2022-2023

A REPORT ON

# INFOSYS VISITORS

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एच आर आई
HRI
ATOMS IN THE SERVICE OF THE NATION

HARISH-CHANDRA
RESEARCH INSTITUTE
CHHATNAG ROAD, JHUNSI, PRAYAGRAJ 211019

## Year 2022-23

**HRI, PRAYAGRAJ**

# INFOSYS VISITOR ANNUAL REPORT

This is a report on Infosys Visitors to Harish-Chandra Research Institute, Prayagraj in the year 2022-23. In this year we had 4 visitors, two in Mathematics and two in Physics. Visitors in Mathematics were, Prof. Jean-Marc Deshouillers and Prof. Ram Murthy and in Physics the visitors were, Prof. Henriette Elvang and Prof. Susha Parameswaran.

Prof. Deshouillers is a professor at the Institute de Mathematique de Bordeaux, France and he works in Number Theory. Prof. Ram Murthy is from Queen's University, Canada and he also works in Number Theory.

Prof. Henriette Elvang is a professor at University of Michigan and she works on Scattering Amplitudes. Prof. Susha Parameswaran is a professor at University of Liverpool, UK and she works on String Phenomenology.

The reports will be presented in anti-chronological order.

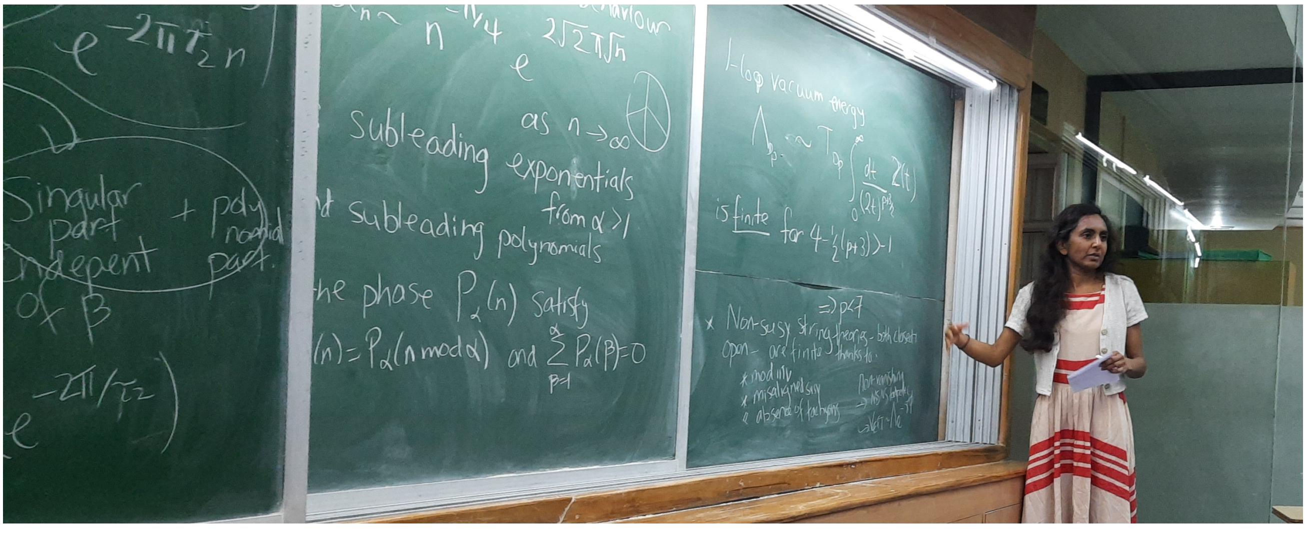

## NON-SUPERSYMMETRIC STRING THEORIES

Prof. Susha Parameswaran is a faculty in the University of Liverpool, UK. She did her Ph.D. from University of Cambridge, UK. She is one of the experts in the field of String phenomenology.

