Effective Field Theory

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Sangam, HRI, 7-16 March, 2024

EFT: References

1. **Infrared singularities and massive fields: T. Appelquist and J. Carazzone ; Phys. Rev. D11, 28565 (1975)**

2. **Effective Gauge Theories: S. Weinberg; Phys.Lett.B 91 (1980) 51-55**

3. **Renormalization and Effective Lagrangians: J. Polchinski; *Nucl.Phys.B* 231 (1984) 269-295**

4. **Effective Field Theory: H. Georgi; Ann.Rev.Nucl.Part.Sci. 43 (1993) 209-252**

5. **On-Shell Effective Field Theory: H. Georgi; Nucl.Phys.B 361 (1991) 339-350**

6. **Effective Lagrangian Analysis of New Interactions and Flavor Conservation: W. Buchmuller and D. Wyler; Nucl.Phys.B 268 (1986) 621-653**

7. **Dimension-Six terms in the Standard Model Lagrangian: B. Grzadkowski, M. Iskrzynski, M. Misiak, and J. Rosiek; JHEP 10 (2010) 085**

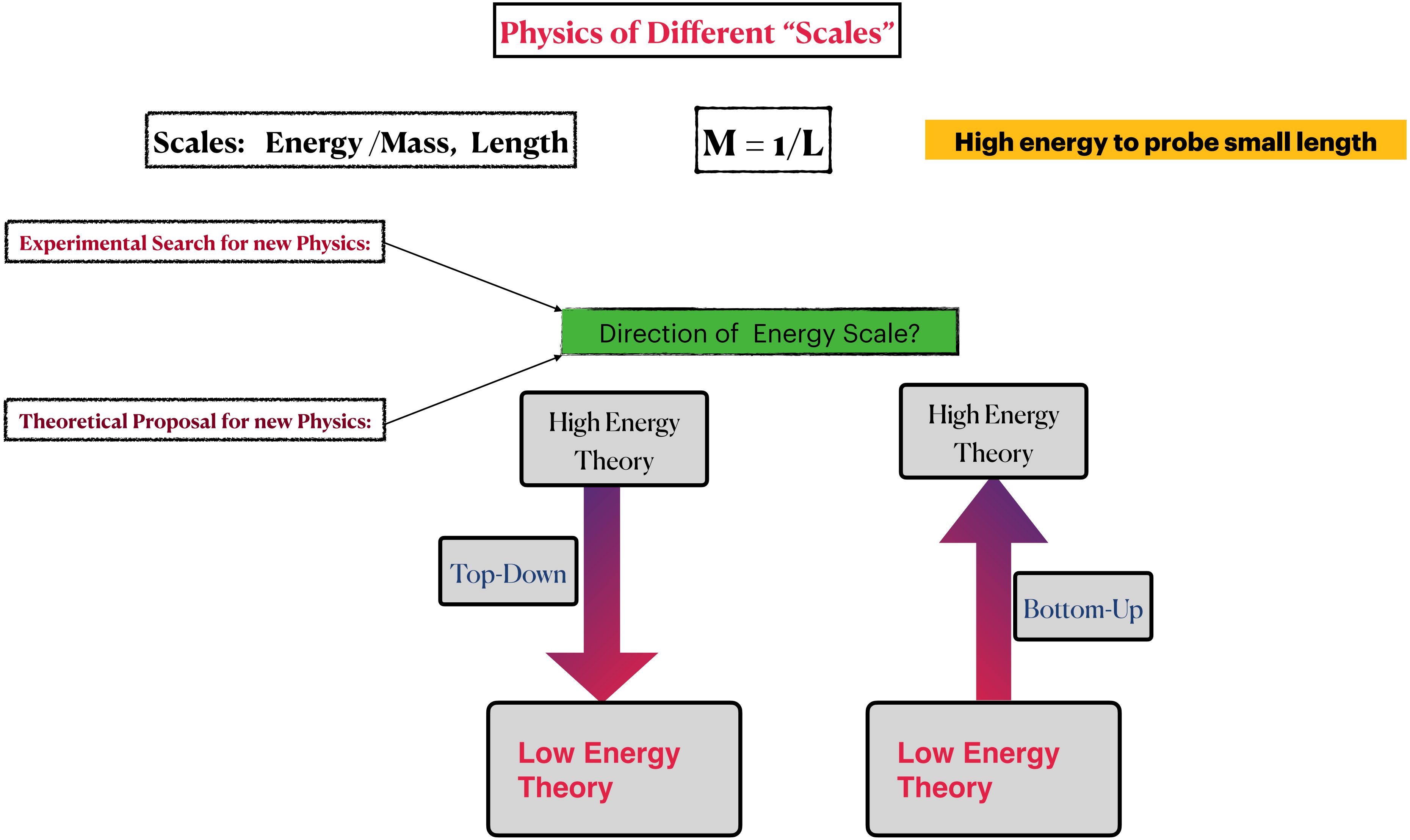
8. **How to use the Standard Model effective field theory: B. Henning et. al. ; JHEP 01 (2016) 023**

