Shannon Entropy as a signature of entanglement dynamics of two cold atoms in an optical double well trap

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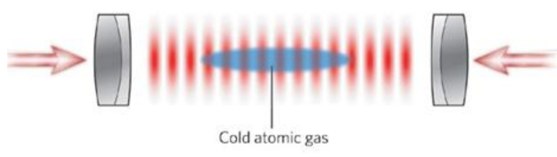
Abstract

We study the coherent dynamics of two neutral cold atoms trapped in an optically created double well and show that the spatial entropy can be a definitive signature of entanglement as an alternative to concurrence. This physical model, involving tunneling and contact interaction, is akin to the Hubbard model which explains the physics of interacting particles in periodic potentials. We show that an interplay between tunneling and contact interaction produces Bell like eigenstates ensuring thereby that the modulated amplitude of concurrence and the Shannon entropy, two quantities with vastly different physical interpretations, develop the similar time dependence. We apply our result to an experimentally realized system.

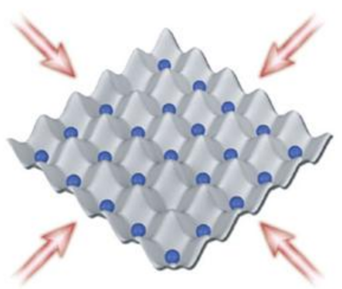
I. Motivation

Cold atoms as a tool for quantum information and scalable quantum computation

Advanced laser cooling and trapping techniques for cold atoms in optical lattices allow for:



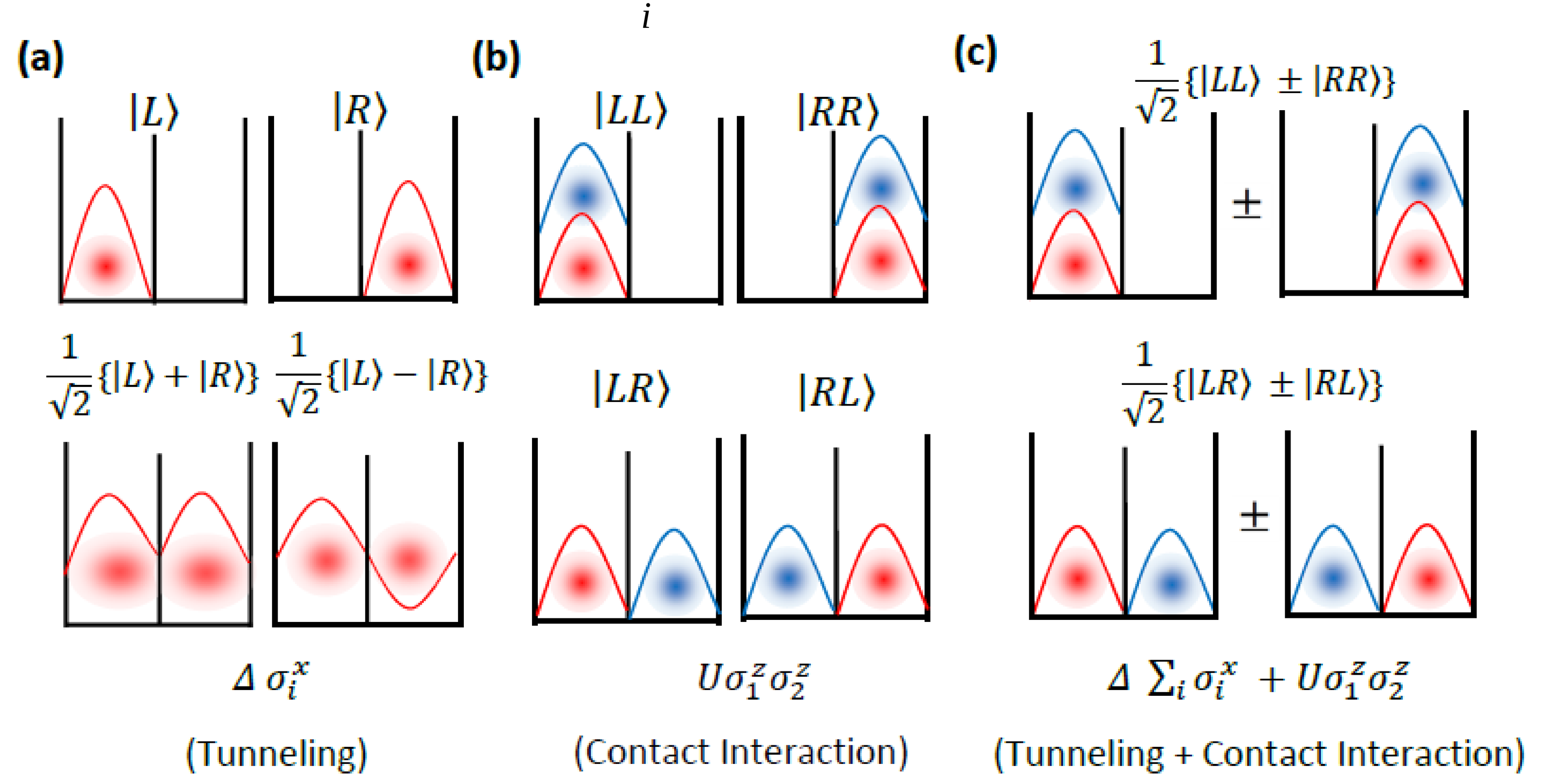
- the experimental engineering of interesting quantum states and Hamiltonians
- the high precision control of inter particle interaction
- the creation and coherent manipulation of entanglement
-the ability to address individual atoms
- the simulation of qubits, quantum gates and small-scale quantum computations
-the quantum simulation of condensed matter physics