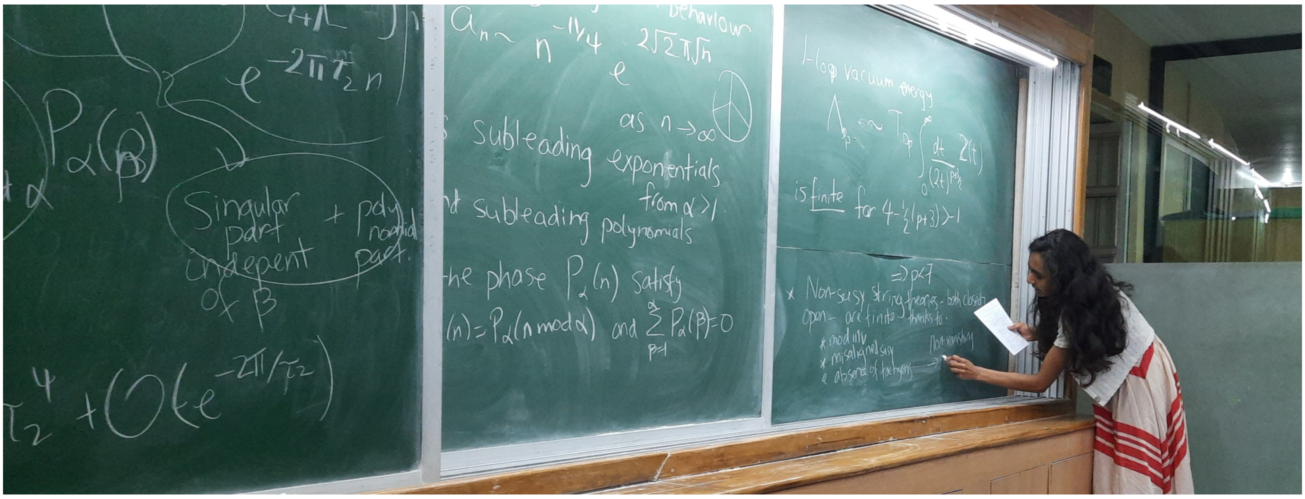
Prof. Parameswaran visited Harish-Chandra Research Institute from April 8 to April 15, 2023. During her visit she gave four lectures on Non-supersymmetric string theories. Each lecture was 90 minutes long and was followed by a discussion session during which non-experts asked questions and she clarified their doubts. These sessions were meant to be an hour long but they often far exceeded those limits. She covered an incredible amount of ground to give a glimpse of non-supersymmetric string theories. She also gave a broad picture of the number of areas, both in physics and mathematics, which feed into the investigations in this field as well as the number areas in physics in which string phenomenology is already having significant impact. Below we give a brief summary of her lectures. As mentioned above the discussion sessions were mostly centred around the topics covered in the lectures.


## SUMMARY OF THE LECTURE COURSE

- In her lectures, Prof. Parameswaran gave an introduction into non-supersymmetric string theories, starting from their construction and rather beautiful formal properties, and reaching contemporary ideas as to their possible relevance to descriptions of our Universe.




- She began with a brief review on how to construct the bosonic and superstrings, leading to the three tachyon-free non-supersymmetric closed string theories. There is a lot of literature on these string theories and it is not humanly possible to cover all of them is a week long course. However, She touched upon all the points that were relevant for the expositions in the subsequent lectures.




- After giving the flavour of the non-supersymmetric string theories without tachyonic instability, she discussed the Scherk-Schwarz symmetry breaking mechanism, and followed it up by a more detailed exposition of some non-supersymmetric open string models. The non-supersymmetric open string models invariably contains non-perturbative solutions called the D-branes. These D-brane configurations gives rise to a novel phenomenon of brane supersymmetry breaking.

- Modular invariance of the partition function is critically important in string theory. Among many implications, absence of the ultra-violet divergences is one such important implication. Prof. Parameswaran then showed how the modular invariance of the partition function is closely connected with mixing of ultra-violet-infra-red energy scales.

- Prof. Parameswaran then introduced the concept of the mis-aligned supersymmetry and proceeded to show how putting together modular invariance, the mixing of energy scales, and the mis-aligned supersymmetry ensures finiteness of all tachyon-free non-supersymmetric string theories. In the end she showed how these investigation give insights into longstanding problems in particle physics and cosmology. *E.g.*, the hierarchy problem, the cosmological constant problem, etc.

- Prof. Parameswaran also discussed the interplay between these investigations and the Swampland programme which puts constraints on the structure of the theory of quantum gravity. Swampland programme is a curious mix of the bottom-up and top-down approaches to the theory of quantum gravity. It assumes the central role of string theory as a fundamental theory and imposes constraints on a possible theory of quantum gravity. It also uses the constrains from experiments and observations to impose low energy compatibility constraints.

- Realistic model building using string compactifications invariably need to deal with the Swampland constraints. Keeping this in mind Prof. Parameswaran provided a reasonably detailed summary of how to account for the Swampland constraints while studying non-supersymmetric string compactifications.

- Covering such a vast and technical topic which has to satisfy multiple physical constraints coming both from high and low energy physics within a set of 4 lectures was monumental task but Prof. Parameswaran did a stupendous job in her lectures.

