Inflation : Baumann (TASI lectures)

Notes based on : "Inflation & String Theory" Baumann & McAllister

(1)

Observations :

type Ia SN , CMB fluctuations by WMAP & PLANCK

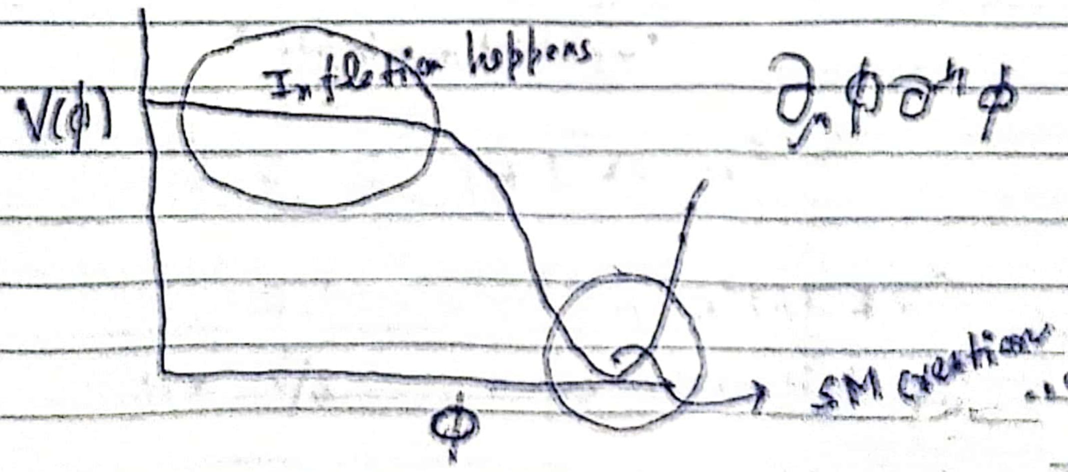
LSS surveys

→ ΛCDM 68 %. DE ; 27%. DM
5% ordinary matter

→ LSS formed by gravitational collapse of primordial fluctuations

→ Primordial fluctuations originated from quantum fluctuations stretched to cosmic scale by exponential expansion (inflation)

→ What is the origin of inflation
→ How do we get $V(\phi)$?



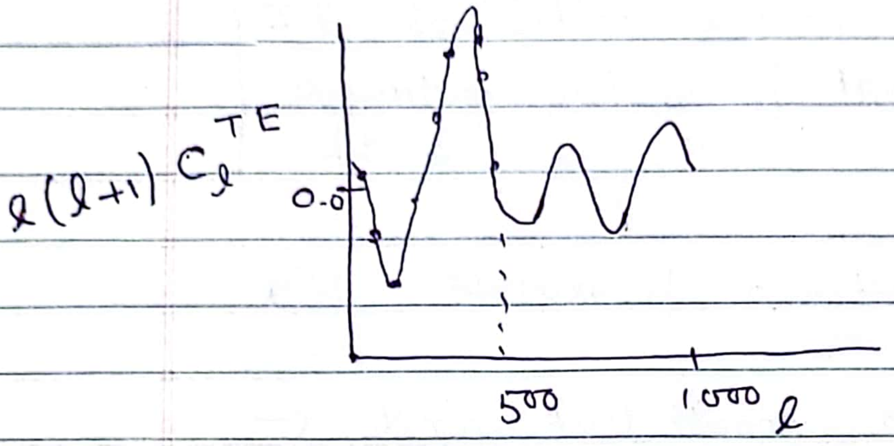
$\partial_\mu \phi \, \partial^\mu \phi$

Goal :

Cosmology without inflation

$\Downarrow$

Causal signal travels from initial singularity to CMB formation $<<<$ CMB scales where phase coherences is seen



CMB $\Rightarrow$ superhorizon perturbations at recombination.

$l < 200$

coherent phase !

Definite $-$ve correlation !

same wavenumber $|\vec{k}|$, but different $\vec{k}$ all gets summed up coherently.

"Horizon problem"

Resolved if $|\dot{H}| \ll H^2$

$$ds^2 = -dt^2 + a^2(t)\, d\vec{x}^2$$

$H \equiv \dot{a}/a$ quasi exponential expansion

$a(t) \propto e^{Ht}$

Homogeneous initial conditions on sub horizon scales are stretched to apparently acausal super horizon ~~[illegible]~~ scales.

$$\delta\phi_{\text{quantum}} \rightarrow \delta H(t, \vec{x}) \rightarrow \delta\rho(t, \vec{x})$$

after inflation

If inflation is correct, CMB probes quantum nature of the inflaton quantum field ✓

~~[illegible]~~ Nature of quantum field?

→ Know full theory or EFT idea!

parametrize "unknown" physics associated with d.o.f at high scale Λ by a collection of higher dimensional operators

↓

non renormalizable irrelevant operators

e.g. $\mathcal{O}/\Lambda$

"Typically" ~~[illegible]~~ effects of irrelevant operators (E/Λ)

and $E \ll \Lambda$, the effect of high energy d.o.f decouples

the

But in some cases low energy observables are strongly affected by irrelevant interactions.

$\Downarrow$

UV sensitive.

$\boxed{\text{• Inflation is UV sensitive! i.e. even if } H_{inf} \ll M_{pl} \text{, dynamics of inflation sensitive to Planck scale physics.}}$

- In every model of inflation, duration of inflation is affected by at least a few non-renormalizable operators

- For large field models, infinite ~~number~~ series of interactions & arbitrarily high dimension affects inflation dynamics.

- EFT approach $\phi \to \phi + c$ shift symmetry ensures radiative stability.