II. Our System

Two cold atoms in an optically created double well

$$H = E_0 I + \Delta \sum_i \sigma_i^x + U\sigma_1^z \otimes \sigma_2^z$$



Two particles in a double well equivalent to **two qubits** with **left** ($|L\rangle$) and **right** ($|R\rangle$) localization representing single qubit states.

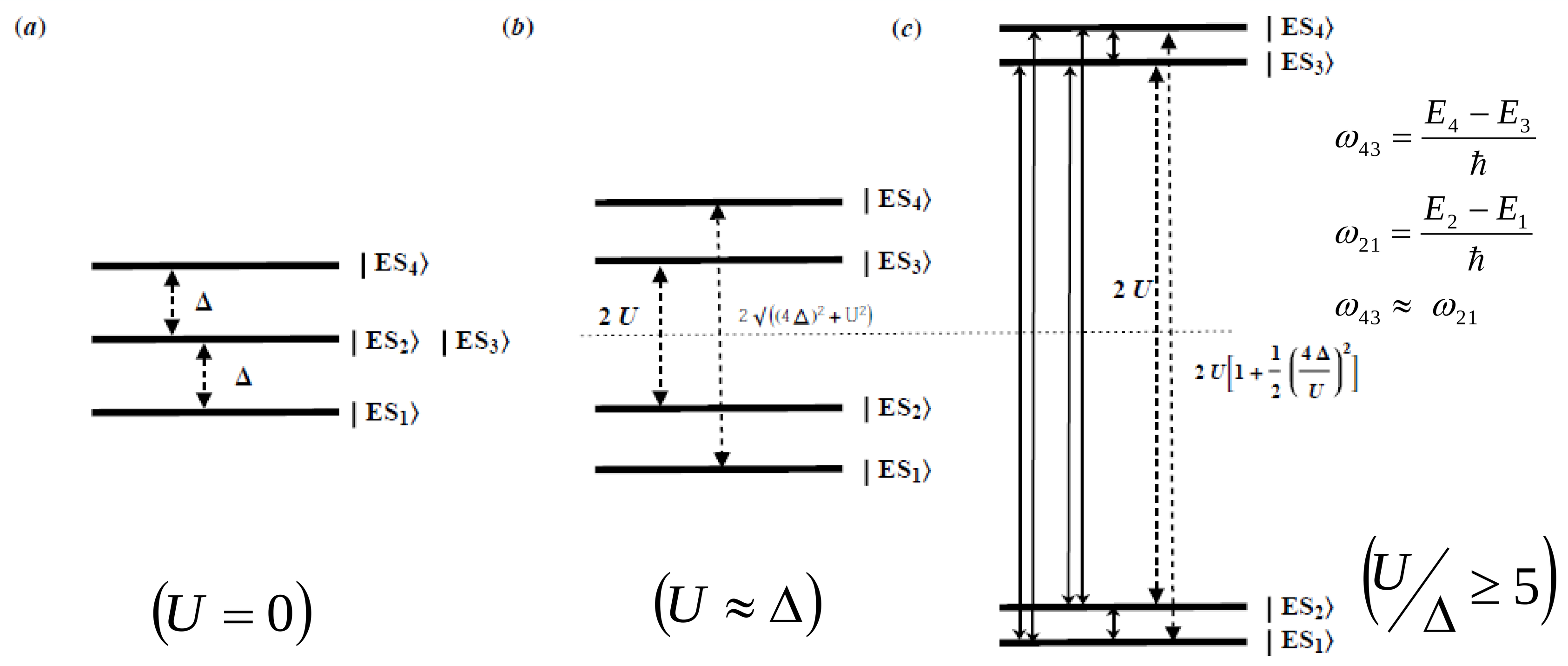
The two particles can **tunnel** between the wells and **interact** with each other.