9. **The Standard Model as an Effective Field Theory: I. Brivio and M. Trott; Phys.Rept. 793 (2019) 1-98**

10. **Introduction to Effective Field Theories: A. Manohar; Les Houches Lect.Notes 108 (2020)**

11. **Exact SMEFT formulation: C. Hays et. al.; JHEP 11 (2020) 087**

12. **Mapping the SMEFT to discoverable models: R. Cepedello, et. al.; JHEP 09 (2022) 229**


Physics of Different "Scales"
Scales: Energy /Mass, Length
M = 1/L
High energy to probe small length
Experimental Search for new Physics:
Direction of Energy Scale?
Theoretical Proposal for new Physics:
High Energy Theory
Top-Down
Low Energy Theory
High Energy Theory
Bottom-Up
Low Energy Theory

Different length scales around us

Base 10 Logarithms of Scales of Typical Objects in the Physical Universe in Units of Meters

Typical objects	Powers of 10 (meters)
Observable Universe (Quasars, etc.)	27
Super-clusters	25
Clusters of galaxies	24
Size of Virgo cluster	23
Distance to Andromeda galaxy	22
Milky Way diameter	21
Distance to Orion arm	19
Distance to the nearest stars	17
Size of the solar system	13
Venus, Earth, and Mars	11
Earth-Moon distance	9
Earth diameter	7
San Francisco	4
Human scale	0
Micro-Organisms / Hair Thickness	-4
Size of a red blood cell	-5
DNA Structure	-8
Carbon Nucleus	-14
Quarks	-16
Planck length	-35

Our interest → Quarks

Source: Google web

Q. Is there a unique theory to encompass the Physics of all length scales?

A. There is no such thing (so far). Theory of Everything is a myth.

We, the Particle Physicist, are interested in a length scale around 10^{-16} meters

smallest sub-atomic "fundamental" particles (observed so far)

Sub-atomic Particles and their dynamics

Important ingredients to build a model/theory:

(i) **Symmetry of the system**

(ii) **Particles and their characteristics under the assigned symmetry**

$(i) + (ii) \Rightarrow$ **Lagrangian (set of operators)**

Task

Using information of Symmetry and Particles construct the Lagrangian

Q. How do we perform the task?

A. One can do it manually looking for all possible *invariants*

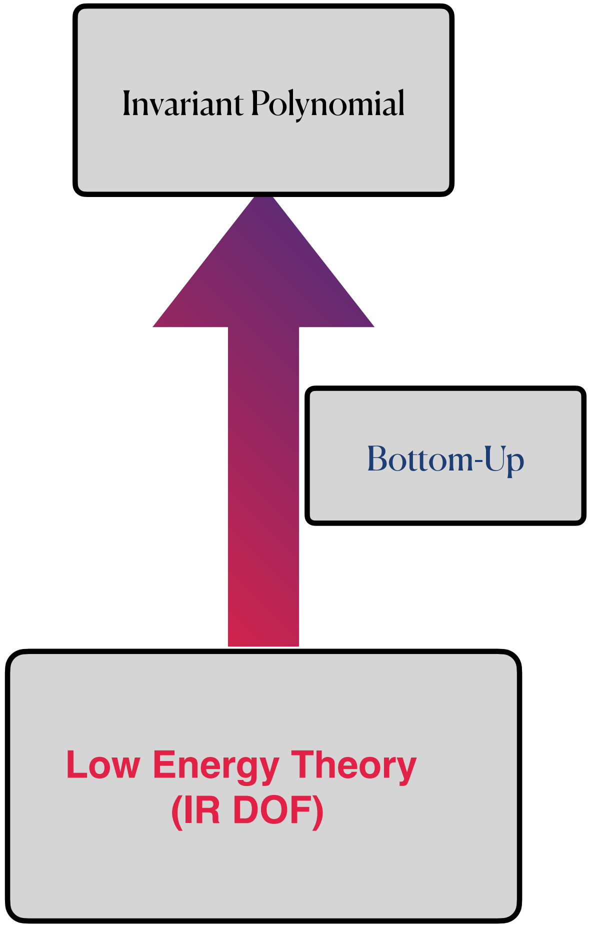
Q. What could be the possible issue with this?