Horizon Problem:

FRW universe

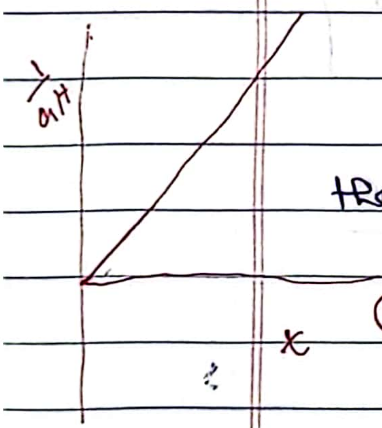
$$ds^2 = -dt^2 + a^2(t)\, d\vec{x}^2$$

$$dt^2 = a^2(\tau)\, d\tau^2 \qquad = a^2(\tau)\left[-d\tau^2 + d\vec{x}^2\right]$$

Maximal comoving distance $|\Delta\vec{x}|$ a particle can travel between τ & $\tau + \Delta\tau$ is

$$|\Delta\vec{x}| = \Delta\tau \quad \text{for any } a(\tau)$$

standard BB cosmology, early universe dominated by radiation and at sufficiently back in time $a \to 0$ singularity



Choose initial singularity at $t = 0$

the $|\Delta x| = \Delta\tau = \displaystyle\int_0^t \frac{dt'}{a(t')} = \int_0^a \frac{da}{Ha^2}$

Comoving particle horizon

$$= \int_{-\infty}^{\ln a(t)} \frac{d\ln a}{aH} = \int (aH)^{-1}\, d(\ln a)$$

where $H = \dfrac{1}{a}\dfrac{da}{dt}$

$$\tau \propto \begin{cases} a & RD \\ a^{1/2} & MD \end{cases}$$

In standard BB cosmology $\ddot{a} < 0$ (deccelaration) and comoving Hubble radius ~~decrease~~ GROWS with time

$\Rightarrow \Delta x$ is dominated (into the integral) by contributions from late times.

$\Downarrow$

Horizon problem

conformal time between recombination to today >> time between $t=0$ to recombination

· At CMB, two points separated by more than 1° were never in causal contact.

$\downarrow$

$\rightarrow$ But they are homogeneous in 10^{-5}

[Moreover fluctuations are correlated on what seem to be superhorizon scales. at recombination.

<u>Postulate</u>: Comoving Hubble radius decreases so that early time gets contribution

$\Downarrow$

Add extra conformal time (negative).

~~If that epoch is long enough~~ $\rightarrow$ all points in CMB becomes within one causal patch

Observed correlation can be explained ✓

- We need shrinking $\left(\frac{1}{aH}\right)$
  (comoving Hubble radius

Inflation: H fixed $\rightarrow$ a grows.

- Ekpyrotic/ Cyclic: Assume a contracts, but H grows s.t $\left(\frac{1}{aH}\right)$ shrinks

Cosmic Inflation :

Comoving Hubble radius decreases.

$$\frac{d}{dt}(aH)^{-1} = -\frac{1}{a}\left[\frac{\dot{H}}{H^2}+1\right] < 0$$

$|\dot{H}| \ll H^2 \Rightarrow \epsilon \equiv -\frac{\dot{H}}{H^2} < 1$ ✓ $\quad \epsilon \to 0$

Definition of inflation ! $\quad$ de Sitter !

$a \sim e^{Ht}$

with $H \sim$ Constan

$$3M_{pl}^2 H^2 = \rho \quad \text{— (A)}$$

$$6M_{pl}^2(\dot{H}+H^2) = -(\rho+3p). \text{ —(B)}$$

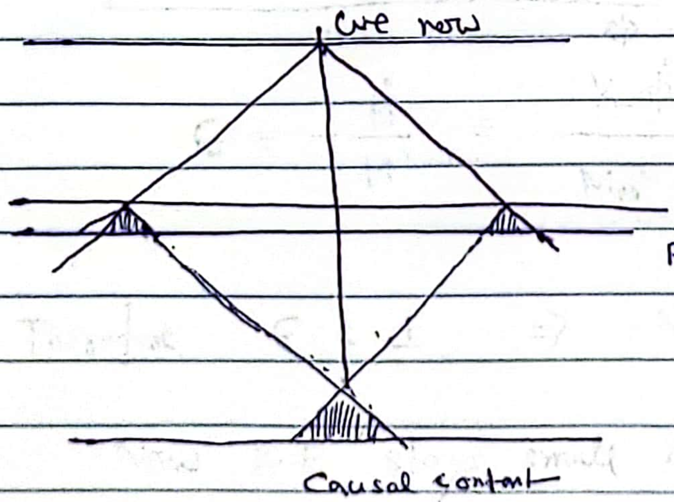
$(A \& B) \rightarrow \quad 2M_{pl}^2\dot{H} = -(\rho+p)$