# PROBABILISTIC AND ANALYTIC METHODS IN ADDITIVE NUMBER THEORY

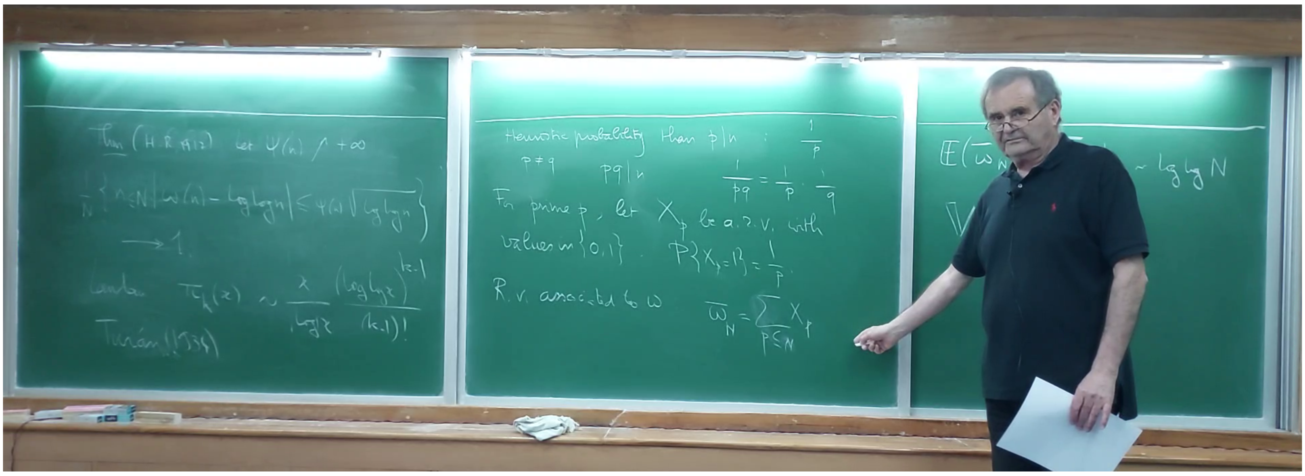
Prof. Jean-Marc Deshouillers is a professor at the Institute de Mathématiques de Bordeaux. He did his Ph.D. from University of Paris VI Pierre et Marie Curie. He is one of the leading experts in Analytic and Combinatorial Number theory. He has worked in analytic and combinatorial number theory, probabilistic number theory, finite automata as well as applied probability and statistics. In these subjects he has published more than 127 papers and made a significant contribution in all these field. Apart from this, he has advised 18 PhD students, many of whom are well established and internationally well recognised leading experts in their area.

## Notable Works

Some of the notable works by Professor Deshouillers includes his work with Henryk Iwaniec on Kuznetsov trace formula, his famous work with R. Balasubramanian and François Dress in 1985 related to Waring's problem, his work with Kawada and Wooley in 2005 related to Waring's problem, resolution of ternary Goldbach's conjecture with Effinger and Herman te Riele under the assumption of Generalised Riemann Hypothesis and his work with Drmota and Müllner towards the resolution of Sarnak's conjecture for automatic sequences in 2015.

He has also refined the probabilistic model of Erdös and Renyi along with François

Hennecart and Bernard Landreau which provides a heuristic understanding of several problems in additive number theory. The model of Erdös and Renyi had defect that it does not account for certain congruence conditions and as a result predicts certain results which we know are false. The model introduced by Deshouillers, Hennecart and Landreau remedies this defect. This work is also related to the Lecture series given by him during his visit at HRI and is explained in a little more details below.



## Visit to HRI

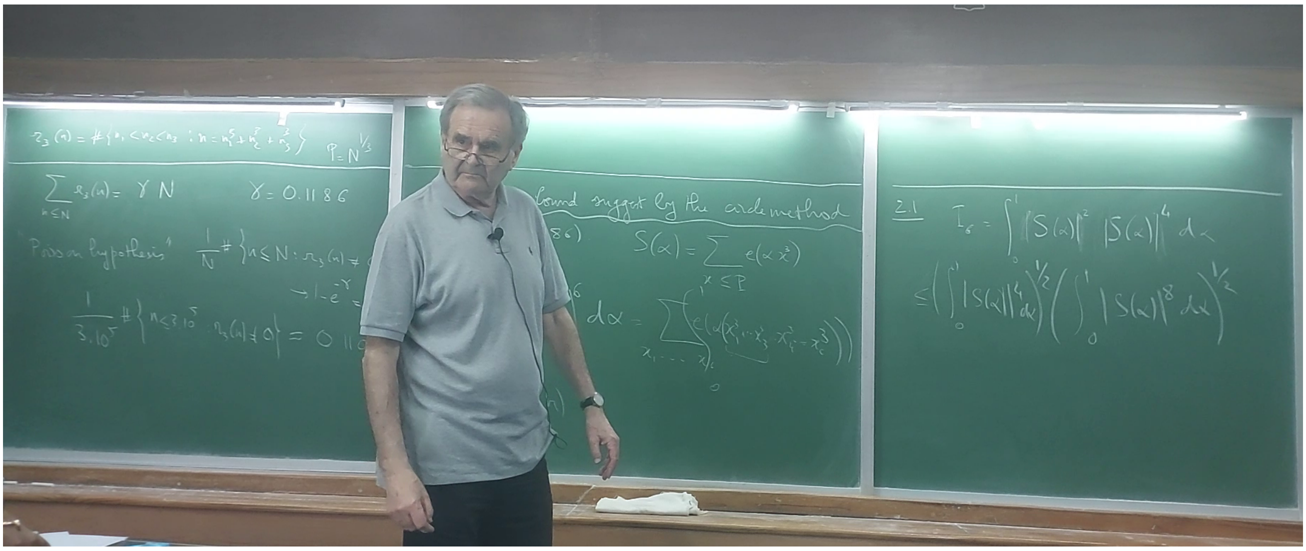
Professor Deshouillers visited HRI during 14th February 2023 to 2nd March 2023. During this visit he interacted with faculty members, students, post-doctoral fellows working at HRI as well as people working in other institutes in India who visited HRI to interact with him during his visit. He gave four lectures in a lecture series titled "Probabilistic and Analytic Methods in Number theory". The duration of each lecture was 90 minutes and it was followed by discussions with him over coffee. Moreover he was available for interaction in office throughout his visit.

