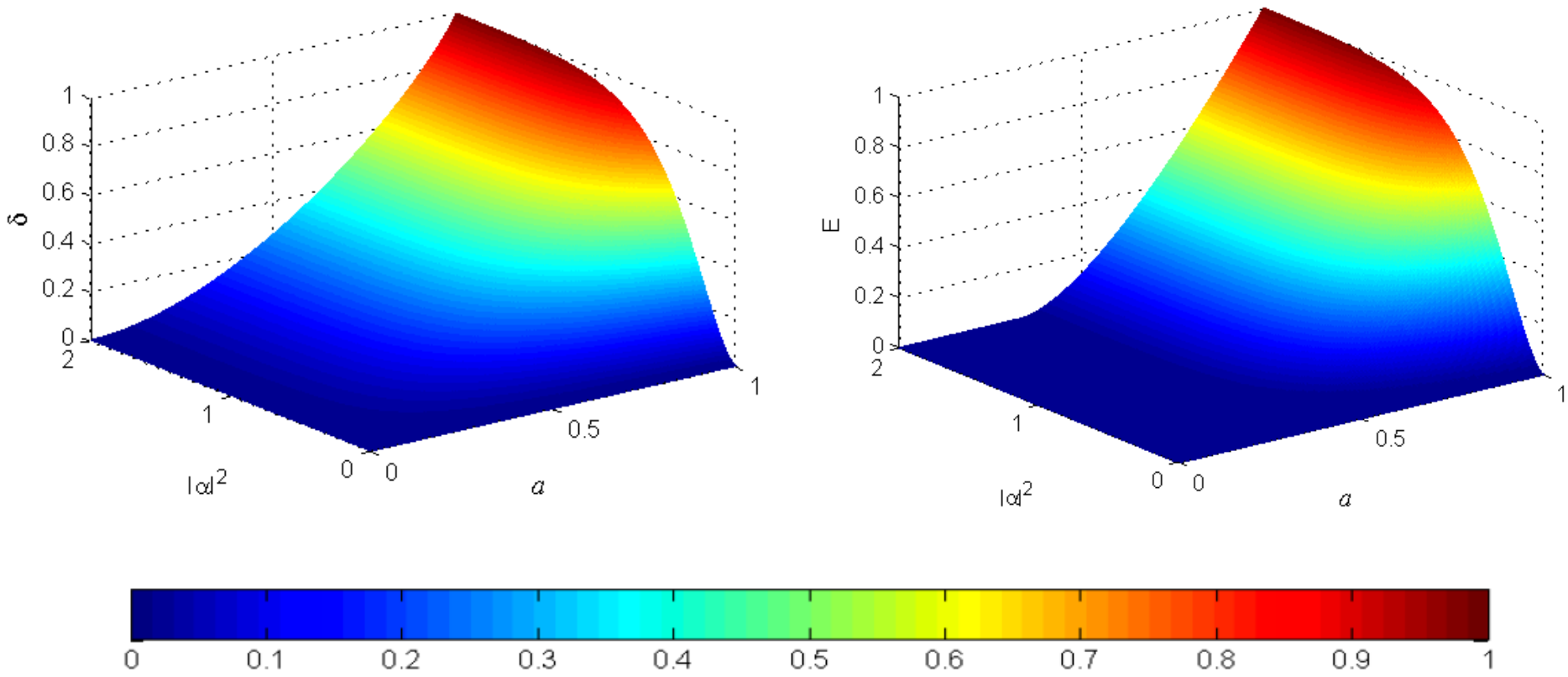
For states $\left|\psi^{+}\right\rangle_{XY}$ and $\left|\phi^{+}\right\rangle_{XY}$ which we call quasi-Werner states, however, the dependence of quantum discord D is seen on θ, and α although it is independent of ϕ. For very small $|\alpha|$, the dependence on θ is very pronounced and D first increases with a and then it decreases. Dependence on θ, however, becomes unnoticeable for $|\alpha|>1$. For this case, however, D increases uniformly with a. This is shown in the figures given here



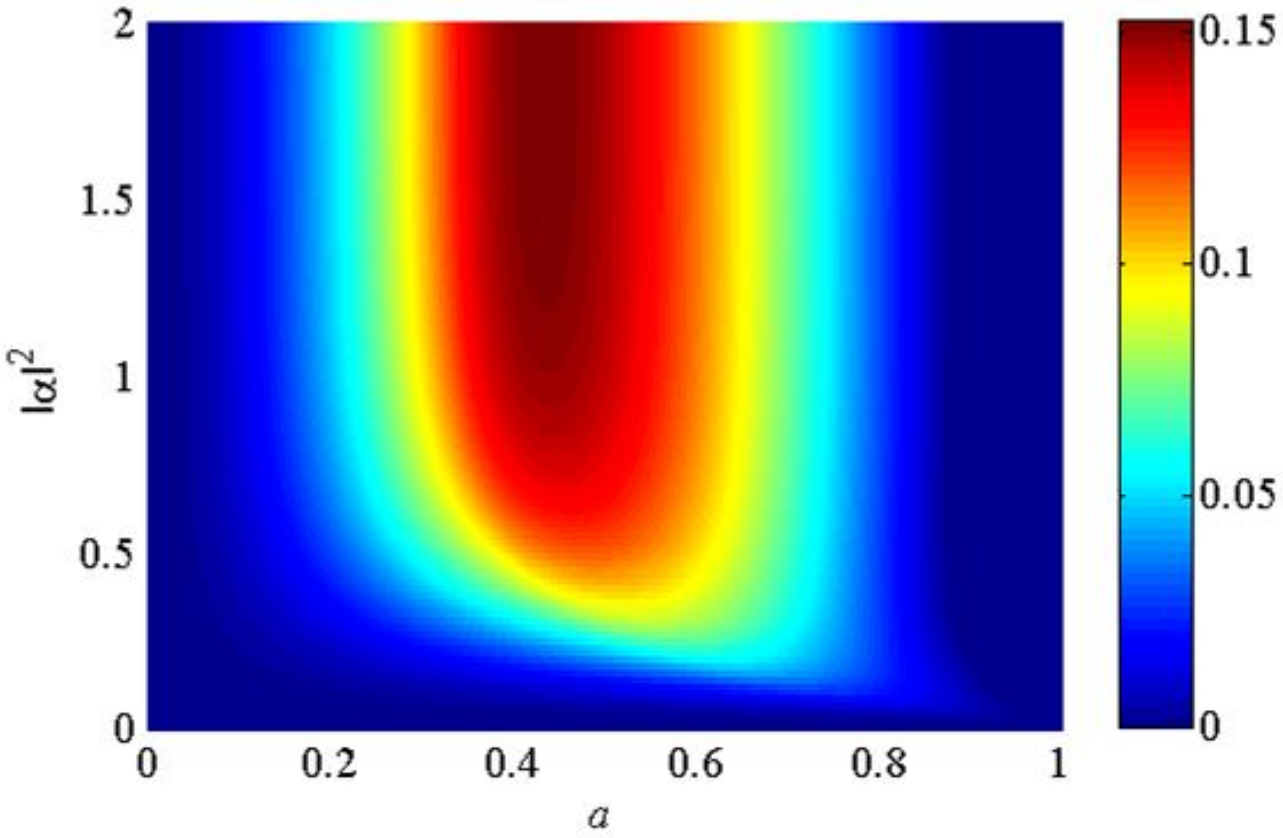
More figures are shown here for higher values of $|\alpha|$.