Then $\quad \epsilon = \frac{3}{2}\left(1+ p/\rho\right).$

Inflation requires $\quad p < -\frac{1}{3}\rho$

violation of SEC

$(\rho+p \geq 0 \;\&\; \rho+3p \geq 0)$



Future reference

$$\tilde{\eta} = \frac{\dot{\epsilon}}{H\epsilon}$$

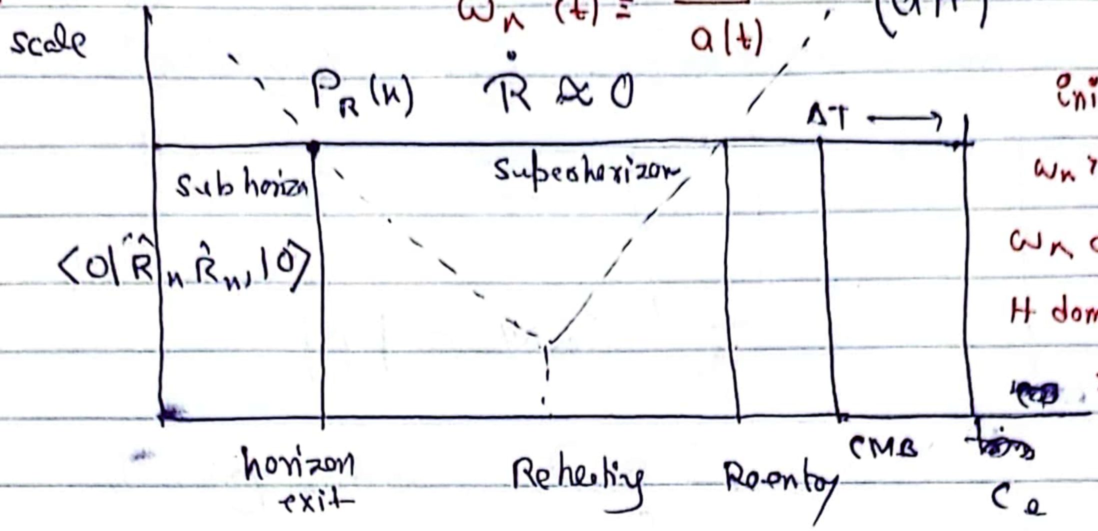
$u = yR$

$y^2 = 2M_{pl}^2 \frac{\epsilon}{c_s^2}$

$$\ddot{u}_k + 3H\dot{u}_k + \frac{c_s^2}{a^2}k^2 u_k = 0$$

8

$$\omega_k(t) \equiv \frac{c_s k}{a(t)}$$



Initially $\omega_k \gg H$ oscillate

ω_k drops, then H dominates ✓

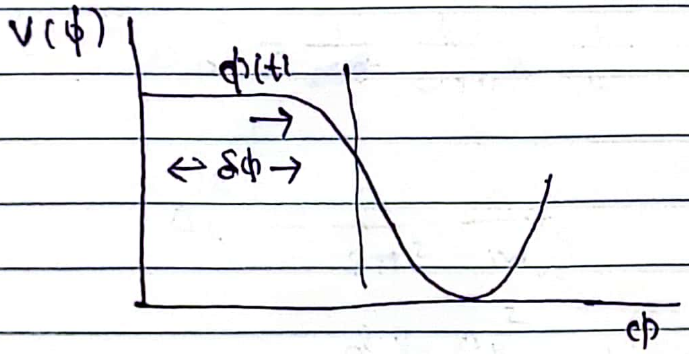
freezes !

Effective Theories of Inflation :

origin of inflation background?

$$|-\dot{H}| \ll H^2 \quad \text{(inflation)}$$

$$S = \int d^4x \sqrt{-g}\left[\frac{M_{pl}^2}{2}R - \frac{1}{2}(\partial\phi)^2 - V(\phi)\right]$$



$$3M_{pl}^2 H^2 = \frac{1}{2}\dot{\phi}^2 + V(\phi)$$

$$\ddot{\phi} + 3H\dot{\phi} = -V'(\phi)$$

$p = \frac{1}{2}\dot{\phi}^2 - V(\phi)$

$\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi)$

$p \approx -\rho$

$$\epsilon \equiv -\frac{\dot{H}}{H^2} = \frac{\frac{1}{2}\dot{\phi}^2}{M_{pl}^2 H^2}$$

Therefore $\epsilon < 1 \Rightarrow V \gg \frac{1}{2}\dot{\phi}^2$ ✓

Now K.E stays small & SR persists if

$$|\ddot{\phi}| \ll 3H|\dot{\phi}|$$

CamScanner

Prolonged SR inflation can be expressed as

$$\epsilon_V = \frac{M_{pl}^2}{2}\left(\frac{V'}{V}\right)^2 \ll 1$$

$$|\eta_V| = M_{pl}^2 \frac{|V''|}{V} \ll 1$$

$\tilde{\eta} \equiv \frac{\dot{\epsilon}}{H\epsilon}$

During SR $\epsilon = \epsilon_V$

$$\eta_V \approx 2\epsilon - \frac{1}{2}\tilde{\eta}$$

Note $|\eta_V| \ll 1 \rightarrow$ small hierarchy between inflaton mass & Hubble scale

$m^2 = V'' \ll 3H^2 \simeq \frac{V}{M_{pl}^2}$

$$\Delta_R^2 = \frac{1}{24\pi^2}\frac{1}{\epsilon}\frac{V}{M_{pl}^4} \quad \left(= \frac{H^4}{8\pi^2 M_{pl}^2 |\dot{H}|}\right)$$

$$\Delta_h^2 = \frac{2}{3\pi^2}\frac{V}{M_{pl}^4} \quad \left(= \frac{2}{\pi^2}\frac{H^2}{M_{pl}^2}\right)$$

$$n_s - 1 \equiv \frac{d \ln \Delta_R^2}{d \ln k}$$

$$= 2\eta_V - 6\epsilon_V$$

observables are calculated at horizon crossing; ie $k_* = aH$

$$r \equiv \frac{\Delta_h^2}{\Delta_R^2} = 16\,\epsilon_V$$

$$\Delta_h^2(k) = \frac{2}{\pi^2}\frac{H^2}{M_{pl}^2}$$

$$\Rightarrow \frac{H}{M_{pl}} = \pi\, \Delta_R(k_*)\sqrt{\frac{r}{2}}$$

Observation: $\Delta_R(k_*) = 4.7\times10^{-5}$

$$\Rightarrow H = 3\times10^{-5}\left(\frac{r}{0.1}\right)^{1/2} M_{pl}$$

i.e for $r \sim 0.1$, expansion rate $\sim 10^{-5}\, M_{pl}$

$$E_{inf} \equiv \left(3H^2 M_{pl}^2\right)^{1/4} = 8\times10^{-3}\left(\frac{r}{0.1}\right)^{1/4} M_{pl}.$$

Reducing r by four order of mag reduce E_{inf} by only one order!