## Summary of the Lecture Course

The lectures dealt with three variants of Waring's problem. For one of this variant, `essentially' a complete answer is known. However, for the other two variants, our knowledge is far from complete. He illustrated the situation with specific examples.

For an instance we know since 1939, that every natural number can be expressed as a sum of 9 cubes and moreover there are natural numbers which can not be expressed as a sum of 8 or fewer cubes. We can ask the another variant of the same problem.

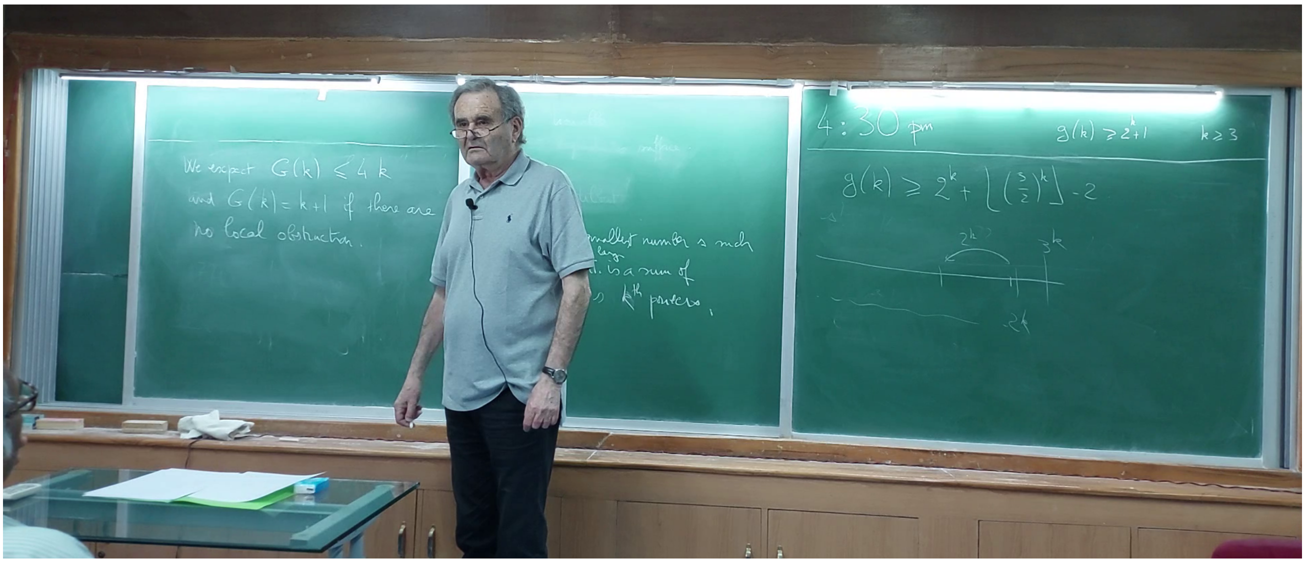
What is the minimum natural number G(3) such that every sufficiently large natural number can be written as a sum of G(3) many or fewer cubes. We expect that G(3) = 4, whereas the best known result is the result proved by Linnik in 1943 which proved that G(3) ≤ 7. We can ask another variant of the same problem. What is the minimum number of summands P(3) required so that the density of the set of natural numbers which can be written as a sum of P(3) or fewer cubes is positive? We expect that P(3) = 3, however this is not known. Moreover it is not entirely clear (even conjecturally) what should be the precise value of the density of the set consisting of those natural numbers which can be written as a sum of three cubes.



In such a situation one defines a probabilistic model in which the set of integers called `pseudo-cubes' (more generally `pseudo k-th powers' for k ≥ 2) mimics several properties of cubes. One studies the properties of the set of integers which in this probabilistic model can be written as a sum of three pseudo-cubes. Then since this pseudo-cubes behaves like cubes, it is reasonable to expect that the cubes and pseudo-cubes satisfy similar properties. The result for these pseudo-cubes on the other hand could be proved using the techniques from the probability theory. In other words, probabilistic model provides a heuristic understanding of several problems in additive number theory. One such probabilistic model is due to Erdös and Renyi. However as mentioned above, this model has a drawback that it does not take care of various congruence conditions. To remedy this, a refinement of this probabilistic model has been developed by Deshouillers along with François Hennecart and Bernard Landreau.

The aim of lecture series by Deshouillers was to explain the known results in additive number theory with focus on cubes and also introduce the probabilistic model mentioned above and explain what can be proved for pseudo-cubes. Since the details of each topic requires a semester course, the focus was to explain the main ideas involved along with a few technical details wherever possible.