Variation of minimum of quantum discord against θ and of the entanglement of formation E with |α| and a are shown in the figures that follow.


δ
1
0.8
0.6
0.4
0.2
0
2
1
0
|α|2
0
0.5
1
a
E
1
0.8
0.6
0.4
0.2
0
2
1
0
|α|2
0
0.5
1
a
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

We also study the difference of δ and E against |α| and a.



We now consider a well-known cavity QED system, of single two-level atom (initially in thermal equilibrium with its environment) interacting with a cavity-fock state. The cavity we choose can have finite dimensionality more than two.

The interaction Hamiltonian is

$$H_{ac} = \beta(a^{\dagger}\sigma_{-} + a\sigma_{+})$$

where

suffix *ac* refers to atom-cavity interaction,

$a^{\dagger}(a)$ is the creation (annihilation) operator of the field mode,

β is the atom field coupling constant,

$\sigma_{\pm}$ and σ_z are the atomic raising, lowering, and inversion operators of the atom.

Time-evolution operator in the interaction picture is described by

$$U(t) = \exp(-iH_{ac}t)$$

We assume that initially atom is in thermal equilibrium represented by density matrix

$$\rho_a(0) = \lambda_0|0\rangle\langle 0| + \lambda_1|1\rangle\langle 1|$$

where

$$\lambda_0 = [1 + e^{-\omega/KT}],\ \lambda_1 = e^{-\omega/KT}/[1 + e^{-\omega/KT}]$$

and the initial state of cavity is n-photon state given by density matrix

$$\rho_c(0) = |n\rangle\langle n|$$

Initially atom-cavity has no quantum correlation, i.e., the atom-cavity joint state is a product state and write

$$\rho_{ac}(0) = \rho_a(0) \otimes \rho_c(0)$$

The state of the atom-cavity system at any arbitrary time is described by

$$\begin{aligned}\rho_{ac}(t) &= U(t)\rho_{ac}(t)U^{\dagger}(t) \\ &= \lambda_0(C_n^2|0,n\rangle\langle 0,n| + S_n^2|1,n-1\rangle\langle 1,n-1|) + \lambda_1(C_{n+1}^2|1,n\rangle\langle 1,n| + S_{n+1}^2|0,n+1\rangle\langle 0,n+1|) \\ &+ i\lambda_0 C_n S_n(|0,n\rangle\langle 1,n-1| - |1,n-1\rangle\langle 0,n|) + i\lambda_1 C_{n+1}S_{n+1}(|1,n\rangle\langle 0,n+1| - |0,n+1\rangle\langle 1,n|).\end{aligned}$$

$$C_n = \cos\psi_n,\ S_n = \sin\psi_n,\ C_{n+1} = \cos\psi_{n+1},\ S_{n+1} = \sin\psi_{n+1}, \psi_n = \beta t\sqrt{n}, \psi_{n+1} = \beta t\sqrt{n+1}.$$

To study the dynamics of quantum discord we perform an ideal von Neumann projection measurement on atom by a complete set of one-dimensional projector

$$\left|\pi_0^a\right\rangle = C\left|0\right\rangle + zS\left|1\right\rangle, \left|\pi_1^a\right\rangle = z^* S\left|0\right\rangle - C\left|1\right\rangle$$

where $C = \cos\theta \quad S = \sin\theta \quad z = \exp(i\phi)$

and satisfies the completeness relation,

$$\sum_{j=0,1} \left|\pi_j^a\right\rangle\left\langle\pi_j^a\right| = I$$

Cavity state after measurement on atom corresponding to outcomes { π_j^a } is

$$\rho_{c\left|\pi_j^a\right.}(t) = [Tr_a(\left|\pi_j^a\right\rangle\left\langle\pi_j^a\right|\rho_{ac}(t)\left|\pi_j^a\right\rangle\left\langle\pi_j^a\right|)]/P_j$$

where P_j is the probability of outcome π_j^a given by

$$P_j = Tr[\left|\pi_j^a\right\rangle\left\langle\pi_j^a\right|\rho_{ac}(t)]$$

The quantum discord for this system is given by

$$\begin{aligned} D(c:a)_{\{\Pi_j^a\}} &= I(c:a) - J(c:a)_{\{\pi_j^a\}} \\ &= S(\rho_a) - S(\rho_{ac}) + S(\rho_{c\left|\{\Pi_j^a\}\right.}), \end{aligned}$$

The expression for quantum discord is given by

$$D(c:a) = -\{(\lambda_0 C_n^2 + \lambda_1 S_{n+1}^2)\log(\lambda_0 C_n^2 + \lambda_1 S_{n+1}^2)$$
$$+ (\lambda_0 S_n^2 + \lambda_1 C_{n+1}^2)\log(\lambda_0 S_n^2 + \lambda_1 C_{n+1}^2)\} + \{\lambda_0 \log\lambda_0 + \lambda_1 \log\lambda_1\}$$
$$-\Bigg[\frac{P_0}{2}(1+\sqrt{1-4y^{\pi_0}})\log(1+\sqrt{1-4y^{\pi_0}}) + (1-\sqrt{1-4y^{\pi_0}})\log(1-\sqrt{1-4y^{\pi_0}})\}$$
$$+\frac{P_1}{2}\{(1+\sqrt{1-4y^{\pi_1}})\log(1+\sqrt{1-4y^{\pi_1}}) + (1-\sqrt{1-4y^{\pi_1}})\log(1-\sqrt{1-4y^{\pi_1}})\}\Bigg],$$

where $P_0 = \lambda_0(C^2 C_n^2 + S^2 S_n^2) + \lambda_1(C^2 S_{n+1}^2 + S^2 C_{n+1}^2)$

$$P_1 = \lambda_0(S^2 C_n^2 + C^2 S_n^2) + \lambda_1(S^2 S_{n+1}^2 + C^2 C_{n+1}^2)$$

$$y^{\pi_0} = \frac{1}{P_0^2}[\lambda_0\lambda_1(C^4 C_n^2 S_{n+1}^2 + S^4 S_n^2 C_{n+1}^2 + C^2 S^2 S_n^2 S_{n+1}^2)]$$

$$y^{\pi_1} = \frac{1}{P_1^2}[\lambda_0\lambda_1(S^4 C_n^2 S_{n+1}^2 + C^4 S_n^2 C_{n+1}^2 + C^2 S^2 S_n^2 S_{n+1}^2)]$$