A. For a complicated framework this could be error prone:

Over and(or) Under estimation of invariants

Q. Is it possible to develop a Tool which will ease out the task and make it error free?

A. Let's explore


Invariant Polynomial
Bottom-Up
Low Energy Theory
(IR DOF)

<u>To proceed further, let's consider the following mapping</u>

Symmetry → "Tag" | **Particles → "variables"**

Invariants → "Clusters" | **Lagrangian → "Polynomial"**

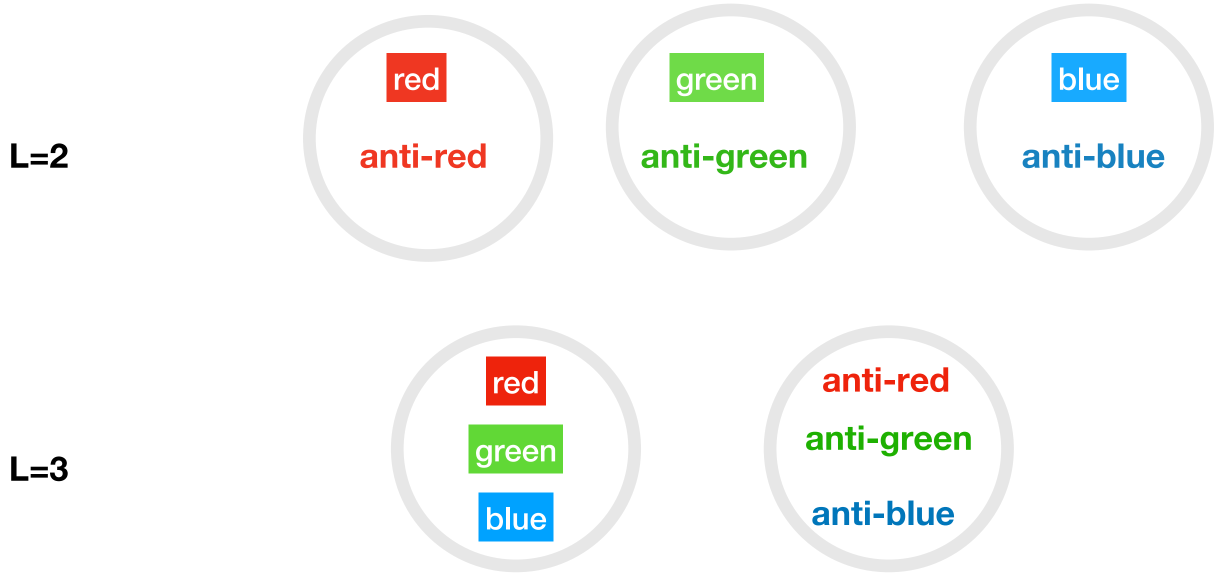
Q. Any "Polynomial" or a "Special One"

A.