With reasable observational goal

$$E_{inf} \sim 10^{-2}\, M_{pl} \sim 10^{16}\ \text{GeV}$$

$$\ll M_{pl}$$

Tests of Inflation:

ΛCDM model — $\omega_b = \Omega_b h^2$

$\omega_c = \Omega_{cdm} h^2$

Ω_Λ

optical depth τ

A_s

n_s

with $\Delta_R^2(k) = A_s \left(\frac{k}{k_*}\right)^{n_s - 1}$

6 parameters (No r)

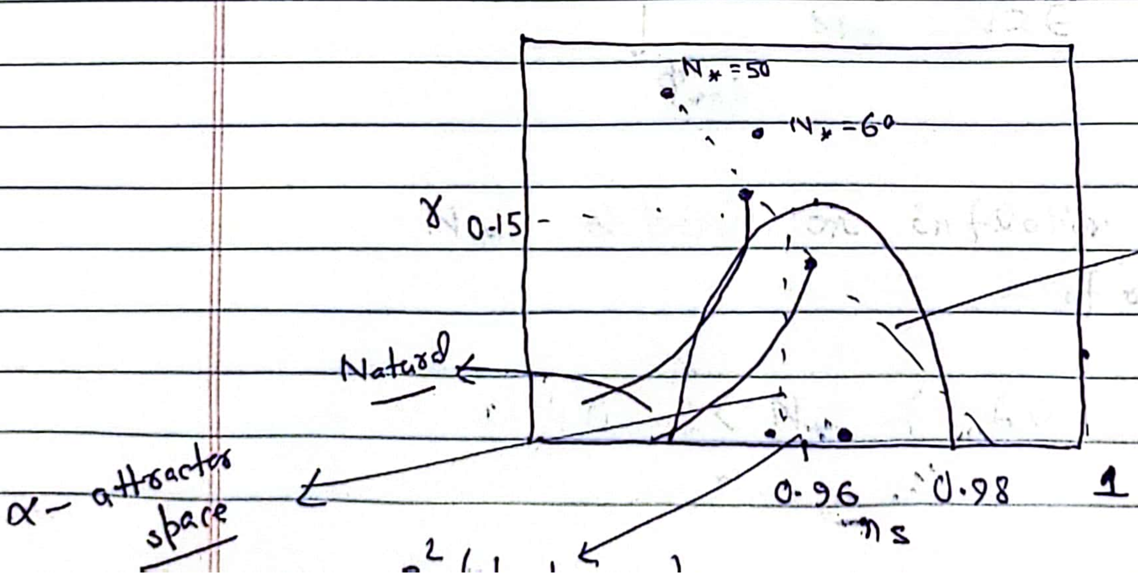
$A_s \sim 2.1 \times 10^{-9}$

$n_s \sim 0.9603 \pm 0.0073$

scale invariance excluded by 6σ

When r included then also

$n_s \sim 0.963 \pm 0.0019$



- $\Omega_K = -0.072 \pm 0.0081$ (Geometry)
- $n_s = \ldots$ Powerspectrum ✓
- Coherent phase

Future ✓

- Tensor fluctuations $(r < 0.05)$
- Non-gaussianity
- Non-adiabatic fluctuation
- Tensor tilt
- Running of scalar spectral index etc ✓

$$N_* = \int_{\phi_{end}}^{\phi_*} \frac{d\phi}{M_{pl}} \frac{1}{\sqrt{2\epsilon}}$$

N_* depends on inflation model & duration of reheating

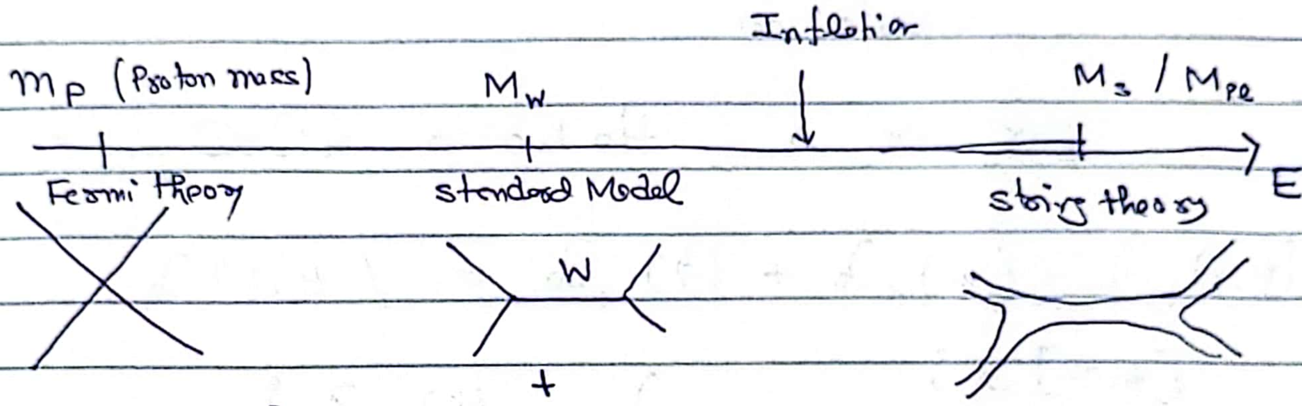
$$40 \lesssim N_* \lesssim 60$$

Inflation In Effective Field Theory :

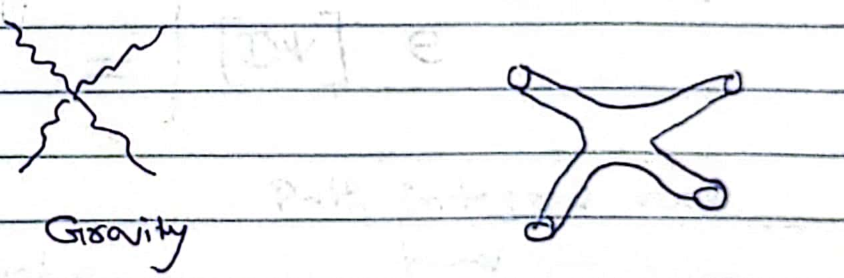
Inflation

↗ starts with low energy IR d.o.f & parametrize our ignorance about UV theory in term of higher-D operator

↘ starts with "complete" UV theory e.g string theory → derive inflation as one of low energy consequences.



$$G_F = (293\ \text{GeV})^{-2}$$



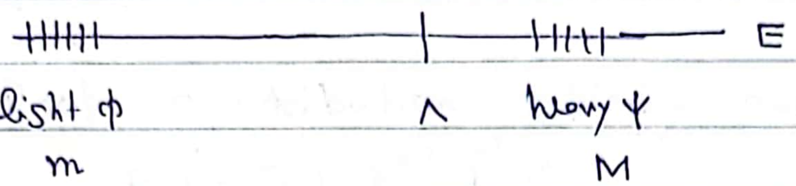
$$M_{Pl} = \frac{1}{\sqrt{8\pi G_N}} = 2.4\times10^{18}\ \text{GeV}$$

Fermi theory becomes invalid at $m \sim M_W$ scale ~ 100 GeV

[similarly (gravity + SM) interaction ok until M_P scale

or Fermi theory ok until $G_F^{-1/2} \sim 293$ GeV

Effective Action:



- Identify relevent D.O.F. at some energy of measurement $\Rightarrow$ particles produced on shell ✓

- Define a cut off $\Lambda >> m_\phi$

$$\mathcal{L}(\phi,\psi) = \mathcal{L}_l(\phi) + \mathcal{L}_h(\psi) + \mathcal{L}_{lh}(\phi,\psi)$$

$$e^{iS_{eff}(\phi)} = \int [D\psi]\, e^{iS[\phi,\psi]}$$

Wilsonian effective action.