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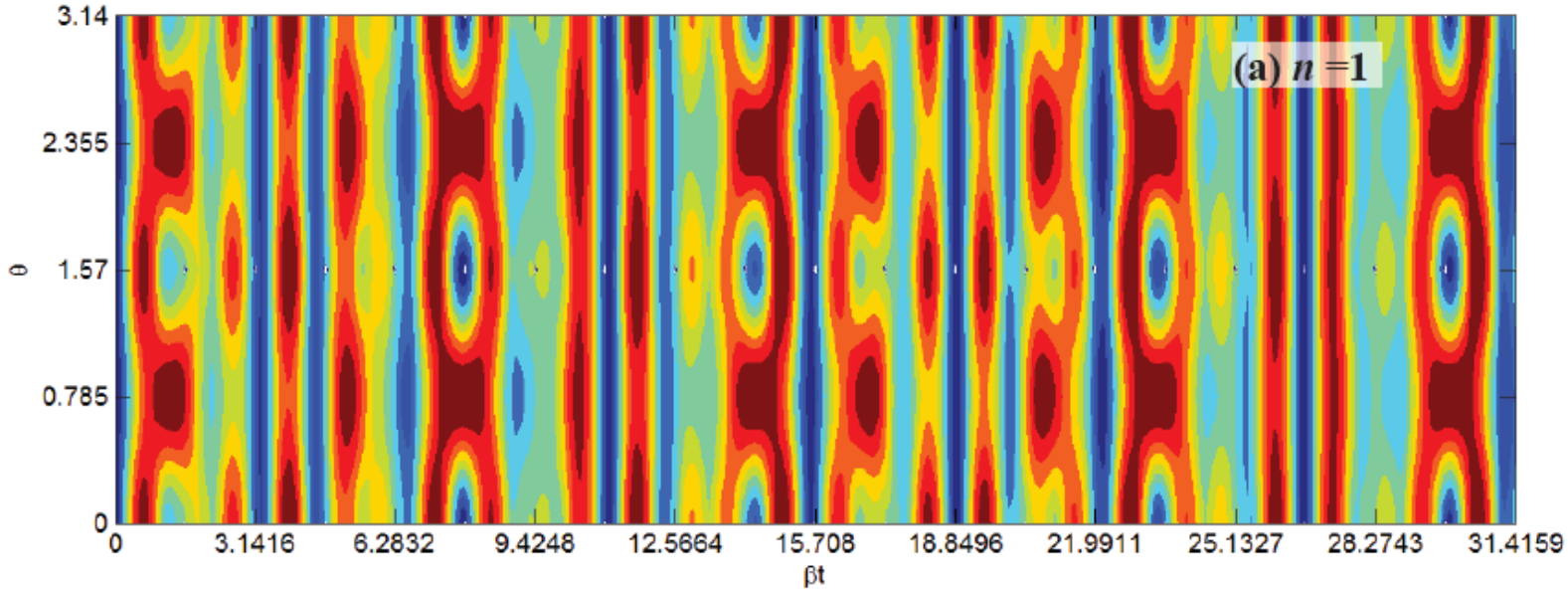
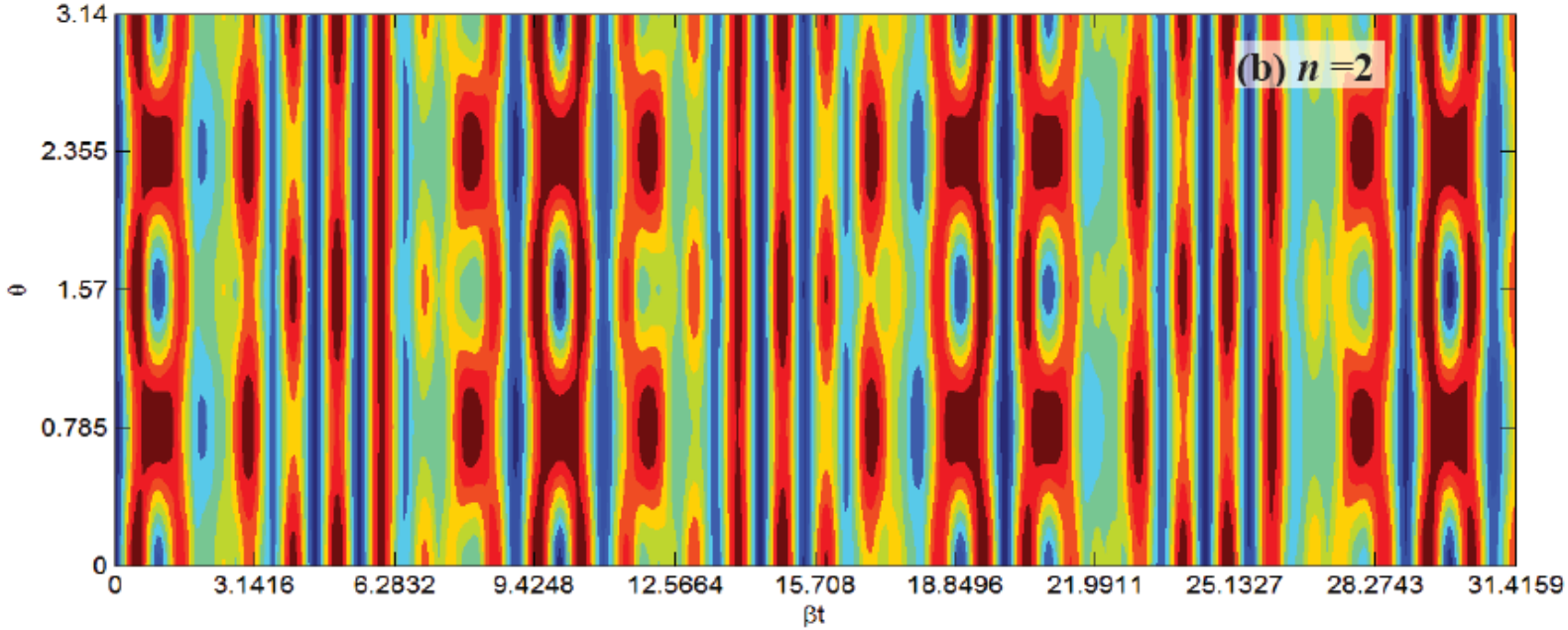




