One bin test (on-off problem)

A counting expt. Observed events $\rightarrow n$, $n = n_s + n_b$

$$E(n) = \boxed{\mu}\,s + \boxed{b}$$

μ ↳ signal strength parameter (p.o.i)

$b \Rightarrow$ nuisance parameter

$s \Rightarrow$ comes from theory.

Two hypotheses : $\mu = 0$ background only

$\mu \neq 0$ signal + background.

null $\mu = 0$: $\quad P(n|b) = \dfrac{e^{-b}\, b^n}{n!}$

$\mu \neq 0$: $\quad P(n|\mu s + b) = \dfrac{e^{-(\mu s+b)}\,(\mu s+b)^n}{n!}$

L

b is constrained by a control measurement

(let's say, an MC estimate of b)

m counts in control sample.

$E(m) = \tau b$ (Assume, τ is known)

MC.

Joint prob of observing (n, m)

$$L(\mu, b) = \underbrace{\left[\frac{(\mu s+b)^n}{n!}\, e^{-(\mu s+b)}\right.}_{\text{real run}} \cdot \underbrace{\left.\frac{(\tau b)^m\, e^{-\tau b}}{m!}\right]}_{\text{auxiliary}}$$

2

When $\mu \neq 0$

$$\left.\frac{\partial L}{\partial \mu}, \frac{\partial L}{\partial b} = 0\right\} \Rightarrow \quad \hat{\mu} = \frac{\overbrace{n - m/\tau}^{\mu s}}{s}, \qquad \hat{b} = \frac{m}{\tau}$$

$E(m) = \tau b$

↳ post-fit estimates

When μ is not allowed to vary, but kept at some specified value (profiling)

$$\hat{\hat{b}} = \frac{(n+m) - (1+\tau)\mu s}{2(1+\tau)} + \left[\frac{(n+m-(1+\tau)\mu s)^2 + 4(1+\tau) m \mu s}{4(1+\tau)^2}\right]^{1/2}$$

in particular, for $\mu = 0$

$$\hat{\hat{b}} = \frac{(n+m)}{2(1+\tau)} + \frac{(n+m)}{2(1+\tau)} = \frac{n+m}{(1+\tau)}$$

: weighted average of the two counts.

We construct a test statistic

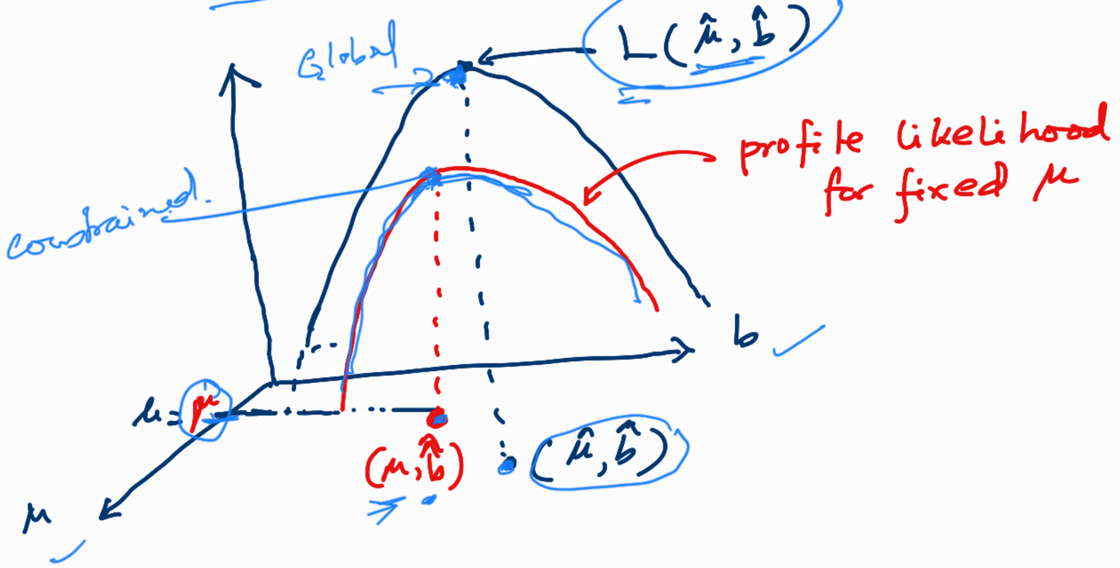
* "statistic means a function of data

$$\boxed{\lambda(\mu)} = \frac{L(\mu, \hat{\hat{b}})}{L(\hat{\mu}, \hat{b})} \begin{array}{l} \leftarrow \text{conditional max} \\ \leftarrow \text{unconditional max} \end{array}$$

$\lambda(0)$

profile L.R.