Path integral over heavy modes

In practice difficult to do the path integral ~~easy~~ and we do matching calculation in perturbation theory is done! etc etc.

- Classical approximation (also used in Feynman diagram language): substitute ... for the ϕ fields

Loop contribution typically non local

$$\phi \left(\Box + M^2 \right)^{-1} \phi$$

$$E \ll M \;\rightarrow\; \frac{\phi}{M^2}\left(1 + \frac{\Box}{M^2} \cdots \right)\phi$$

local ✓

Locally

$$\mathcal{L}_{eff}[\phi] = \mathcal{L}_0(\phi) + \sum_i C_i(g) \frac{O_i[\phi]}{M^{\delta_i - 4}}$$

dimensionless → carry energy content

O_i operators of dimension $\delta_i > 4$

O_i operators typically ... consistent with the symmetries of ... theory.

Fine tuning : → Absense of any operator allowed by UV symmtry but not present at $\mathcal{L}_{eff}[\phi]$

or very small coefficient of that operator ✓ ✓

$$\mathcal{L}_{eff}[\phi] = \mathcal{L}_{l}(\phi] + \sum c_i \frac{\Theta_i}{M_l^{\delta_i - 4}}$$

↗ Renormilizable

↓ non renormilizable ✓

Even if UV theory renormilable in eff theory non-renormalizabe terms arise !

$[\Theta_i] < 4$ relevent operators dominates in IR , small at UV

$[\Theta_i] > 4$ irrelevent operators Dominant at UV & small in IR

at low energy observables contributions

$$\sim (E/M)^{\delta_i - 4}$$

i.e for low energy effects would be small or you need to know only a few lower dimensional operators !

· If low energy observables are not so sensetive to UV physics → you can do this safely —

→ If it is sensetive (as in inflation), it can be a probe!

A Toy Model :

UV theory
$$\mathcal{L}[\phi, \psi] = -\frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2 - \frac{1}{4!}\lambda\phi^4$$

$M \gg m$
$$-\frac{1}{2}(\partial\psi)^2 - \frac{1}{2}M^2\psi^2$$

two parameter λ, g ✓
$$-\frac{1}{4}g\phi^2\psi^2$$

Renormalizable operators

Z_2 symmetry $\quad \phi \to -\phi$

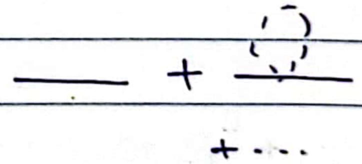
$$\mathcal{L}_{eff}[\phi] = -\frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m_R^2\phi^2 - \frac{1}{4!}\lambda_R\phi^4$$

$$-\sum_{i=1}^{\infty}\left(\frac{c_i(g)}{M^{2i}}\phi^{4+2i} + \underbrace{\frac{d_i(g)}{M^{2i}}(\partial\phi)^2\phi^{2i}}\right)$$

~~also maintains $\phi \to -\phi$~~ non-renormalizable operator.

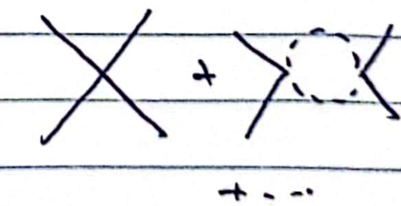
In cut-off regularization

$$m_R^2 = m^2 + \frac{g}{32\pi^2}\left(\Lambda^2 - M^2 L\right) =$$



Cut off scale !

$$\lambda_R = \lambda - \frac{3g^2}{32\pi^2}L + \cdots =$$



$$L \equiv \ln\left(\Lambda^2/\mu^2\right)$$

$\mu \equiv$ Renormalization scale !

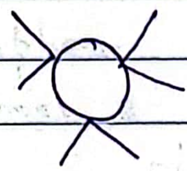
Dimensional
$$\left\{\begin{aligned} m_R^2 &= m^2 + \frac{g}{32\pi^2}\left(-M^2 L\right) \\ \lambda_R &= \lambda - \frac{3g^2}{32\pi^2} L \end{aligned}\right\} \quad L \to \frac{1}{\epsilon} + 0.577 - \ln(4\pi)$$

Λ^2 dependence scheme dependent

$\longrightarrow$ NOT physical

NOTE, M^2 dependence (in m_R^2) is physical & comes with the same coupling as like unphysical Λ^2 dependence.

Common practice: Use cut off Λ as a proxy for physical mass dependence of heavy particles!

Also c_1 =  $+ \cdots \sim g^3 + \cdots$

Trouble with light scalars:

bare mass gets contribution $\sim M$ say ✓

m_R to be $O(m)$, you need cancellation

$m_R^2 \sim m^2 - g M^2$

light mass unnatural.

Now $m_R \ll M$ if $g \ll 1$

So if in UV theory $g \ll 1$ for some symmetry reason, then renormalized mass can be small!