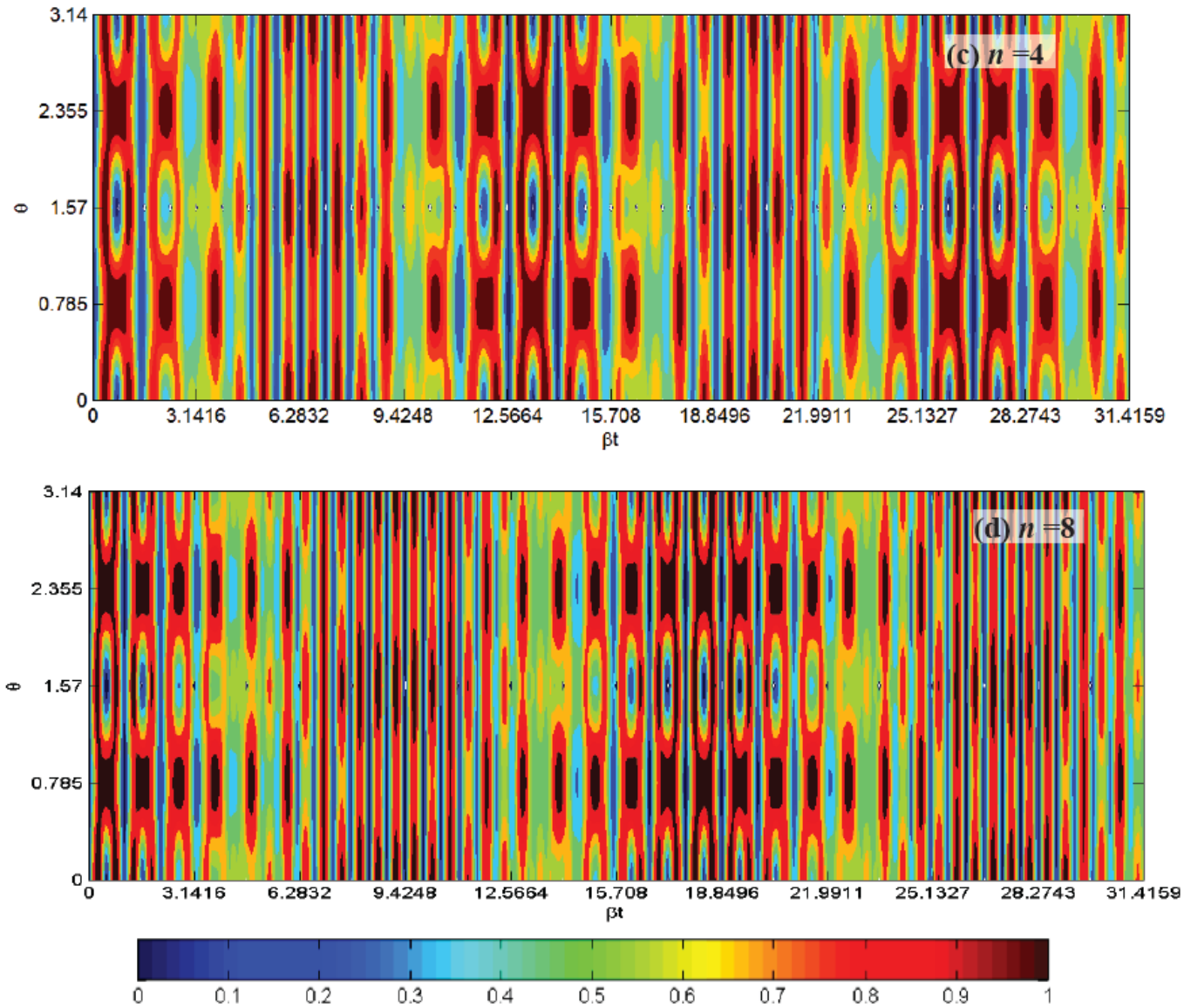


Fig.1 Quantum discord D(c:a) with respect to interaction time and measurement parameter for n=1,2,4,8 number of photons initially in cavity.

Consider the case $\lambda_0 = \lambda_1 = 0.5$, i.e., at initial time $(t = 0)$ the state of the atom is

$\rho_a(0) = (1/2)(|0\rangle\langle 0| + |1\rangle\langle 1|)$ that correspond to the limit of very high temperatures.

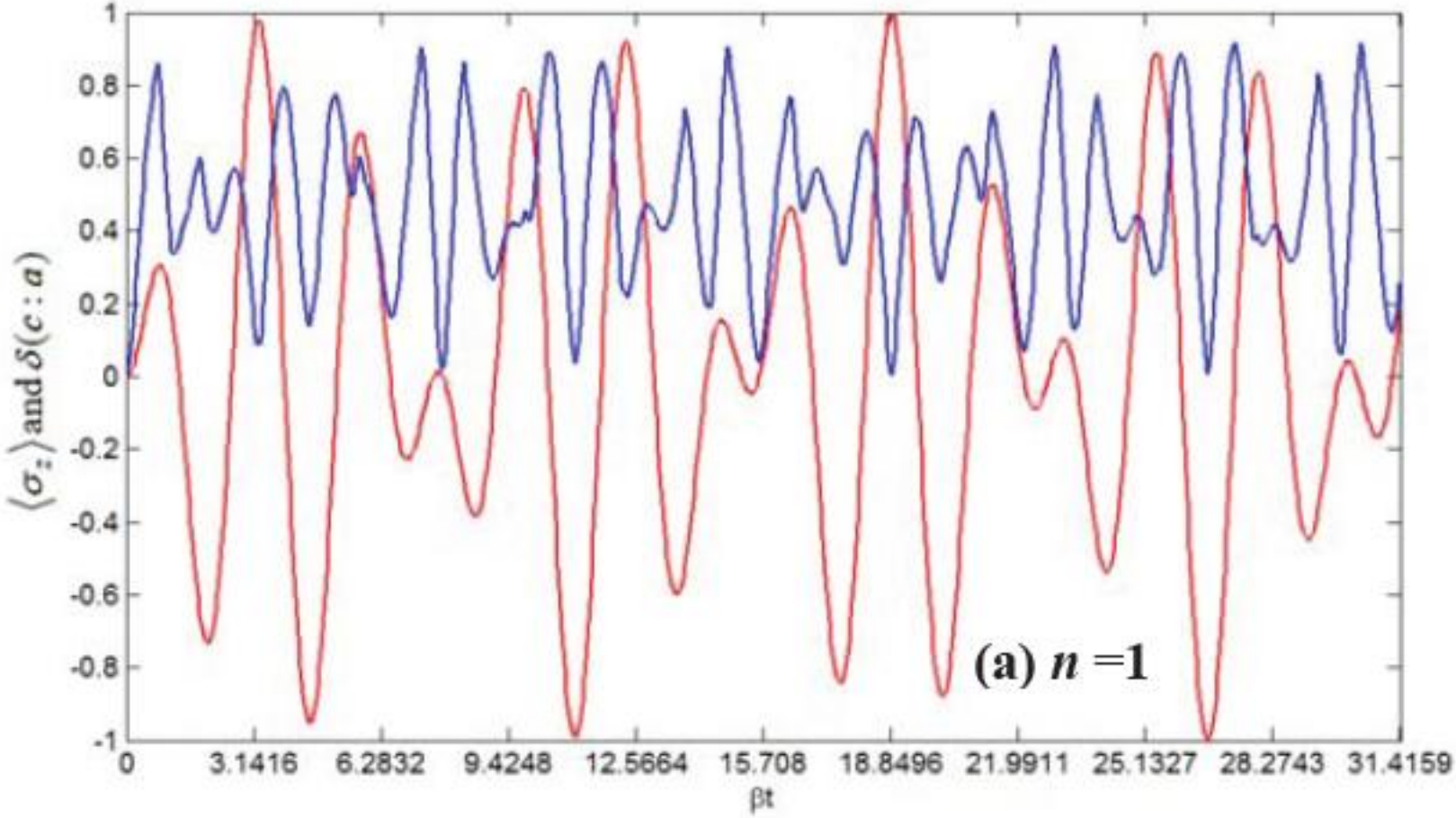
With this initial condition on atom, Figs 1(a), (b), (c), and (d) show the contour plot of quantum discord with respect to interaction time (βt) and measurement parameter θ , for initial photonic Fock state of cavity with n =1, 2, 4 and 8 numbers of photons, respectively.