Obviously, $0 \leq \lambda(\mu) \leq 1$

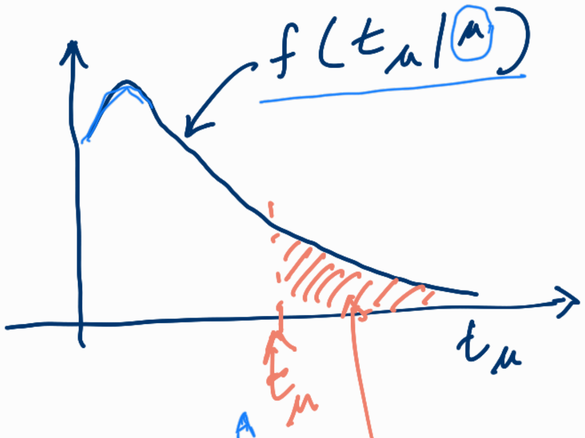


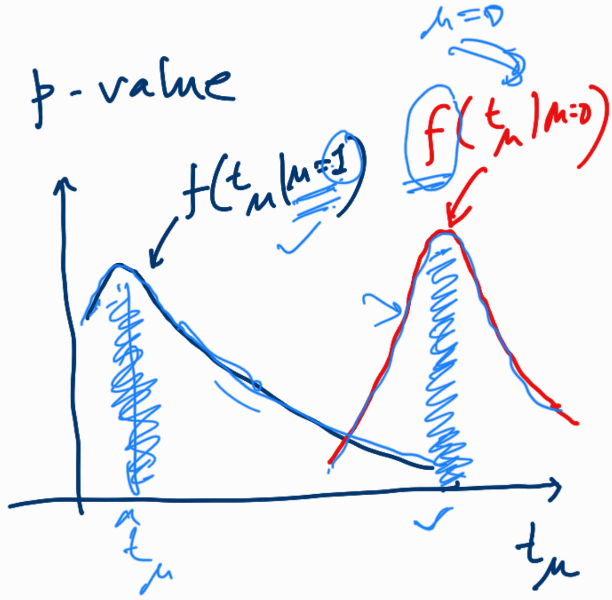
$$\boxed{t_\mu = -2 \ln \lambda(\mu)}$$

$\to 0$

statistic, random.

Distribution of t_μ and p-value





$-0'5$

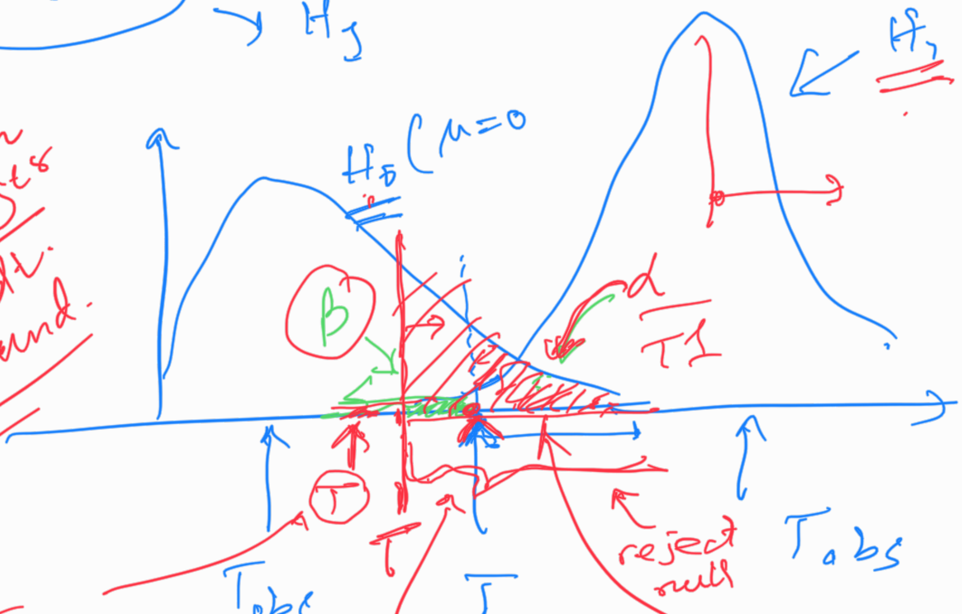
$$p_\mu = \int_{\hat{t}_\mu}^{\infty} f(t_\mu|\mu)\,dt_\mu$$

$T(\vec{x}) \to H_0$
$\quad\;\; \to H_1$

Casella-Berger

Brandt.

Sigmund.



α power

$(1-\beta)$
largest

T2.

p-value $(T_{obs}|H_0)$

p-value $(T_{obs}|H_1)$

critical region.

Neyman-Pearson

UMP