Talk

- One of the most important questions in SFT is understanding how the fluctuations of one background can rearrange themselves to create other background.
- In open bosonic SFT, the question is whether the field equations of a reference D-brane $BCFT_0$ has a classical solution representing some other D-brane $BCFT_*$.
- About the simplest kind of solution we can imagine in this respect takes the form

$$\Psi = \underbrace{\Psi_{tv}}_{BCFT_0} - \underbrace{\Sigma}_{BCFT_{0*}} \underbrace{\Psi_{tv}}_{BCFT_*} \underbrace{\bar{\Sigma}}_{\in BCFT_{*0}} \in BCFT_0$$

where $BCFT_{0*}$ is the state space of a stretched string between $BCFT_0$ and $BCFT_*$ and $BCFT_{*0}$ is the reverse.
- The left/right multiplication $\Sigma(\cdots)\bar{\Sigma}$ is effectively a map from the state space $BCFT_*$ into the state space $BCFT_0$ of our reference D-brane
- In the first term, Ψ_{tv} is a tachyon vacuum solution in $BCFT_0$, and represents the annihilation of the reference D-brane.
- In the second term, $-\Psi_{tv}$ represents the perturbative vacuum in $BCFT_*$ relative to the tachyon vacuum Ψ_{tv} in $BCFT_*$. It represents the creation of the D-brane $BCFT_*$ out of the tachyon vacuum. The appearance of $\Sigma(\cdots)\bar{\Sigma}$ is necessary to express this configuration in terms of the d.o.f. of $BCFT_0$.
- In this expression, string fields in different state spaces appear. We ~~will~~ note that the position of the string field in relation to $\Sigma, \bar{\Sigma}$ implies the state space it occupies. In expressions where $\Sigma, \bar{\Sigma}$ are absent, this can create ambiguity but I will try to be clear.
- The solution Ψ satisfies the EOM if ~~we have~~ $\Psi_{tv}, \Sigma, \bar{\Sigma}$ satisfy the following properties:

$$Q\Psi_{tv} + \Psi_{tv}^2 = 0$$

$$Q_{\Psi_{tv}} \Sigma = Q_{\Psi_{tv}} \bar{\Sigma} = 0$$

$$\bar{\Sigma}\Sigma = 1$$

- We may expand the action around the solution

$$\Psi + \varphi_{\circledcirc}$$

where $\varphi_{\circledcirc} \in BCFT_{\circledcirc}$ represents a fluctuation around the background set by the solution Ψ. However, the natural fluctuations of this background are represented by string field $\varphi_{*} \in BCFT_{*}$. We may guess that $\varphi_{\circledcirc}$ and φ_{*} are related by

$$\varphi_{\circledcirc} = \Sigma \varphi_{*} \bar{\Sigma}$$

and we can show that

$$S_{\circledcirc}[\Psi + \varphi_{\circledcirc}] = S_{\circledcirc}[\Psi] + S_{*}[\varphi_{*}]$$

where $S_{\circledcirc}$ is the action formulated in $BCFT_{\circledcirc}$ and S_{*} is the action formulated in $BCFT_{*}$. This would give a simple proof of nonperturbative (global) background independence in open bosonic SFT.

- A few years ago, C. Maccaferri and I ~~realized~~ were able to realize this structure provided that $BCFT_{\circledcirc}$ and $BCFT_{*}$ can be related by bcc operators $\sigma, \bar{\sigma}$ with OPE:

$$\bar{\sigma}(x)\sigma(0) = 1 + \dots$$

This condition is not very convenient, since typical bcc operators have nonvanishing conformal weight, and their OPEs have singularities. For time independent backgrounds, however, this kind of OPE can be achieved if we assume that the $BCFT_{*}$ background carries a constant, timelike gauge ~~field~~ potential of a certain critical value. Since the constant gauge potential is not observable, any value defines the same $BCFT_{*}$ and we are free to choose it at our convenience.