- Higgs mass unnaturalness
- Light inflaton mass "

} →

Note Divergences appear only in the renormalization of mass & coupling constant & $M \to \infty$ limit, effects of heavy particle disappear

$$\mathcal{L}_{eff} = -\frac{1}{2}(\partial\phi)^2 - \frac{1}{2} m_R^2 \phi^2 - \frac{1}{4!}\lambda_R \phi^4$$

Bottom Up: Parameterizing ignorance

We may not know UV theory, so we cannot "derive" $\mathcal{L}_{eff}$ by integrating out fields
↓

① – Make symmetry assumption of UV physics

② – Write most general $\mathcal{L}$ consistent with symmetries

$$\mathcal{L}_{eff}[\phi] = \mathcal{L}_1(\phi) + \sum_i c_i \frac{O_i(\phi)}{\Lambda^{\delta_i - 4}}$$

* Point ① is not trivial as all low energy symmetry may NOT be realized at UV

* How do we know c_i & Λ?

↓

"Naturalness" may guide !!

Naturalness :

Top Down : → Wilson coefficient $\sim O(1)$
if $\Lambda \sim M$ (heavy mass)

But once you know UV theory, you in principle calculate c_i's given paramet of UV theory.

Bottom Up :

You have an effective theory (low energy)

$\downarrow$ Naturalness

"predict" / "infer" the plausible scale M of new physics

M is lightest D.O.F that is part of UV theory.

Say we have $\mathcal{L}_{eff} = \mathcal{L}_d [\phi] + \sum c_i \frac{O_i[\phi]}{\Lambda_i^{\delta_i - 4}}$

$\downarrow$

e.g SM.

What is Λ ? At what scale we expect new physics to show up?

~~Calculate loop correction~~ $\mathcal{L}_2(\phi)$, ~~with Λ being the cut-off scal~~

Calculate loop corrections to the parameters of $\mathcal{L}_2(\phi)$ by taking Λ as cut off.
$\downarrow$
Renormalizable

Parameters are bottom-up natural as long as their measured values are larger than the loop corrections

Increase Λ, and at some point Λ_{max} parameter values become unnatural.

New physics kicks in $\Lambda \lesssim \Lambda_{max}$ and

modify effective theory & explain why the measured parameter is so small

"Unnatural" alternative is several cancellation between loops, bare term etc?

Electroweak Scale:

$$\Delta m_H^2 \sim \frac{y_t^2}{(4\pi)^2} \Lambda^2$$

with $y_t \sim 1$, $m_H \sim 125$ GeV

we need from "naturalness" $\Lambda < 1.5$ TeV

New physics should come around 1 TeV

$\downarrow$

SUSY, Technicolor etc ✓

Dark Energy:

$$\langle T_{\mu\nu} \rangle = -\rho_{vac}\, g_{\mu\nu}$$

The vacuum of QFT with local Lorentz invariance

Quantum contributions to vacuum

$$\Delta\rho_{vac} \sim \Lambda^4$$

$$\rho_{vac}^{observed} \sim \left(10^{-3}\, eV\right)^4$$

Naturalness $\Lambda_{new\,phys} \lesssim 10^{-3}$ eV

nothing like observed! ✓

Symmetries:

Is m (light scalar mass) $<< H$
natural ?

— SUSY in flat space :

In absence of symmetry $\Delta m^2 \alpha \Lambda^2$
Unbroken SUSY $\Delta m^2 = 0$
(NOT renormalised)

But SUSY is broken during inflation
$\Delta m^2 \sim H^2$

— Global symmetries in flat space :

Consider global internal ~~sym~~ symmetry

Say $\mathcal{L}_I(\phi)$ has spontaneously broken global $U(1)$
↓
Angular dirⁿ (Goldstone) $\phi \to \phi +$ constant
shift symmetry

Forbids mass term.

Breaks shift symmetry mildly (approximate)

$$\Delta V = \frac{1}{2} m^2 \phi^2 \qquad m \ll \Lambda$$

$$\Delta m^2 \propto m^2$$

(loop correction to tree level.)

Also $m \to 0$, the exact symmetry is restored & ϕ becomes massless.

$\Downarrow$ Technically natural!

smallness of symmetry breaking controls the renormalization

Inflation in EFT:

$$S_{eff}[\phi] = \int d^4x \sqrt{-g} \left[\frac{M_{pl}^2}{2} R + \mathcal{L}_\phi(\phi) + \sum c_i \frac{\mathcal{O}_i}{\Lambda^{\delta_i - 4}} \right]$$

Max value of $\Lambda \sim M_{pl}$ ✓

Even during horizon exit $\omega = H$

$$\omega = \frac{c_s k}{a}$$

minimal cut off $\Lambda \gtrsim H$

All fields of mass $\lesssim H$ are part of EFT

Point Even irrelevant operators makes zeroth order dynamics altered!

UV sensitivity: –

1. η – problem – radiative corrections ✓

$$\Delta m^2 \sim \Lambda^2 \quad \text{with } \Lambda > H$$

$$\Delta \eta \sim \frac{\Lambda^2}{H^2} \gtrsim 1$$ (sustained SR inflation not

Introduce SUSY

BUT inflationary background break SUSY spontaneously

as $\rho_{inf} > 0$

For $\omega > H$ (ie $\lambda < H^{-1}$)

high frequency modes cancellation between bosons fermion works. but for $\omega \lesssim H$ do effect susy breaking.

$$\Delta m^2 \sim M_{susy} \sim H^2$$

$$\Delta \eta \sim 1$$

small field model $\quad \epsilon << \eta$

$$\eta \approx \frac{1}{2}(n_s - 1) \sim 0.02$$

Global symmetries :

Inf $\mathcal{L}_e(\phi)$ respects $\phi \to \phi + \text{const}$

i.e no relevant or marginal operators that violate above ~~sym~~ symmetry

Loop does not derive corrections to Λ rather $\propto$ small breaking parameter.