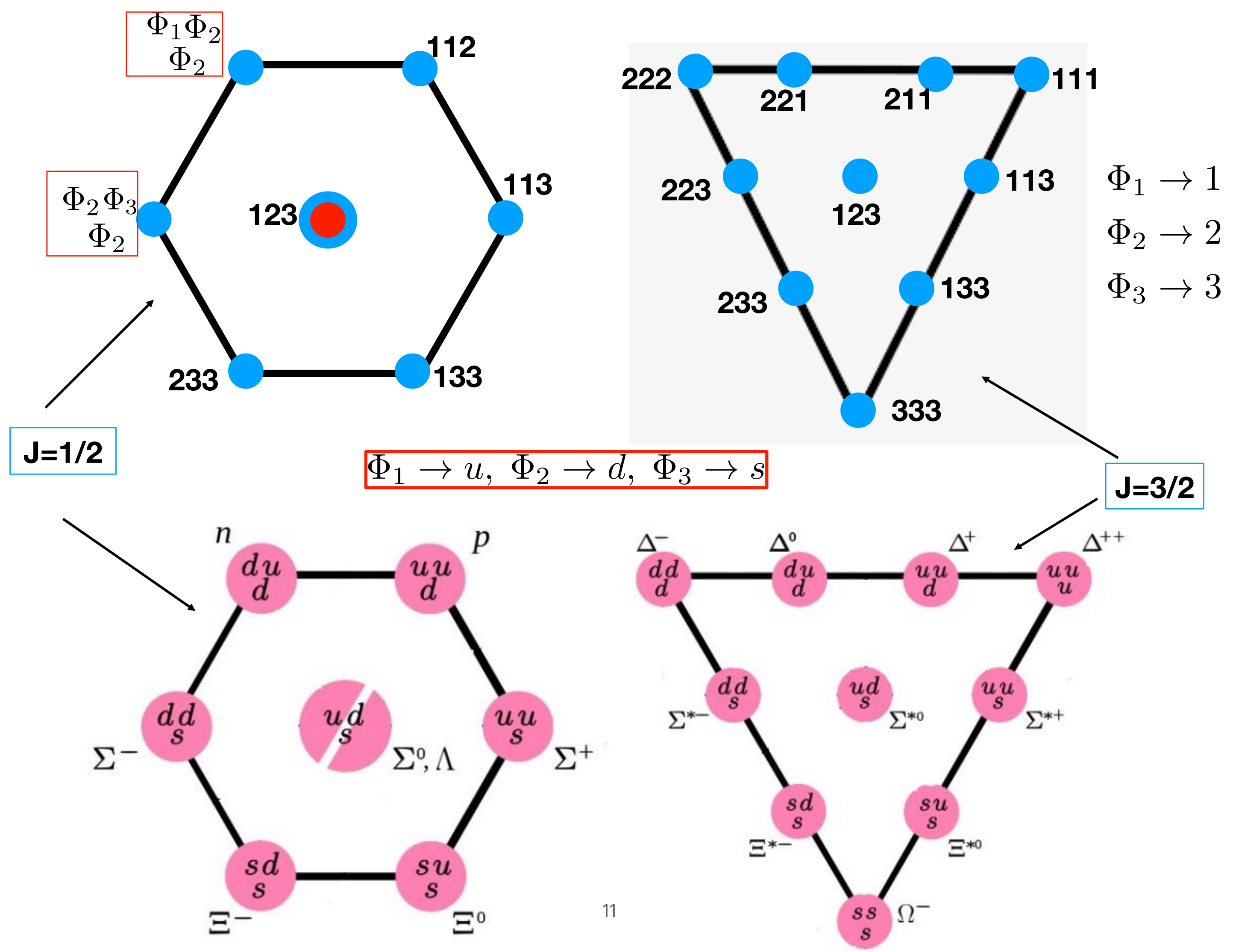
Toy Example.....

Tags	Φ_1	Φ_1^*	Φ_2	Φ_2^*	Φ_3	Φ_3^*
L	**1**	**1**	**1**	**1**	**1**	**1**
Spin (J)	**+1/2**	**+1/2**	**+1/2**	**+1/2**	**+1/2**	**+1/2**
	-1/2	**-1/2**	**-1/2**	**-1/2**	**-1/2**	**-1/2**
Color (C)	red	**anti-red**	red	**anti-red**	red	**anti-red**
	green	**anti-green**	green	**anti-green**	green	**anti-green**
	blue	**anti-blue**	blue	**anti-blue**	blue	**anti-blue**
Electric charge (Q)	+2/3	-2/3	-1/3	+1/3	-1/3	+1/3