This resolution has a number of serious drawbacks, however.

1. In $BCFT_{\circledcirc}$ and $BCFT_{*}$, we need a ~~free boson~~ timelike free boson with Neumann boundary conditions which allows us to turn on a gauge field. This rules out time dependent backgrounds or instanton backgrounds. Also, the timelike gauge potential breaks symmetries, such as Lorentz invariance, which we may prefer to have manifest in the solution.

2. bcc operators of this form typically lead to associativity anomalies in the string field algebra. For example, if $\sigma, \bar{\sigma}$ are normalized so that $\bar{\sigma}\sigma = 1$, multiplication in the opposite order gives

$$\sigma\bar{\sigma} = \text{constant} \neq 1$$

where the constant is given by the ratio of the norms of the $SL(2,\mathbb{R})$ vacuums in the two theories. While this does not appear to pose a serious problem for the solution itself, it appears to prevent a useful generalization of the idea to superstring field theory. Another issue is that degrees of freedom do not map in a very nice way between different backgrounds. ~~For example, consider three BCFTs $BCFT_0$, $BCFT_1$, and $BCFT_2$.~~ ~~We may construct solutions relating these backgrounds with fields $\Sigma_{02}, \bar{\Sigma}_{02}$; $\Sigma_{01}, \bar{\Sigma}_{01}$; and $\Sigma_{12}, \bar{\Sigma}_{12}$~~

For example, consider solutions relating 3 backgrounds $BCFT_1$, $BCFT_2$, $BCFT_3$; we have $\Sigma_{23}, \bar{\Sigma}_{23}$ relating $BCFT_3$ to $BCFT_2$, and $\Sigma_{12}, \bar{\Sigma}_{12}$ relating $BCFT_2$ to $BCFT_1$. Translating degrees of freedom from $BCFT_3$ to $BCFT_2$, and $BCFT_2$ to $BCFT_1$ involves states of the form

$$\Sigma_{12}\,\Sigma_{23}\,\varphi\,\bar{\Sigma}_{23}\,\bar{\Sigma}_{12}$$

but products of Σ's are generically undefined; even if we have achieved $\bar{\sigma}_{12}\sigma_{12} = \bar{\sigma}_{23}\sigma_{23} = 1$, we do not know anything about the OPEs of σ_{12} and σ_{23}, which could be vanishing or divergent.

- ~~In summary, while it is possible to realize a solution in this way, this approach is limited and singular.~~

- In this lecture we consider a different realization of the solution which resolves these difficulties.

- As a first step, we need to consider more carefully how the identities $Q_{\Psi_{tv}}\Sigma = Q_{\Psi_{tv}}\bar{\Sigma} = 0$ and $\bar{\Sigma}\Sigma = 1$ can be realized.

- First, let us assume that $\Sigma, \bar{\Sigma}$ leave the homotopy operator at the tachyon vacuum invariant:

$$\bar{\Sigma} A \Sigma = A$$

Since $\bar{\Sigma}, \Sigma$ are killed by $Q_{\Psi_{tv}}$, this implies $\bar{\Sigma}\Sigma = 1$. In principle, we could have a $Q_{\Psi_{tv}}$ exact term on the right hand side; however, for the type of states considered in analytic calculations, states at ghost number -2 do not appear, so this identity is not too strong an assumption.

- In addition, we assume that the homotopy operator is nilpotent:

$$A^2 = 0$$

This is true for analytic tachyon vacuum solutions in the KBc subalgebra, again because states at ghost # -2 do not appear.