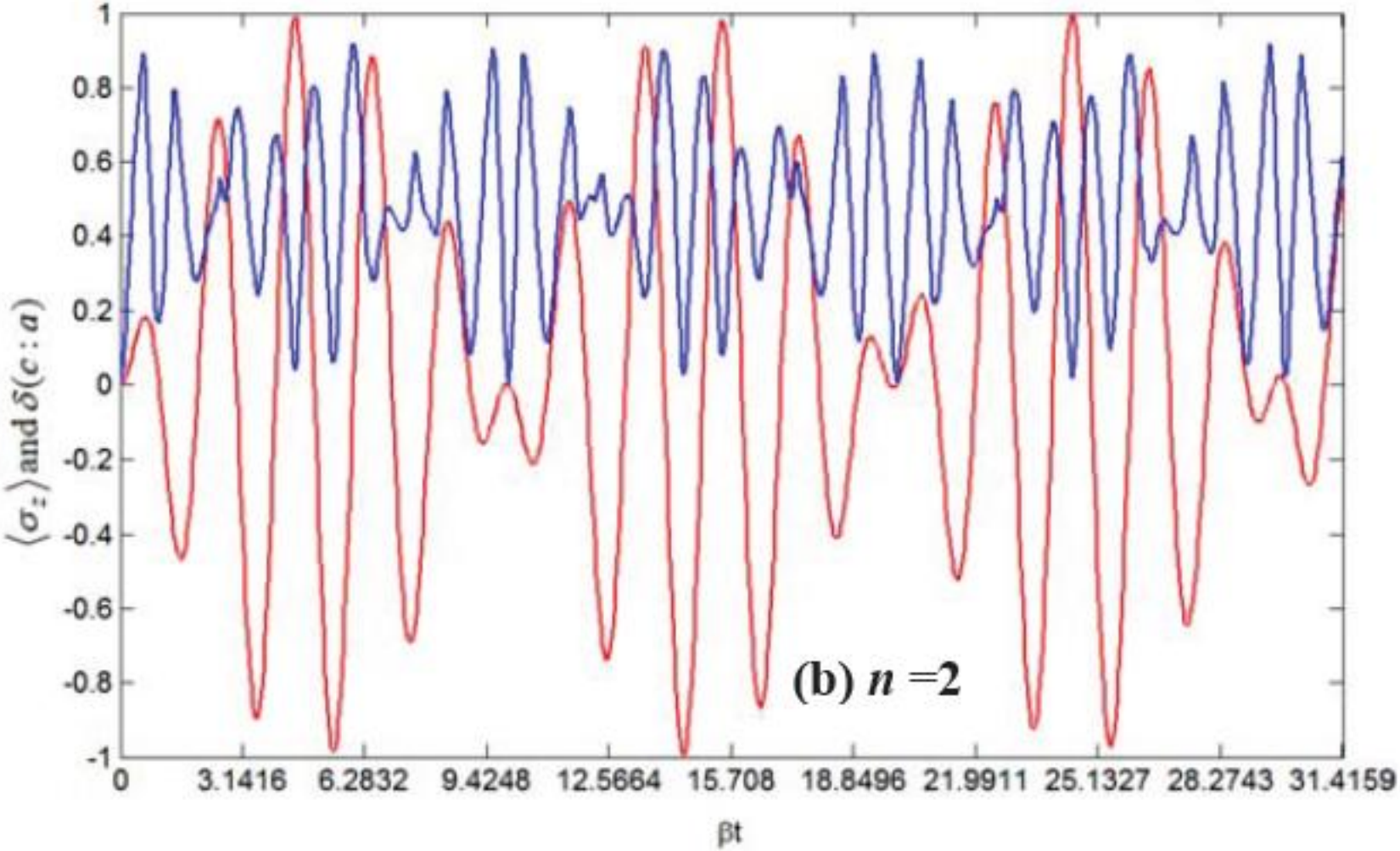
Fig. 1 (a) shows the interesting case that there are certain ranges of interaction times for which the quantum correlation or discord between atom and the cavity do not vanish for any measurement basis (i.e., for any value of measurement parameter θ).