Tunneling strength Δ, controlled by the barrier height.

Contact Interaction, $U = 2\hbar\sqrt{\omega_y \omega_z}\; a_s\, \delta(x_1 - x_2)$

m: mass of atom, ω_y, ω_z **:** transverse confinement frequencies
a_s **:** s-wave scattering length

III. Eigenvalue spectrum



IV. Eigenstates

$$|ES_1\rangle = -p\left(\frac{1}{\sqrt{2}}\big[|LR\rangle + |RL\rangle\big]\right) + q\left(\frac{1}{\sqrt{2}}\big[|LL\rangle + |RR\rangle\big]\right) \qquad |ES_2\rangle = \left(\frac{1}{\sqrt{2}}\big[|LR\rangle - |RL\rangle\big]\right)$$

$$|ES_3\rangle = \left(\frac{1}{\sqrt{2}}\big[|LL\rangle - |RR\rangle\big]\right) \qquad |ES_4\rangle = p\left(\frac{1}{\sqrt{2}}\big[|LL\rangle + |RR\rangle\big]\right) + q\left(\frac{1}{\sqrt{2}}\big[|LR\rangle + |RL\rangle\big]\right), \quad p > q$$

When *(U/Δ ≥ 5), p>>q* , the eigenstates become **Bell states** in positional basis.

V. Time Evolution

Eigenvalue spectrum (III & IV) drives the time evolution of an initial state.

$$\psi(x_1, x_2, 0) = \sum_{i=1,4} c_i |ES_i\rangle$$

When ***(U/Δ ≥ 5)***, eigenstates *{ESi}'s* become nearly **Bell states**. In this parameter regime for a general initial state, we calculate:

(a) Time dependent Shannon Entropy
(b) Square of the time dependent Concurrence

Shannon Entropy:

$$\begin{aligned} S_H(t) &= -\{P_{LL}Log_2(P_{LL}) + P_{LR}Log_2(P_{LR}) + P_{RL}Log_2(P_{RL}) + P_{RR}Log_2(P_{RR})\} \\ &\approx A\left\{\boxed{\alpha_1 \cos(2\omega_{21}t) + \alpha_2 \cos(2\omega_{43}t)}\right\} + C_0 \end{aligned}$$

where P_{LL} , P_{LR}, P_{RL} and P_{RR} are the probabilities of localization.
Note that $\boldsymbol{S_H(t)}$ contains only the low frequency terms **(ω_{21} , ω_{43}).**

Square of Concurrence:

$$\begin{aligned} |C|^2 &= C_1 + P_1 \cos(2\omega_{21}t) + P_2 \cos(2\omega_{43}t) + P_3 \cos(2\omega_{42}t) \\ &\quad + P_4 \cos(2\omega_{41}t) + P_5 \cos(2\omega_{32}t) + P_6 \cos(2\omega_{31}t) \end{aligned}$$

$$|C|^2 \approx B\left\{\boxed{\alpha_1 \cos(2\omega_{21}t) + \alpha_2 \cos(2\omega_{43}t)} + \beta\right\} \times \{\gamma \cos(2\omega_{31}t + \theta) + 1\}$$