- In the first lecture, Prof. Deshouillers introduced the circle method. This method was introduced by Hardy, Littlewood and Ramanujan and is one of the main methods to prove several problems in additive number theory. He started with classical examples, explained the technical difficulties to study those examples and then illustrated the method with the help of an example which is technically easier to study, but already explain the main ideas of the method. He also explained the ideas introduced by Vinogradov which leads not only to new results but simplifies the method considerably. He also explained the main ideas used in proving the celebrated result of Vinogradov that every sufficiently large odd natural number can be written as a sum of three primes and serious difficulties in extending the method to prove Goldbach conjecture.




- In the second lecture Prof. Deshouillers explained how the probabilistic methods leads to solution of certain number theoretic questions. He started with recalling some basic facts from Probability theory and then used this method to prove the celebrated result of Hardy-Ramanujan that `on average' the number of prime factors of any natural number n is roughly log(log(n)). He then introduced the Kubilius model which generalises the result of Hardy-Ramanujan and is a foundation of the area `Probabilistic number theory'. He then explained the method of Erdös which uses probability theory to prove the existence of certain type of subsets of natural numbers satisfying certain properties. It is difficult to prove the existence of such sets without using probability theory and in some situation there is no known proof without using probability theory.

- In the third lecture he started the discussion with finishing the proof of some results discussed in the second lecture. He then discussed the probabilistic model of Erdös and Renyi and discussed its limitations and mentioned the model he introduced with François Hennecart and Bernard Landreau. He then explained the basic upper bounds related to Waring's problem and Hua's inequality.

- In the final lecture he obtained an upper bound for the sixth moment for an exponential sums related to cubes and used this to obtain a lower bound for the density of the sets of natural numbers which can be written as a sum of three cubes. He then mentioned the conjecture if Hooley which provides the upper bound for the sixth moment of this exponential sum and its implication for the lower bound for the the density of the sets of natural numbers which can be written as a sum of three cubes. He then explained the Erdös-Renyi model and its limitations. He explained the probabilistic model introduced by him along with Hennecart and Landreau and its implication for the for the density of the sets of natural numbers which can be written as a sum of three cubes. The density predicted by their model is slightly larger in compare to the lower bound for the density implied by Hooley's conjecture. He finished the lecture series by explaining a result of his along with Cilleruelo and an open question related to the possible improvement of their result.

## Work done during the visit

During the visit, Jean-Marc Deshouillers also worked with Gyan Prakash, a faculty member at HRI. In a joint work with François Hennecart, Bernard Landreau and Gyan Prakash they proved that given any integer $H \geq 1$, there exists an interval $[n, n + H]$ of length H such that every integer $m \in [n, n + H]$ such that m is not congruent to $\pm 4$ modulo 9 can be written as a sum of three `pseudo-cubes'.

# PROBABILITY AND NUMBER THEORY

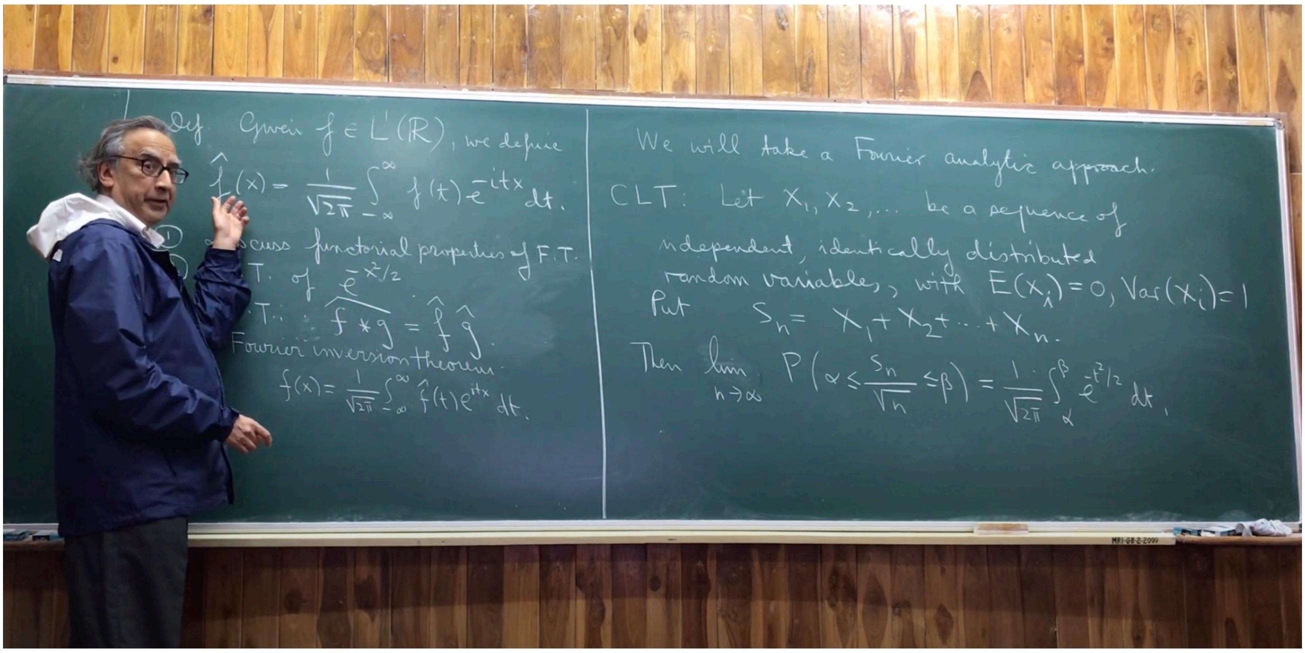
Prof. Ram Murthy is A. V. Douglas Distinguished University Professor and Queen's Research Chair at the Queen's University, Canada. He did his Ph.D. from Massachusetts Institute of Technology. Prof. Ram Murthy is a recipient of several accolades. He received the Coxeter-James prize in 1988, was elected to the Royal Society of Canada in 1990. He became a fellow of Indian National Science Academy in 2008 and was inducted as a fellow of American Mathematical Society in 2012.