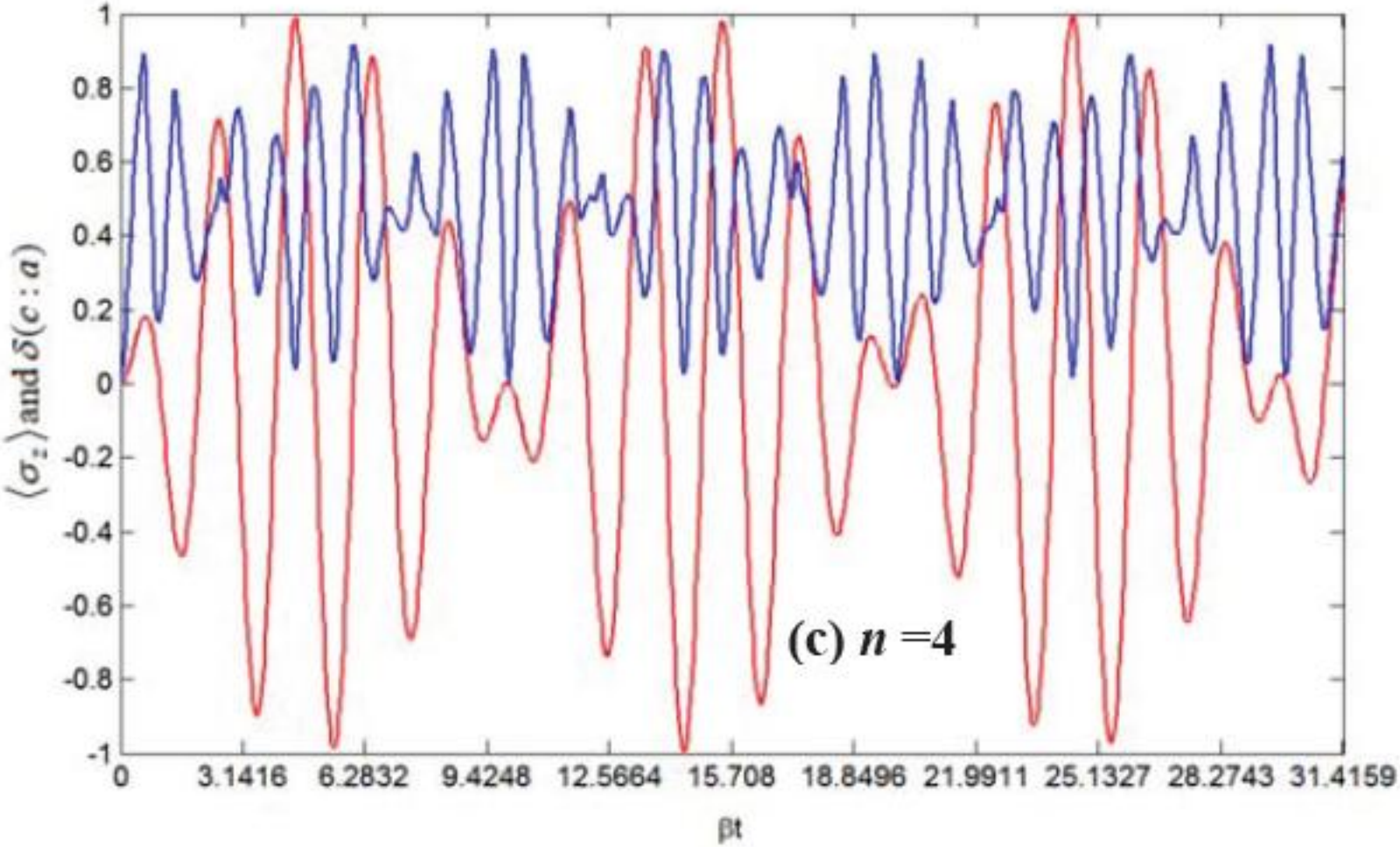
Figs. 1 (a-d) also show that, for any measurement basis, the quantum discord is rapidly oscillating in nature in a complex way with respect to interaction time.

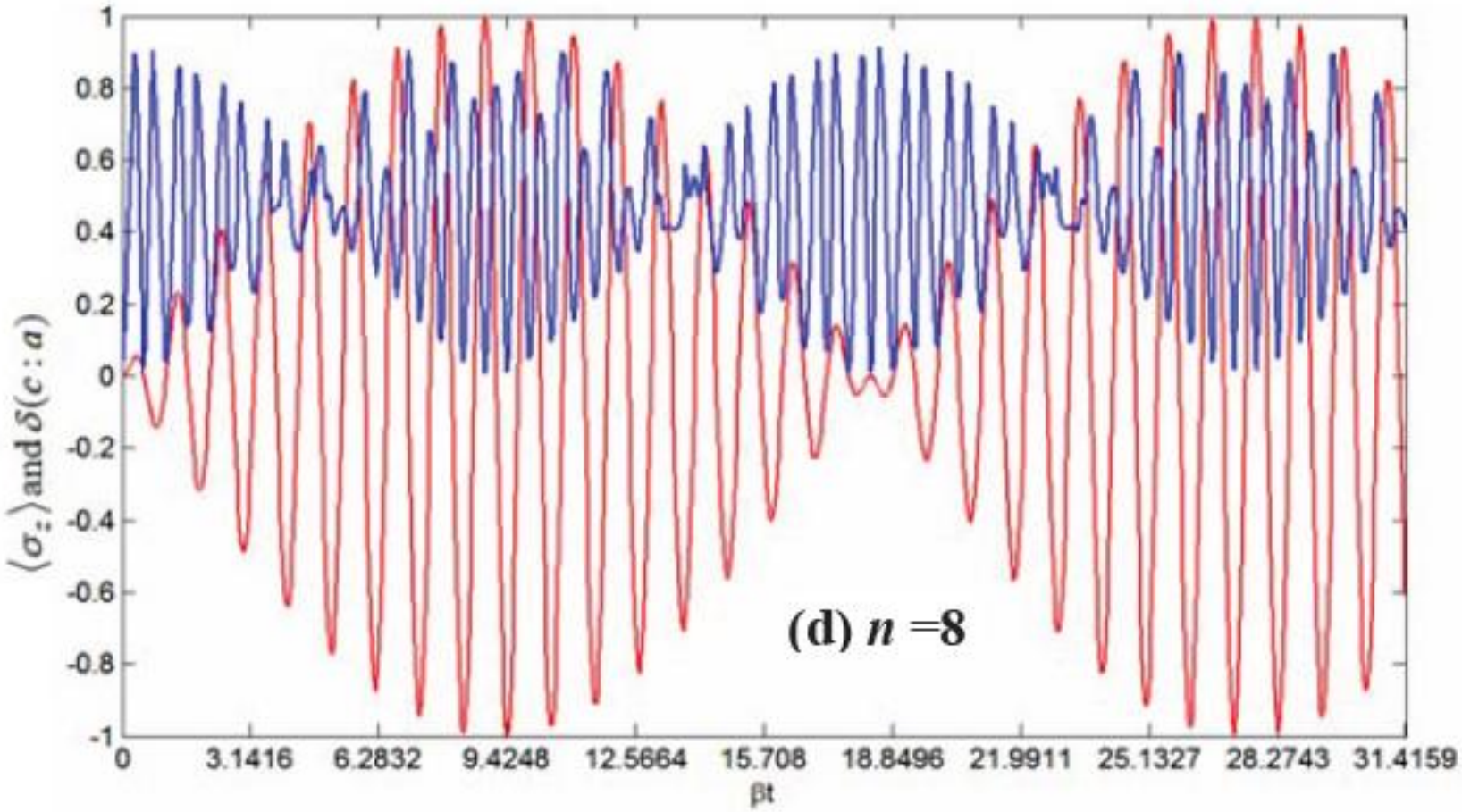
However, the frequency of oscillation increases with increase in numbers of photon initially in cavity.