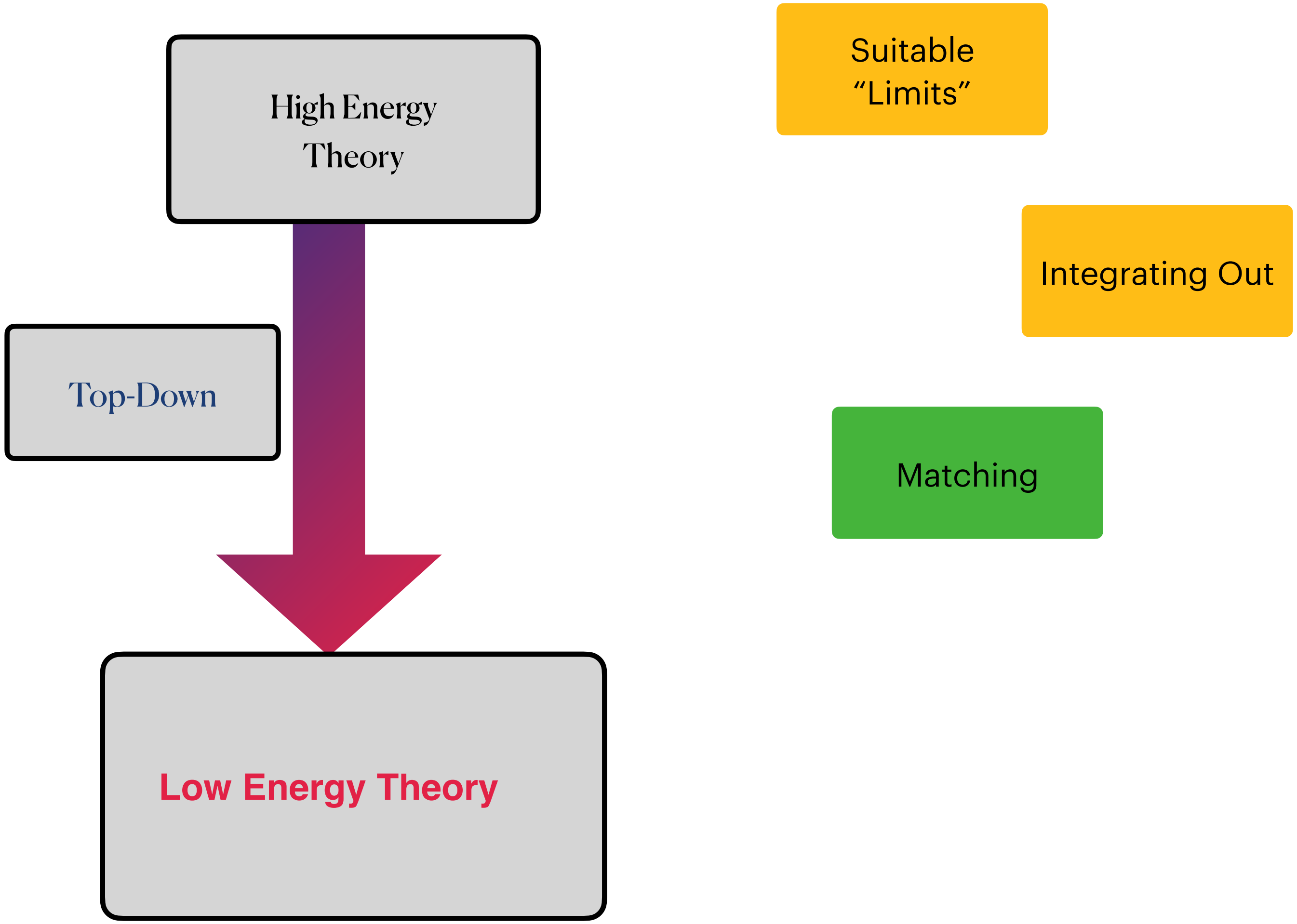
White (invariant) Clusters



Examples: $\Phi_1 \Phi_1^*$, $\Phi_2\Phi_2\Phi_2$, ...



Φ1Φ2 Φ2
Φ2Φ3 Φ2
112
113
123
133
233
222
221
211
111
223
233
333
Φ1 → 1
Φ2 → 2
Φ3 → 3
J=1/2
J=3/2
Φ1 → u, Φ2 → d, Φ3 → s
n
p
Σ−
Σ0, Λ
Σ+
Ξ−
Ξ0
Δ−
Δ0
Δ+
Δ++
Σ*−
Σ*0
Σ*+
Ξ*−
Ξ*0
Ω−


High Energy Theory
Top-Down
Low Energy Theory
Suitable "Limits"
Integrating Out
Matching

“Limits” and Matching ...