η - problem II :

Say at low energ $\phi \to \phi + c$ realised!
then mass radiatevely stable fo small breaking

BUT the symmetry may be broken at UV and can not be addresed by lookin $\mathcal{L}_{eff}$ at low energ

~~An Effect case~~. Say $V_e(\phi)$ respect approximat shift symmetry!

$$\mathcal{O}_6 = c\, V_e(\phi) \frac{\phi^2}{\Lambda^2} \quad \text{breaks shift symmetry}$$

Now if $\phi < \Lambda$

ΔV due to $\mathcal{O}_6 \ll V(\phi)$.

potential is not changed much!

$$\Delta\eta \approx 2c \left(\frac{M_{pl}}{\Lambda}\right)^2$$

ie $c \sim \mathcal{O}(1)$ & $\Lambda < M_{pl}$ loose η

Overall $V_e(\phi)$ cancells, i.e problem independent of [illegible]

In general $\mathcal{O}_\delta = c \langle V \rangle \left(\frac{\phi}{\Lambda}\right)^{\delta-4}$ — (A)

Vacuum energy during inflation!

$$\Delta\eta \approx c(\delta-4)(\delta-5)\left(\frac{M_{pl}}{\Lambda}\right)^2\left(\frac{\phi}{\Lambda}\right)^{\delta-6}$$

If $\Lambda = M_{pl}$ & $\phi < \Lambda$, $\delta \gg 6$ negligible.

Small field models upto dimension six ok!

if $\phi \ll \Lambda$, but with $\Lambda < M_{pl}$, $\delta \geq 6$ will also contribute.

→ Model dependent

- Why operator (A) or dim 6 operator does not exist, require understanding of UV.

- When a symmetry is assumed in EFT, one must demonstrate that symmetry survives non-renormalizable corrections in the context of UV completion!

η - problem in SUGRA:

Assume ϕ says single complex scalar in a chiral multiplet

$$\mathcal{L} = -K_{\phi\bar{\phi}}\, \partial_\mu \phi \partial^\mu \bar{\phi} - e^{K/M_{pl}^2} \left[K^{\phi\bar{\phi}} D_\phi W \overline{D_\phi W} - \frac{3}{M_{pl}^2} |W|^2 \right]$$

$$D_i W = \partial_i W + \frac{1}{M_{pl}^2} W \partial_i K$$

Expand around reference location $\phi = 0$

$$K(\phi) = K(\phi \neq 0) + K_{\phi\bar{\phi}}(0)\, \phi \bar{\phi}$$

$$\mathcal{L} = \partial_\mu \phi \partial^\mu \bar{\phi} - V(0) \left(1 + \frac{\phi\bar{\phi}}{M_{pl}^2} + \dots \right)$$

$$m_\phi^2 = \frac{V(0)}{M_{pl}^2} + \dots = 3H^2 + \dots$$

$$\eta = 1 + \dots$$

Solution: $\phi \rightarrow \phi + iCM$

$$K = K(\phi + \phi^*) = K(\phi_R)$$

$\mathrm{Im}(\phi) \equiv \varphi$ is free of e^K

$W = m\phi^2$ breaks shift symmetry

does not work! ✓

$$W = m\chi\Phi$$

assume 'm' breaks shift symmetry in more fundamental theory!

$$V(\phi, \chi) = \frac{1}{2}m^2\varphi^2 + m^2\chi^2$$

Natural Inflation: Pseudoscalar axion

Perturbative level axion enjoys a continuous shift symmetry

broken nonperturbatively to discrete symm leading to

$$V(\phi) = \frac{V_0}{2}\left(1 - \cos\left(\frac{\phi}{f}\right)\right)$$

- Coherent Phases & Super horizon Perturbations:

☆ Peaks of TT spectrum:

Inflation produces nearly scale invariant spectrum

$$\langle R_{\bar{k}} R_{\bar{k}'} \rangle = (2\pi)^3 \delta(\bar{k}+\bar{k}') P_R(k)$$

where $k^3 P_R(k) \propto k^{n_s-1}$ $n_s \simeq 1$

You can measure the spectrum & find $n_s \simeq 1$

↓

This part broadly agrees!

☆ striking part ⇒ All fourier modes have same phase

Consider a fourier mode $\lambda = \cancel{k}/\cancel{2\pi}$
$= 2\pi/k$

- inside horizon mode oscillates
- stretched
→ exit horizon

$$\lambda_{phy} = \lambda a > H^{-1}$$

Amplitude freezes

Re enter

Once inside horizon primordial fluctuations source density fluctuations

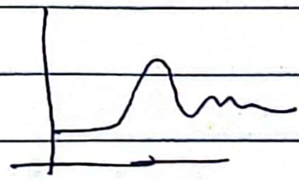
$$\ddot{\delta} - c_s^2 \nabla^2 \delta = F_S[\mathcal{R}]$$

Gravitational source term!

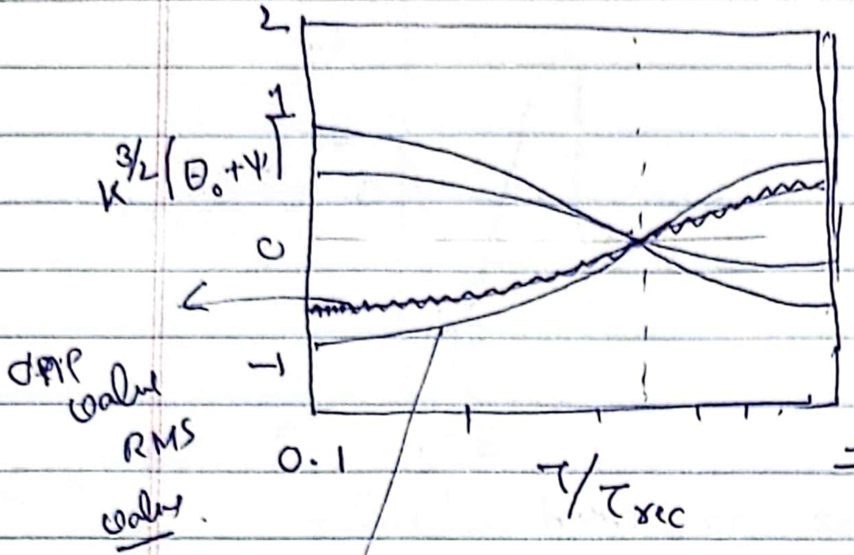
Fluctuations oscillates (Photon + Baryon) strongly coupled in the plasma → At recombination, we get the snapshot of fluctuations.