- Since $Q_{\Psi_{tv}}$ has no cohomology, without loss of generality we can assume

$$\Sigma = Q_{\Psi_{tv}}(A\underset{\uparrow_{BCFT_{0*}}}{\Theta}) \qquad \bar{\Sigma} = Q_{\Psi_{tv}}(\underset{\uparrow_{BCFT_{*0}}}{\bar{\Theta}}A)$$

~~We may of cours~~ This ansatz solves the condition of $Q_{\Psi_{tv}}$ invariance of $\Sigma, \bar{\Sigma}$. We may of course choose $\Theta, \bar{\Theta}$ to be equal to Σ and $\bar{\Sigma}$, but this is not necessary. In particular, $\Theta, \bar{\Theta}$ do not need to be killed by $Q_{\Psi_{tv}}$.

- One can then show that the condition $\bar{\Sigma} A \Sigma = A$ implies a similar condition on $\Theta, \bar{\Theta}$:

$$\bar{\Theta} A \Theta = A$$

Since $\Theta, \bar{\Theta}$ are not $Q_{\Psi_{tv}}$ invariant, this does not imply $\bar{\Theta}\Theta = 1$.

- ~~Next we make an assumpti~~ To solve this equation, let us make a choice of tachyon vacuum for realizing the solution. We assume the simple tachyon vacuum

$$\Psi_{tv} = c(1+K)Bc\frac{1}{1+K} \qquad A = \frac{B}{1+K}$$

our interest in this case is partly motivated by the fact that the simple solution has a controlled generalization to superstring field theory; at this time, Schnabl's solution does not.

- So we must solve the equation

$$\bar{\Theta}\frac{B}{1+K}\Theta = \frac{B}{1+K}$$

- To solve this we introduce additionally $\theta \in BCFT_{0*}$ and $\bar{\theta} \in BCFT_{*0}$ satisfying

$$\bar{\theta} B \theta = B$$

- We may then find a solution for $\Theta, \bar{\Theta}$ in the form

$$\bar{\Theta} = \bar{\theta} \qquad \Theta = (1+K)\theta\frac{1}{1+K}$$

- Therefore, a useful realization of the solution relies on the construtuion of $\theta, \bar\theta$ satisfying $\bar\theta B \theta = B$.
- The solution provided earlier by Carlo and myself ammounts to

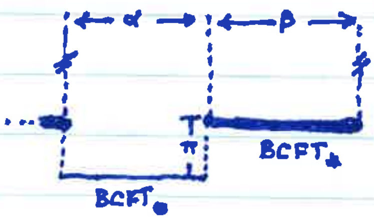
$$\theta = \sigma \qquad \bar\theta = \bar\sigma$$

where the string fields $\sigma, \bar\sigma$ are boundary insertions of bcc operators satisfying

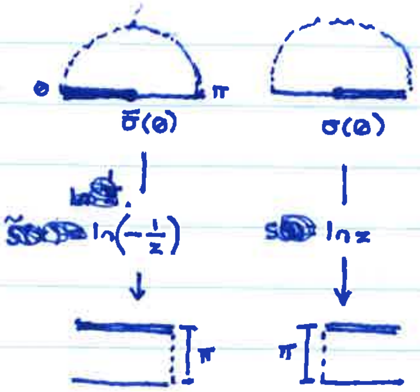
$$\lim_{x \to 0} \bar\sigma(x)\sigma(0) = 1$$

on an infinitessimally thin strip.

- Note that $[B, \sigma] = 0$ since σ is a matter operator. Therefor $\bar\theta B \theta = B$ follows from the opE of $\bar\sigma$ and σ.
- Within the subalgebra of wedge states with insertions it seems impossible to find ~~another kind of solution~~ a better solution; wedge widths add under star multiplication, and since B has width 0, this implies $\theta, \bar\theta$ have width 0. Since $\theta, \bar\theta$ must change the boundary condition, the OPE of the bcc operators must be regular.
- So the only resolution is to leave the algebra of wedge states. We have investigated the following idea.
- Consider wedge states $\Omega^\alpha_\bullet$ and Ω^β_* containing boundary conditions of $BCFT_\bullet$ and $BCFT_*$. Usually we multiply them by gluing the strips side by side; now we propose to glue them so that the ~~strip of Ω^β_* is shi~~ open string boundary of Ω^β_* is shifted vertically relative to $\Omega^\alpha_\bullet$. We may choose the distance to be π. Taking the trace defines a correlation function on a kind of cylinder, but the boundary of the cylinder is broken into two vertically displaced components. Evaluating the correlator requires specifying boundary conditions