This is also evident by the fact that the density of the fringes that represents the variation of quantum discord increases as the number of photons increase from n=1 to n=8.


1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
$\langle\sigma_z\rangle$ and $\delta(c:a)$
0
3.1416
6.2832
9.4248
12.5664
15.708
18.8496
21.9911
25.1327
28.2743
31.4159
βt
(a) n =1


1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
$\langle\sigma_z\rangle$ and $\delta(c:a)$
0
3.1416
6.2832
9.4248
12.5664
15.708
18.8496
21.9911
25.1327
28.2743
31.4159
βt
(b) n =2


1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
$\langle\sigma_z\rangle$ and $\delta(c:a)$
0
3.1416
6.2832
9.4248
12.5664
15.708
18.8496
21.9911
25.1327
28.2743
31.4159
βt
(c) n =4


1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
$\langle\sigma_z\rangle$ and $\delta(c:a)$
0
3.1416
6.2832
9.4248
12.5664
15.708
18.8496
21.9911
25.1327
28.2743
31.4159
βt
(d) n =8

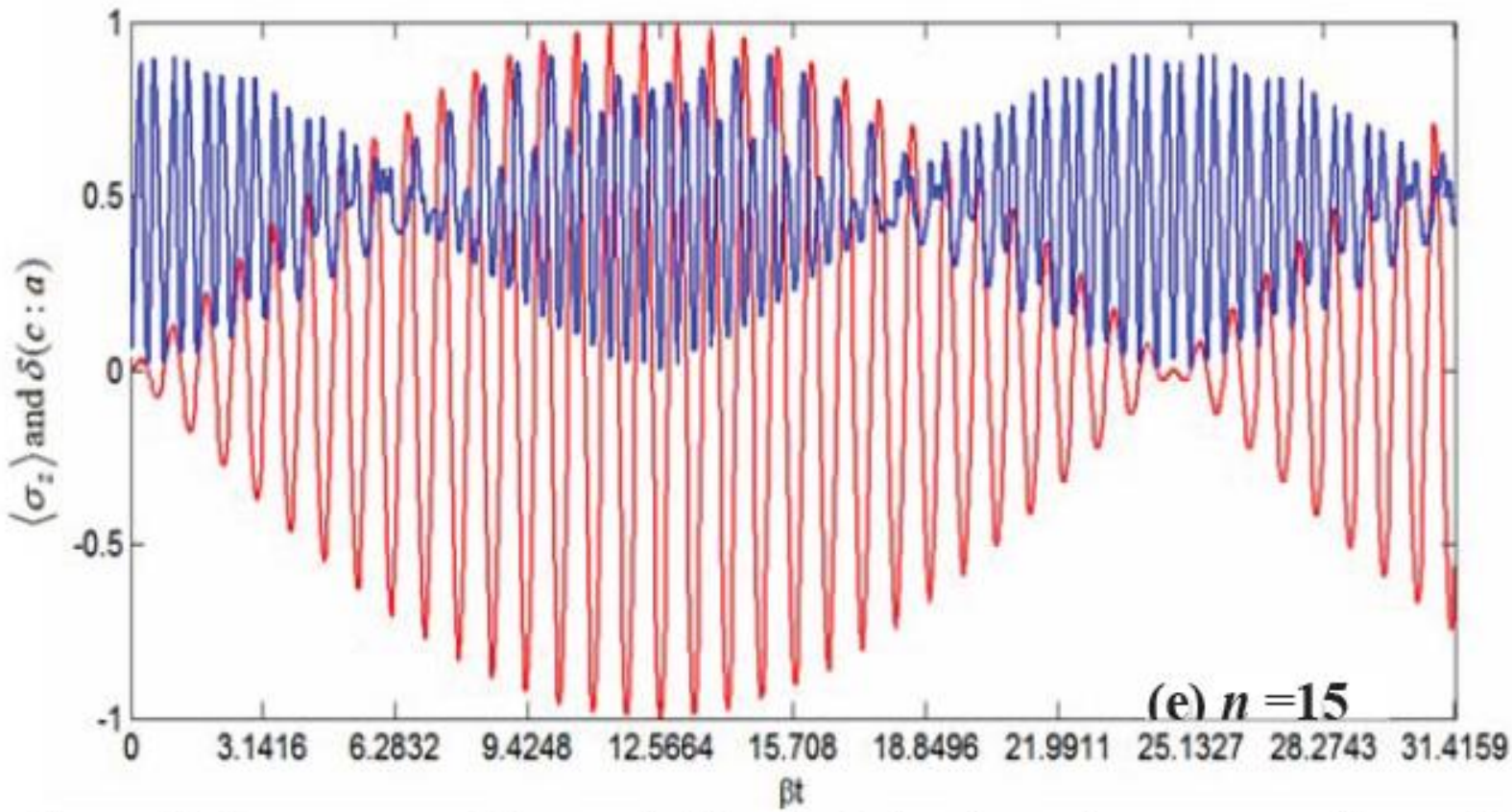


Fig 2 Minimum value of Quantum discord δ(c:a) (blue) and mean value of inversion operator (red) with respect to interaction time for n =1, 2, 4, 8, 15 number of photons initially in cavity.

We also note that the quantum discord is showing phenomenon of beats with interaction time.

Figs. 2 (a-e) show the minimum value of quantum discord over the measurement basis and the mean value of inversion operator for n=1, 2, 4, 8, 15 with respect to interaction time.

It is clear that both the minimum quantum discord and inversion operator exhibit the phenomenon of beats with interaction time.

The frequency of oscillation increases rapidly with increasing number of photons.

The mean inversion operator is given by $\langle \sigma_z \rangle = \sin(\beta t[\sqrt{n+1}+\sqrt{n}])\sin(\beta t[\sqrt{n+1}-\sqrt{n}])$

The first factor involves larger frequency and gives phase of the oscillations.