Professor M. Ram Murty gave a course on Probability and Number Theory. He gave 10 sets of lectures on this topic and the final goal of this course is to discuss the Li's criterion for the Riemann Hypothesis.

He started the course with the celebrated theorem of Hardy and Ramanujan in 1917 which talks about the normal order of the number of distinct prime factors of natural numbers. Then Turan proved a Chebyshev inequality type for the number of distinct prime divisors of natural numbers. In 1937, Erdos-Kac proved the central limit theorem for the number of distinct prime factors of natural numbers. These results paved the birth of an area called ``Probabilistic Number Theory''. In the first lecture, he completely proved the Hardy- Ramanujan, Turan and Erdos-Kac theorems.

In the second lecture, he started with the elementary prime distribution result, namely, Bertrand Postulate, and gave proof of this result. He reviewed the basics of Probability theory. These are the topics covered in the second lecture.

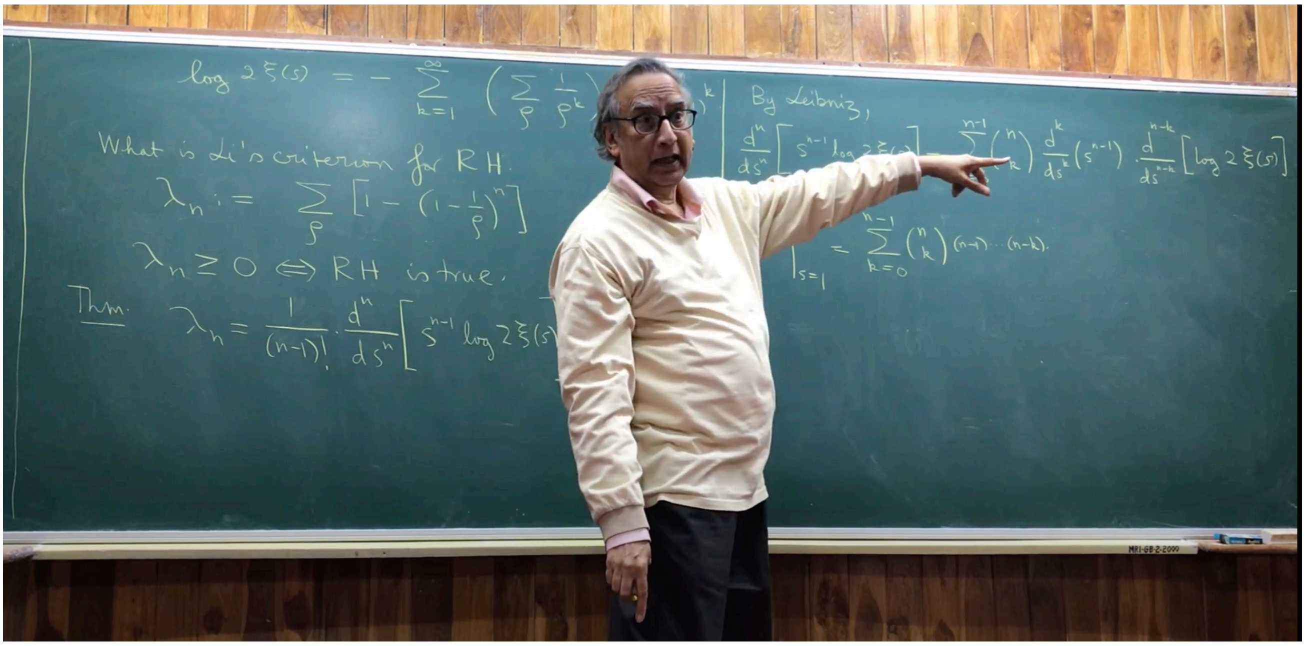
In the third lecture, he was discussing the algebra of distributions. For instance, what is the distribution function for the sum or product of the random variables? In order to do this, he reviewed measure theory and Fubini's theorem for product measures, Lebesgue dominated convergence theorem and differentiation under the integral sign. Then he proved the distribution function for the product of random variables.