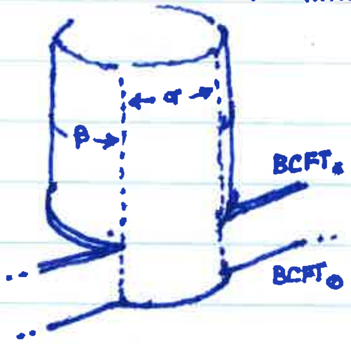


for the path integral on the ~~left/right edges~~ portion of the left/right edges of $\Omega^\alpha_\bullet$ left untouched by gluing to Ω^β_*. These boundary conditions effectively define an open string state in $BCFT_{*\bullet}$ on the left edge, and a state $BCFT_{\bullet *}$ on the right edge. Such states may be specified by inserting bcc operators $\bar\sigma(0)$ and $\sigma(0)$ at the origin of respective half-disks. We may normalize the operators so that their 2-point function on the UHP is unity:



$$\langle I\circ\bar\sigma(0)\ \sigma(0)\rangle^{\text{matter}}_{\text{UHP}} = 1$$

To complete the picture we map the unit half disks to the string propagator frame, where they appear as infinitely long rectangular strips of hight π. These strips can then be glued onto the free portions on the left and right vertical edges of Ω^{α}_{θ}; the result is a funny-looking surface composed of a cylinder attached to two infinitely long strips. An obvious question is whether one can compute correlation functions on such a surface.



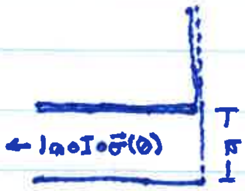
~~In fact it is possible with a Schwarz-Christoffel mapp to map to the UHP using a Schwarz-Christoffel map. The mapping can even~~

In fact it is possible to derive this surface explicitly by conformal transformation of the UHP using a Schwarz-Christoffel map; the transformation can actually be given in closed form using inverse trigonometric functions. Unfortunately, as is often the case with Schwarz-Christoffel maps, the inverse transformation from the surface to the UHP cannot be expressed in terms of elementary functions. This is an annoyance, but calculations are still possible.

- This picture leads to a new definition of θ and $\bar\theta$ in terms of what we call "flag" and "antiflag" states:

$\theta = |\sigma] =$

$\bar\theta = [\bar\sigma| =$



- Like the identity string field, a flag state is characterized by a delta function overlap between the left and right halves of the string. However for example in $|\sigma]$ a point a distance y_L above the boundary on the left half of the string is identified with a point y_R above the boundary on the right half of the string through

$$y_L = \pi + y_R$$

This leaves the interval $0 < y_L < \pi$ for us to attach a state in the string propagator frame.

- The crucial property which follows from this definition is

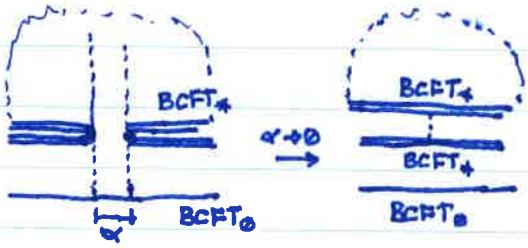
$$\bar{\theta}\theta = [\bar{\sigma}|\sigma] = 1$$

In particular, this fact follows regardless of the OPEs and conformal transformation properties of $\sigma, \bar{\sigma}$.