Scattering in 1D:

Square well potential

$$\begin{aligned} V(x) &= -\frac{\alpha^2}{2m\Delta^2}; \quad 0 \le x \le \Delta \\ &= 0; \quad otherwise \end{aligned}$$

Potential width Δ, depth $\frac{\alpha^2}{2m\Delta^2}$,

α, g are dimensionless parameters.

$$\begin{aligned} R = (1-T) &= [\frac{4k^2\kappa^2 csc^2(\kappa\Delta)}{(k^2-\kappa^2)^2} + 1]^{-1} \\ &\simeq 1 - 4\frac{\Delta^2 k^2}{\alpha^2 \sin^2\alpha} + \mathcal{O}(\Delta^4 k^4) \end{aligned}$$

δ-function potential

$$V(x) = -\frac{g}{2m\Delta}\delta(x)$$

$$k = \sqrt{2mE}, \kappa = \sqrt{k^2 + \frac{\alpha^2}{\Delta^2}}$$

$$\begin{aligned} R = (1-T) &= (1 + 4k^2\Delta^2/g^2)^{-1} \\ &= 1 - 4k^2\Delta^2/g^2 + O(k4) \end{aligned}$$

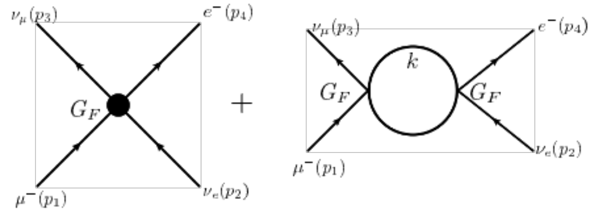
Matching: $g = \alpha \sin\alpha$

Lectures on effective field theory , David B. Kaplan

Effective 4-Fermi theory vs SM

H. Georgi annual review '93
A.V. Manohar [1804.05863]
Halzen & Martin – Quarks and Leptons

muon decay: $\mu^- \to e^- \bar{\nu}_e \nu_\mu$

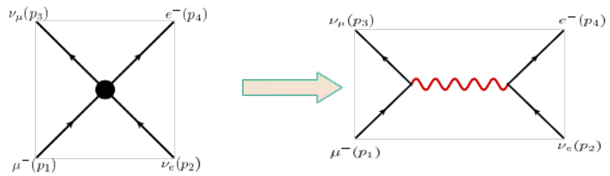


mass dim. of the coupling $[G_F] = -2$

$$\sim G_F^2 \int^\Lambda d^4k \, \frac{1}{k}\frac{1}{k} \sim G_F^2 \Lambda^2$$

$$\mathcal{M} \sim \underbrace{G_F}_{\text{lowest order}} + \underbrace{G_F^2 \Lambda^2}_{\text{next order}} + \mathcal{O}(G_F^3)$$

muon decay: $\mu^- \to e^- \bar{\nu}_e \nu_\mu$



$$\mathcal{M} = \left(\frac{g}{\sqrt{2}}\overline{u}(p_3)\,\gamma^\sigma \frac{1}{2}(1-\gamma^5)u(p_1)\right) \frac{1}{M_W^2 - q^2} \left(\frac{g}{\sqrt{2}}\overline{u}(p_4)\,\gamma_\sigma \frac{1}{2}(1-\gamma^5)v(p_2)\right)$$

$$\sim \frac{g^2}{8M_W^2}\left(\overline{u}(p_3)\,\gamma^\sigma(1-\gamma^5)u(p_1)\right)\left(\overline{u}(p_4)\,\gamma_\sigma(1-\gamma^5)v(p_2)\right)$$

Calculate same process from both theory

$$\frac{1}{q^2 - M_W^2} \sim \frac{1}{M_W^2}$$

$$\boxed{q^2 << M_W^2}$$

Matching condition:

$$\frac{G_F}{\sqrt{2}} \equiv \frac{g^2}{8M_W^2}$$