In the fourth lecture, he proved the central limit theorem. In order to prove this theorem, he recalled Fourier series, Fourier inversion formula. In the next lecture, he continued to provide appllications of central limit theorem. First, he talked about characteristic function of random variables and computed the characteristic function for the sum of random variables using Levy's continuity theorem. Using De Moivre - Laplace result, he proved a result of Ramanujan, as an application of central limit theorem.  In order to prove the Erdos-Kac theorem as the application of the central limit theorem, one needs a result of Lindeberg-Feller which he proved in the end of the talk.

In the 6th lecture, he proved the classical version of the Erdos-Kac theorem. Then he went on to give the idea of the ``all-purpose'' Erdos-Kac theorem which is applicable in a wider context, namely, in the distribution of prime factors in the sequence of shifted primes, or polynomial values, modular forms like Ramanujan Tau function and other related problems.

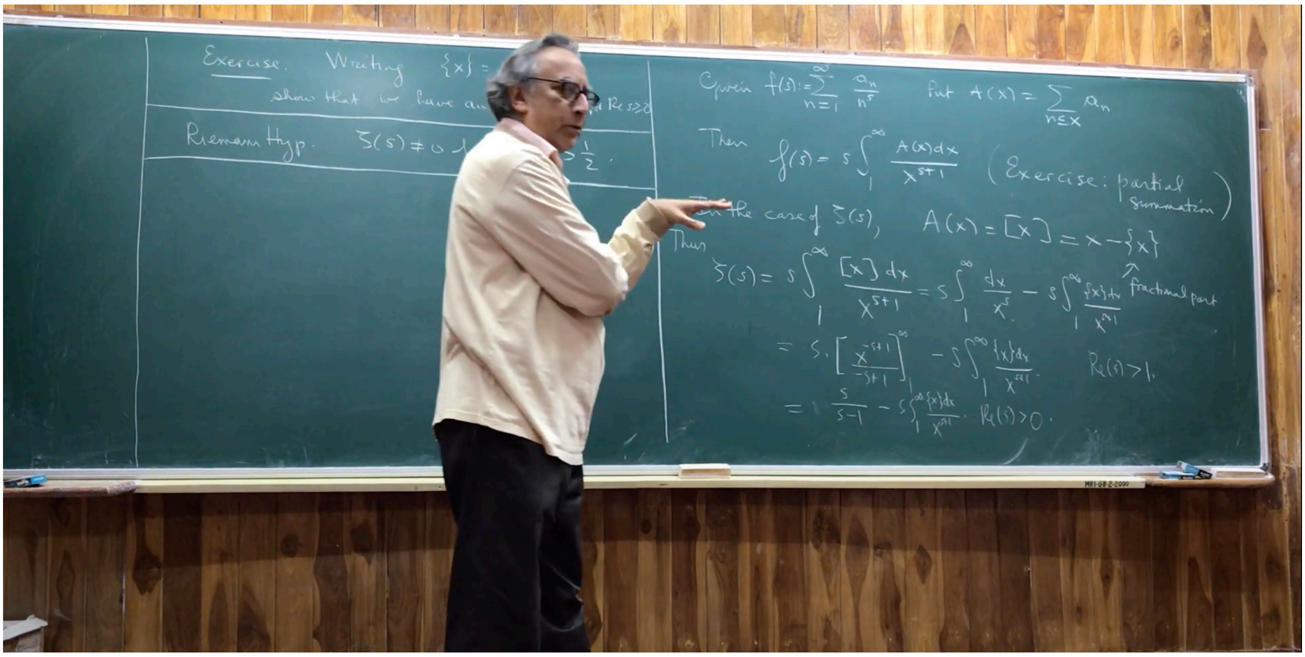
In his 7th lecture, he discussed the Bipartite Sieve which will be useful in predicting that a random polynomial with integer coefficients is irreducible with probability 1. This particular sieve includes the Turan-Kubilius sieve.




In the 8th lecture, he introduced the Riemann Zeta function and its analytic continuation to the entire complex plane except a simple pole at $s=1$. He introduced the Riemann Hypothesis.

In the 9th lecture, he gave a probabilistic intrepretation of the Riemann hypothesis. This intrepretation will lead to Li's criterion. He motivated this by stating the following problem, ``What is the probability that the greatest common divisor of $n_1, n_2, \ldots, n_k$ is $n$, when $n_1, n_2, \ldots, n_k$ are randomly chosen?''

In the 10th lecture, he introduced and proved the Li's criterion. He also provided various times this criterion was applied to prove many equivalent form of the Riemann Hypothesis.