- The mechanism can be understood as follows. We seperate $[\bar{\sigma}|\ |\sigma]$ by a wedge state with small width:

$$[\bar{\sigma}|\Omega^\alpha|\sigma]$$

In the limit $\alpha \to 0$ we get a surface with degeneration. Effectively,



the propagator strip detaches from the rest of the surface and defines an independent correlator which appears multiplied with the correlator on the remainder of the surface. The correlator on the strip is the 2-point function of $\sigma, \bar{\sigma}$, which we have normalized to unity. Therefore

$$\lim_{\alpha \to 0} [\bar{\sigma}|\Omega^\alpha|\sigma] = 1$$

- However, $[\bar{\sigma}|\sigma] = 1$ is not the property we need to have a solution. We must have

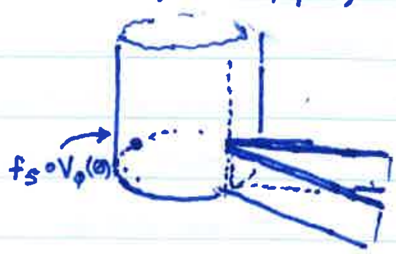
$$[\bar{\sigma}|B|\sigma] = B$$

This is nontrivial since B does not commute with the flag states. Nevertheless, we have confirmed that this property holds.

- A second important point is the nature of the state defined by multiplying flags in the opposite order

$$\theta\bar{\theta} = |\sigma][\bar{\sigma}|$$

This is most easily understood by contracting with a test state in the sliver frame. The result is a correlation function on a cylinder with a cut



of hieght π glued to two propagator strips. Importantly, this state is not proportional to the identity string field, and ~~does~~ therefore does not lead to an associativity anomaly. The state, however multiplies like a projector

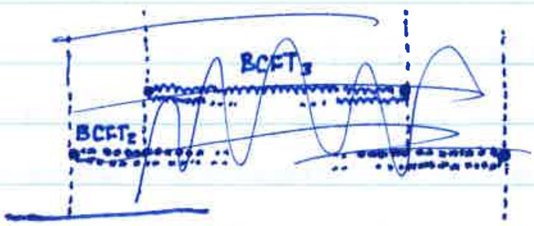
$$(|\sigma][\bar{\sigma}|)(|\sigma][\bar{\sigma}|) = |\sigma][\bar{\sigma}|$$

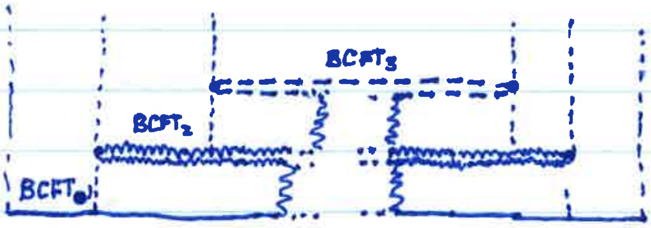
- ~~The flag states states therefore multiply like a partial isometry U, V non-unit~~
- Therefore a flag state can be understood as a non-unitary isometry U

$$U^*U = 1 \qquad UU^* = \text{projector} \neq 1$$

The relevance ~~on~~ of non-unitary isometries to SFT was conjectured long ago by M. Schnabl.

- Another benefit of the flag states is that mapping ~~between~~ degrees of freedom between backgrounds becomes a more well-defined operation. For example, mapping d.o.f. from $BCFT_3$ to $BCFT_2$ and then to $BCFT_1$ will involve multi-flag states.





It is clear that in this situation there is no divergence from collisions of bcc operators

- Therefore, flag states provide a natural ~~resolution to~~ solution for the difficulties encountered ~~in the previous Paulis~~ with bcc ops with regular OPE.
- As an application of this framework, we can show that it is possible to construct multiple D-brane solutions within the universal sector, something which could not be achieved in the previous approach due to the necessity of the timelike gauge potential.