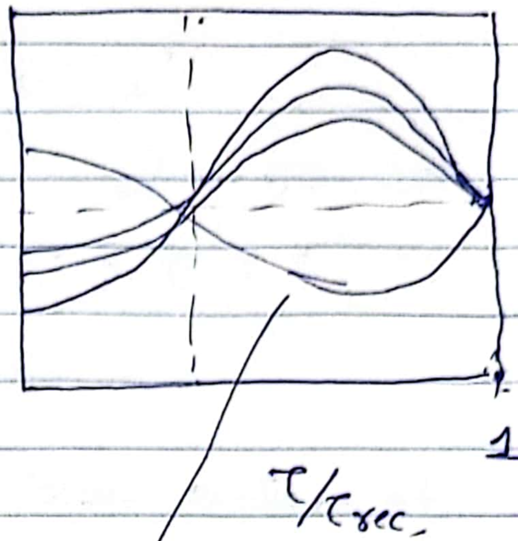
- Modes of a CERTAIN wavelength would be captured with different amplitude
- If ALL ~~~~ modes are in phase, they interfere coherently ✓
  → will be captured at max, min or zero.

This is what we see



Inflation produces that coherent initial phase!





1st CMB peak.

λ corresponding to first trough

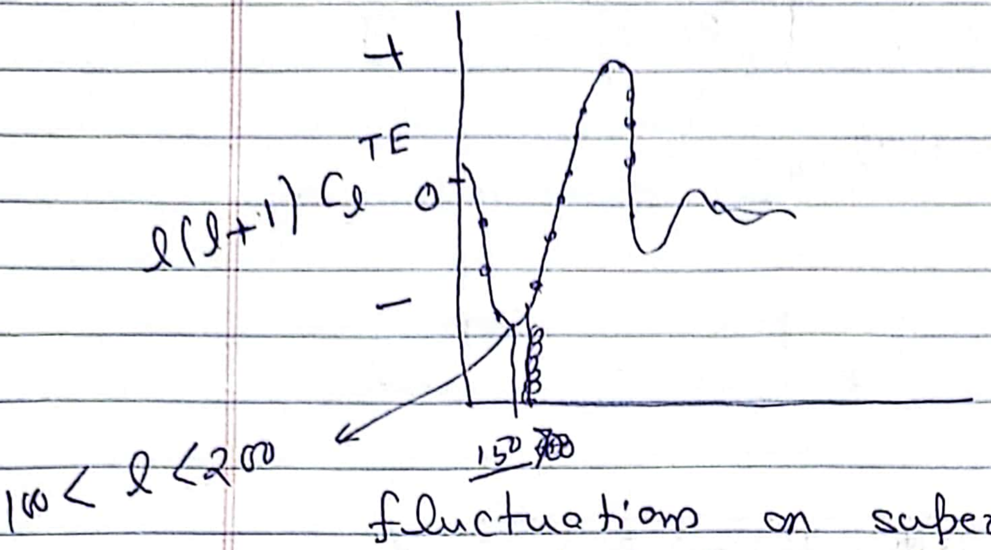
λ corresponding to first peak

With random phase → "White noise"

☆ $l < 100$ TE spectrum :

Note CMB peak etc for $l > 200$
these modes are within horizon during CMB

→ It is in principle possible to do some local physics after horizon reentry and make them in phase.
(Finely tuned) → 90's



$\theta > 1°$

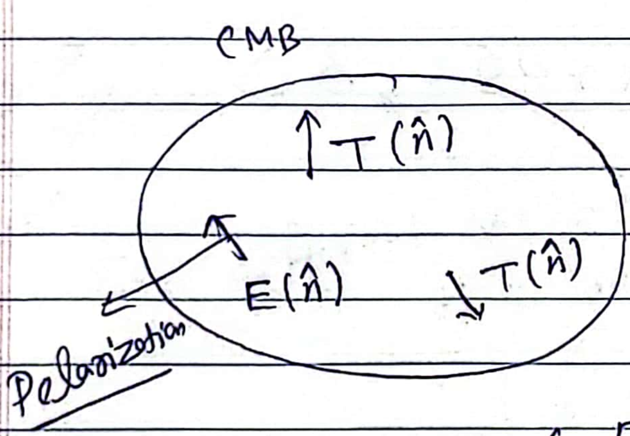
$l \sim 50 - 200$

$5° > \theta > 1°$ ✓

fluctuations on superhorizon scales at the epoch of decoupling ✓

−ve peak around $100 < l < 200$ ✓

Anti correlation is also a result of phase coherence → But now scales are super Hubble → no causal mechanism.



~~At large scale~~

T & polarization anti correlated

F.T. of E

$$\left\langle \Theta^{T}_{lm}\, \Theta^{E}_{lm} \right\rangle \sim C_l^{TE}$$

$\Delta T \sim \delta_{\gamma}$ (density of electron, baryon, photon (monopole) leading plasma)

Enrico Pajer : ICTP ~~[illegible]~~ Cosmology School (Online) 37

$E \sim (\partial v_\gamma)$ (dipole) ↙ leading order

↓ gradient of the velocity of fluid

$C_\ell^{TE} \sim \langle \delta_\gamma (\partial v_\gamma) \rangle \quad < 0$

↑ observations.

$\delta_\gamma = A \cos(\omega t + \phi)\, e^{ik\cdot x}$

Continuity eq^n $\partial \theta_\gamma = - \dot{\delta}_\gamma$

$= - A\omega \sin(\omega t + \phi)\, e^{ikx}$

A & ϕ stochastic & variables

Averaging over the whole possible was the universe was

$\langle \delta_\gamma \, \partial v_\gamma \rangle = \langle A A^* \rangle \int_0^{2\pi} \frac{d\phi}{2\pi} \sin(\,)\cos(\,) = 0$

Say different realisation of the universe phases are random ✓

i.e ϕ is random between 0 & 2π

zero ~~[illegible]~~ → NOT negative

~~[illegible]~~ ℓ_{H} ~~(Hubble at loss scattering) ~ 70~~ ✓

$\ell = 150$ is just twice as short → entered just