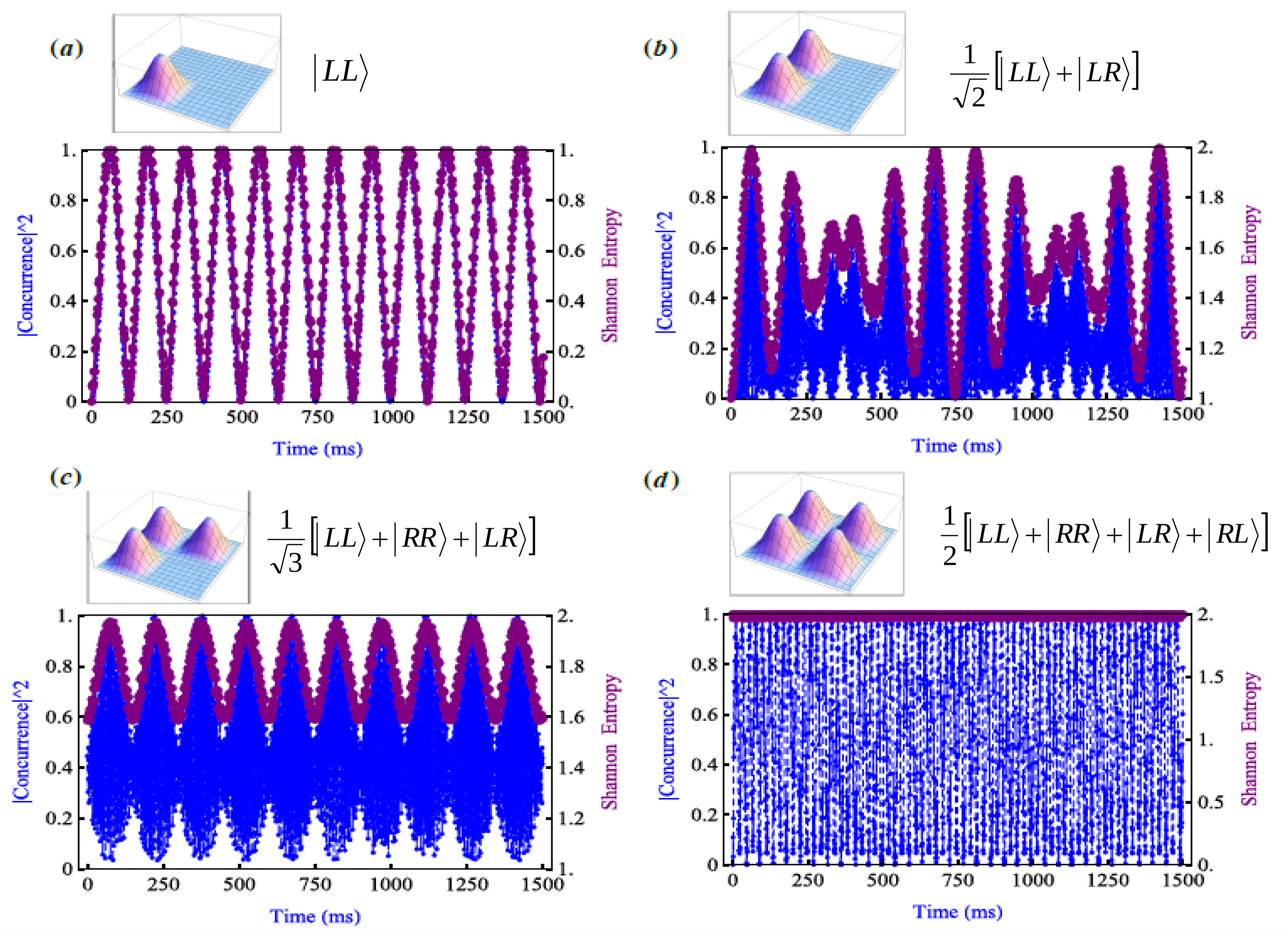
Note that the square of concurrence is a beat like phenomena with its envelope having a time dependence similar to that of the spatial entropy.

VI. Applying our result to a real experiment[3]

Murman et al. have trapped two ultracold Lithium atoms (each in two different hyperfine states) into the motional ground state of an optical dipole trap mimicking a double well.

Experimental parameters***:***

U/h ≈ 650 Hz,
Δ/h ≈ 67 Hz,
U/Δ ≈10



VII. Conclusions

When contact interaction dominates tunneling, Bell like eigenstates are produced ensuring thereby that the modulated amplitude of concurrence and the Shannon entropy, two quantities with vastly different physical interpretations, develop the similar time dependence. This makes Shannon entropy an alternative novel measure of entanglement.

VIII. References

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