# DOUBLE-COPY BOOTSTRAP

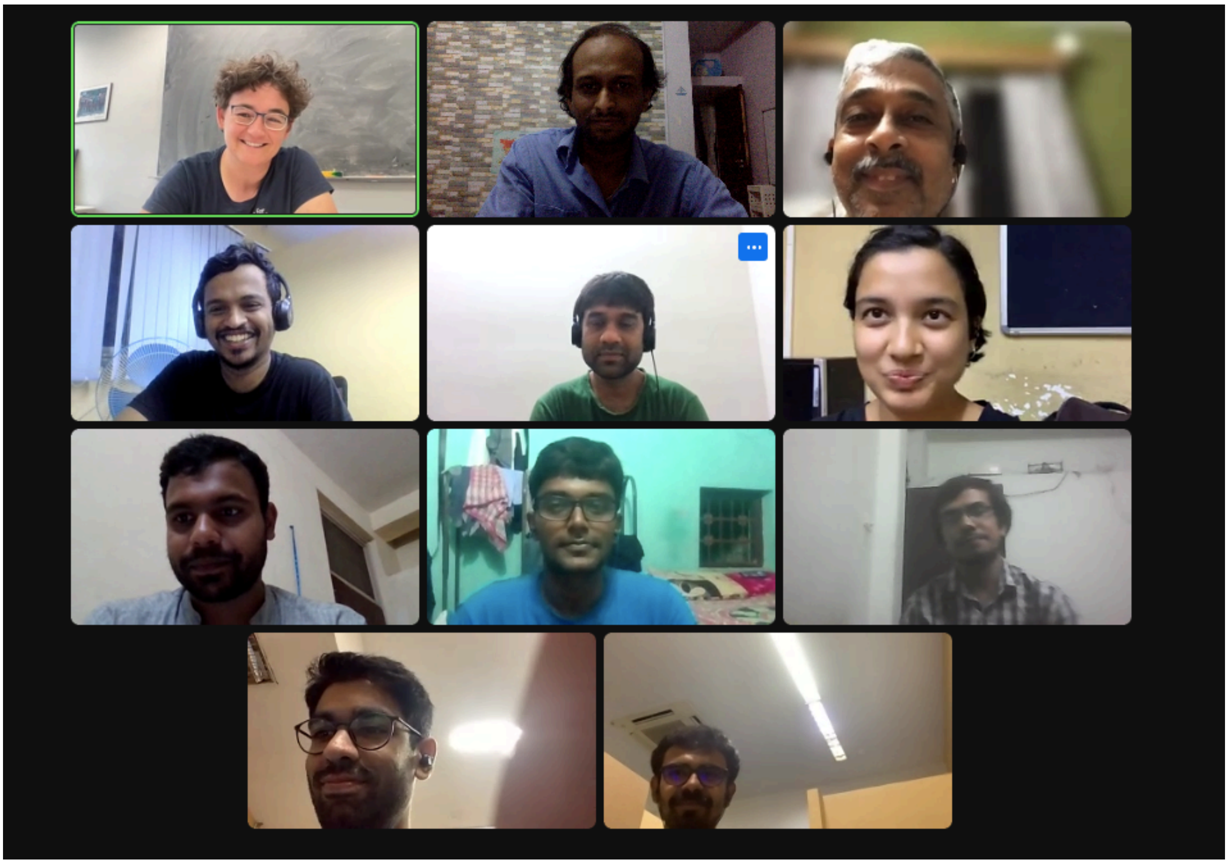
Prof. Henriette Elvang is a professor at University of Michigan, Ann Arbor, USA. She did her Ph.D. from University of California, Santa Barbara. She is one of the experts in the on-shell scattering amplitudes formalism. With Prof. Yu-tin Huang, she has written a treatise on field theory scattering amplitudes which has become a must read not only for all the aspirants but for the practitioners as well.

Prof. Elvang gave lectures on double copy bootstrap in the online mode from July 20 to 25, 2022. This was a three lecture course with each lecture being of 90 minutes duration. In reality the duration of each lecture was much more and each lecture was followed up by a discussion session. Prof. Elvang generously provided time for these online interactions. The string theory group at HRI already has some core expertise in the field of scattering amplitudes and her lectures on double copy formalism were supposed to initiate interests in application of the scattering amplitudes formalism to study amplitudes in the theory of gravity.

That the double copy of open string scattering amplitudes give closed string amplitudes was known long before, which is known as the KLT relation. However, the idea of relating gluon amplitudes to graviton amplitudes in field theory using the double copy formalism took off much later. We now give a brief summary of her lectures.

## SUMMARY OF THE LECTURE COURSE

- Prof. Elvang started with a quick recap of the scattering amplitude techniques mostly for setting up the notation. It was done in this way because she had already provided material necessary for her lectures. We had also had joint sessions prior to these lectures so that the younger members had gone through the requisite material for these lectures. She then introduced the field theory version of the double copy formalism.



- In the second lecture her approach took a turn in the bootstrap direction to set up equations for the field theory version of the double copy. The idea was to set up these equations in the effective field theory and see how much of the KLT relations can be recovered from the low energy double copy formulation in what is known as the bottom-up approach.

- In the final lecture, Prof. Elvang talked about the recent progress in the double copy bootstrap and deliberated on open problems. Each lecture was followed by a discussion session but due to large time difference between USA and India, these sessions were relatively short.