- Given $BCFT_0$, we can create two copies of $BCFT_0$ using row and column vectors

$$\theta = \left(\frac{1}{\sqrt{13}} |L^m_{-2}] \quad \frac{1}{\sqrt{130}} |L^m_{-4}] \right) \qquad \bar{\theta} = \begin{pmatrix} \frac{1}{\sqrt{13}} [L^m_{-2}| \\ \frac{1}{\sqrt{130}} [L^m_{-4}| \end{pmatrix}$$

where in this context L^m_{-2} and L^m_{-4} refer to the "bcc operators" obtained by acting matter virasoros L^m_{-2}, L^m_{-4} on the $SL(2,\mathbb{R})$ vacuum. Of course these are just Fock states in $BCFT_0$ and do not change the boundary condition

- One may easily show that

$$\bar{\theta}\theta = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

which is the identity matrix for the Chan-Paton factors of strings living on two copies of $BCFT_0$.

- There are some subtleties with the solution, however due to the singular geometry at the "slit" where the flag joins the rest of the surface.

- For example, one can show that, if c is a boundary insertion of the c ghost,

$$c[\bar{\sigma}| = 0$$

which leads to an associativity anomaly

$$0 = (c[\bar{\sigma}|)|\sigma] \neq c([\bar{\sigma}|\sigma]) = c$$

For some choices of tachyon vacuum such products do not appear in the solution. Unfortunately, for the simple solution the anomaly can lead to ambiguities. The resolution we have found is to change the usual definition of c so that it represents ~~as~~ c insertion away from the boundary. This, unfortunately, creates problems with the reality condition

- String fields such as

$$(Q[\bar{\sigma}|)|\sigma]$$

are divergent, since the BRST operator creates c-ghosts on the bcc operator which do not like being seperated from the remainder of the surface ~~open~~ in the degeneration limit.

- Such divergent string fields appear in intermediate steps when verifying formal properties such as $\bar{\Sigma}\Sigma = 1$.

- However we have checked that divergences cancel, and lead to no anomaly, after regularization of the product as $\bar{\Sigma}\Omega^{\alpha}\Sigma$ for small α.
- It is hard to tell whether such singularities indicate that the solution itself may in some sense be poorly behaved, or whether they are an artifact of the way we perform analytic calculations.
- For this reason, we are also interested in investigating other solutions.
- One important recent revelation, which we found somewhat shocking, is that the difficult identity $\bar{\Sigma}\Sigma = 1$ is not needed to have a solution at all; the only required property is that $\Sigma, \bar{\Sigma}$ are killed by $Q_{\Psi_{tv}}$, which is easily solved by expressing $\Sigma, \bar{\Sigma}$ in Q_{TV} exact form.
- It follows from this that it is always possible to find a string field $\alpha \in BCFT_*$ with the property that

$$Q_{\Psi_{tv}}\alpha = 1 - \bar{\Sigma}\Sigma$$

So far, we have been assuming $\alpha = 0$.

- We may then generalize the solution as

$$\Psi = \Psi_{tv} - \Sigma\,\Psi_{tv}\frac{1}{1-\alpha\Psi_{tv}}\bar{\Sigma}$$

- The state $\frac{1}{1-\alpha\Psi_{tv}}$ may be defined by a geometric series, and in general the series will generate a pair of bcc operators for each power of α.
- This makes it more difficult to compute, for example, the Fock space expansion of the solution. It also raises important questions about convergence of the series.
- However it turns out that from the perspective of $BCFT_*$, the second term can be viewed as a gauge transformation of the solution $-\Psi_{tv} \in BCFT_*$ around the tachyon vacuum.
- Therefore, the question of whether the geometric series converges is closely tied to the question of whether this gauge transformation is singular.
- We have developed some tools for addressing this kind of question using the concepts of boundary condition changing projectors and phantom terms.

- From this perspective it seems that there is no obstruction to constructing solutions for any background that are as regular as needed, for example in the Fock space expansion.
- It will be interesting to explore these questions further.