The mean frequency of these oscillations is $\beta(\sqrt{n+1}+\sqrt{n})$ and the mean periodic time is

.

$$2\pi/\beta(\sqrt{n+1}+\sqrt{n})$$

The amplitude of oscillations oscillates itself periodically and the beat frequency is

$2\beta(\sqrt{n+1}-\sqrt{n}) = 2\beta/(\sqrt{n+1}+\sqrt{n})$ and the beat period is $(\pi/\beta)(\sqrt{n+1}+\sqrt{n})$

In one beat-period number of oscillations is

$$(\sqrt{n+1}+\sqrt{n})/(\sqrt{n+1}-\sqrt{n}) = \tfrac{1}{2}[2n+1+2\sqrt{n(n+1)}]$$

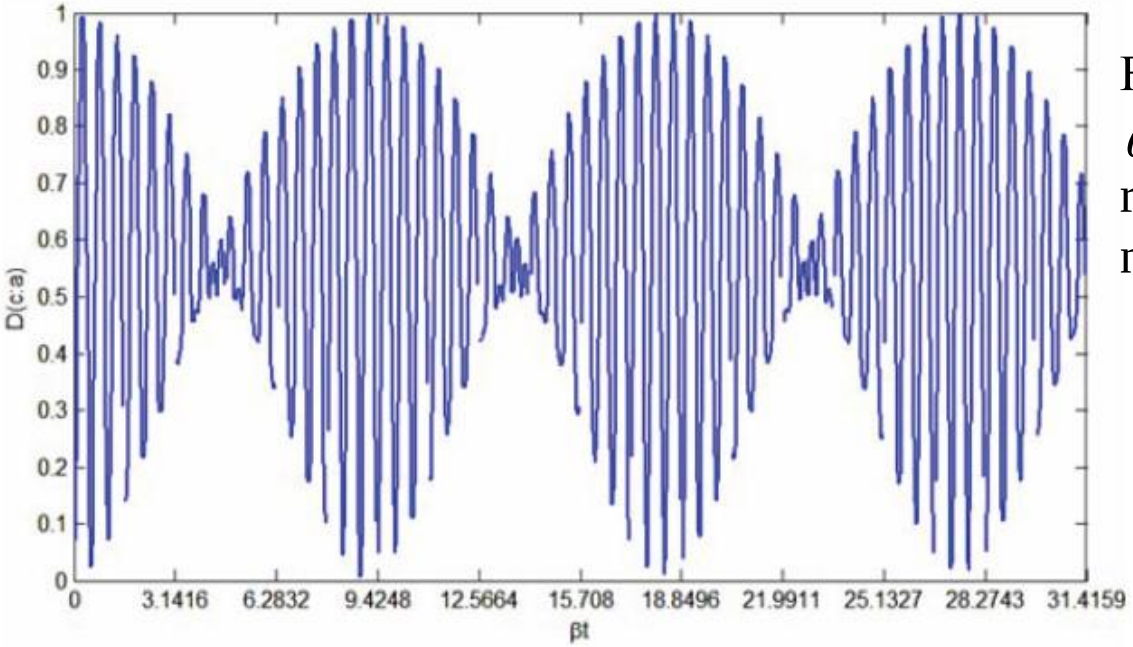
For $n >> 1$, the periodic time and the beat period are $(\pi/\beta)\sqrt{n}$ and $(2\pi/\beta)\sqrt{n}$, respectively, i.e., during two minima of amplitude of the oscillations, about 2n oscillations take place.

The cases for n=1, 2, 4, 8 and 15 are shown in Figs 2(a-d).

For largest n, n=1 in Fig 2d, the number of oscillations in one beat period is nearly 31 while the expression $\frac{1}{2}[2n+1+2\sqrt{n(n+1)}]$ gives 30.99 and the approximate result gives 2n=30 oscillations.

Quantum discord D is a periodic function of θ with the period $\pi/2$.

There is no general periodicity with the interaction time.



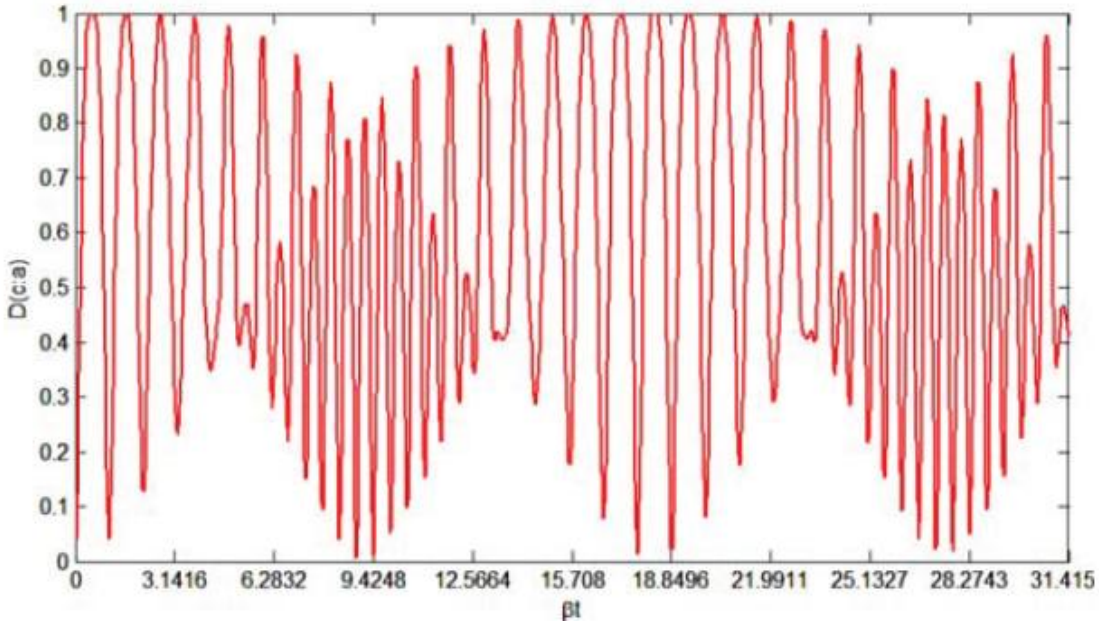


Fig 3 Quantum discord δ(c:a) for $\theta = \pi/2$ (blue) and $\theta = \pi/4$ (red) with respect to interaction time for n=8 number of photons initially in cavity.

For $\theta = \pi/2$ and $\theta = \pi/4$ behaviour of D is shown in Fig. 3.

For $\theta = \pi/2$ oscillations of D occur with a period which is 1/2 of the period for $\langle \sigma_z \rangle$

The beat period is also one half of the corresponding value for $\langle \sigma_z \rangle$.

For $\theta = \pi/4$ the oscillatory behaviour of D is very complicated and interesting. For some intervals the period is same as that for $\langle \sigma_z \rangle$ while for remaining intervals it is one half of that for $\langle \sigma_z \rangle$.

For minimum (against θ) quantum discord δ , it is seen that its behaviour is oscillatory with βt and there appear beats in the amplitude.

The period of oscillation of δ is about one half of that for oscillations of $\langle \sigma_z \rangle$ as is clear from Figs &.2 (a)-(e).

The beat period of δ is also about one half of that for $\langle \sigma_z \rangle$.

One other interesting observation is that, for even beats, the maxima of are alternately high and low although this behaviour is absent for minima (Fig. 2(d) and (e) for n = 8 or 15).

In addition to this we also see that the high maxima in even beats show a dip.

Thank You for your kind attention